Previous IPE

SOLVED PAPERS

MARCH -2025(TS)

PREVIOUS PAPERS

IPE: MARCH-2025(TS)

Time : 3 Hours

MATHS-2A

Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: $10 \times 2 = 20$

1. If $(a + ib)^2 = x + iy$ find $(x^2 + y^2)$.

2. If Arg $\overline{z}_1$, Arg z_2 are $\pi/5, \pi/3$ then find Arg(z_1) + Arg (z_2)

3. If A,B,C are angles of a triangle, x = cisA, y = cisB, Z = cisC, then find the value of xyz.

4. For what value of x, the expression $x^2-5x-14$ is positive.

5. If α, β, γ are the roots of $4x^3 - 6x^2 + 7x + 3 = 0$, then find the value of $\alpha\beta + \beta\gamma + \gamma\alpha$

6. If ${}^{(n+1)}P_5 : {}^{n}P_5 = 3 : 2$ then find n.

7. Find the number of permutations that can be made by using all the letters of the word INTERMEDIATE.

8. Find the set E of x for which the binomial expansion $(3-4x)^{3/4}$ is valid.

9. Find the mean deviation about median for the data 6,7,10,12,13,4,12,16

10. The probability that a person chosen at random is left-handed (in hand writing) is 0.1. What is the probability that in a group of 10 people, there is one who is left-handed?

SECTION-B

II. Answer any FIVE of the following SAQs: $5 \times 4 = 20$

11. Show that the four points in the Argand plane represented by the complex numbers $2 + i$, $4 + 3i$, $2 + 5i$, $3i$ are the vertices of a square.

12. Determine the range of the expression $\dfrac{x + 2}{2x^2 + 3x + 6}$.

13. If the letters of the word 'PRISON' are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "PRISON"

14. Find the number of ways of forming a committee of 5 members out of 6 Indians and 5 Americans so that always the Indians will be in majority in the committee.

15. Resolve $\dfrac{x^3}{(x-a)(x-b)(x-c)}$ into partial fractions

16. A,B,C are three horses in a race. The probability of A to win the race is twice that of B, and probability of B is twice that of C. What are the probabilities of A,B, and C to win the race?

17. A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.

SECTION-C

III. Answer any FIVE of the following LAQs: **$5 \times 7 = 35$**

18. If n is a positive integer then show that $(1+i)^{2n} + (1-i)^{2n} = 2^{n+1}\cos(n\pi/2)$

19. Find the polynomial equation whose roots are the translates of the roots of the equation $x^5 - 4x^4 + 3x^2 - 4x + 6 = 0$ by -3.

20. If the coefficients of 4 consecutive terms in the expansion of $(1+x)^n$ are a_1, a_2, a_3, a_4 respectively, then show that $\frac{a_1}{a_1+a_2} + \frac{a_3}{a_3+a_4} = \frac{2a_2}{a_2+a_3}$

21. If $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + ...$, then prove that $9x^2 + 24x = 11$

22. Find the mean deviation about mean for the following data:

x_i	2	5	7	8	10	35
f_i	6	8	10	6	8	2

23. Three boxes numbered I, II, III contain the balls as follows :
One box is randomly selected and a ball is drawn from it.
If the ball is red, then find the probability that it is from BOX - II

	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

24.

$X = x_i$	−3	−2	−1	0	1	2	3
$P(X = x_i)$	1/9	1/9	1/9	1/3	1/9	1/9	1/9

is the probability distribution of a random variable X. Find the variance of X.

IPE TS MARCH-2025
SOLUTIONS

SECTION-A

1. If $(a + ib)^2 = x + iy$ find $(x^2 + y^2)$.

Sol: Given $(a + ib)^2 = x+iy \Rightarrow (a+ib)(a+ib) = x+iy$. Taking mod on both sides $|a+ib|\,|a+ib| = |x+iy|$

$\Rightarrow \sqrt{a^2+b^2}.\sqrt{a^2+b^2} = \sqrt{x^2+y^2} \Rightarrow a^2+b^2 = \sqrt{x^2+y^2}$.

Squaring on both sides, we get $(a^2+b^2)^2 = x^2+y^2$ $\qquad \therefore\ x^2+y^2 = (a^2+b^2)^2$

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2. If $\text{Arg}\,\bar{z}_1, \text{Arg}\,z_2$ are $\pi/5, \pi/3$ then find $\text{Arg}(z_1) + \text{Arg}(z_2)$

Sol: Given that $\text{Arg}\,\bar{z}_1 = \frac{\pi}{5} \Rightarrow \text{Arg}\,z_1 = -\frac{\pi}{5}$. Also $\text{Arg}\,z_2 = \frac{\pi}{3}$

$\therefore\ \text{Arg}(z_1)+\text{Arg}(z_2) = -\frac{\pi}{5} + \frac{\pi}{3} = \frac{\pi}{3} - \frac{\pi}{5} = \frac{5\pi - 3\pi}{15} = \frac{2\pi}{15}$

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3. If A, B, C are angles of a triangle, x = cisA, y = cisB, Z = cisC, then find the value of xyz.

Sol: In a triangle we have, $A + B + C = 180^\circ$.

$\therefore\ xyz = \text{cis}A.\text{cis}B.\text{cis}C = \text{cis}(A + B + C)$

$= \text{cis}180^\circ = \cos180^\circ + i\sin180^\circ = -1 + i(0) = -1$

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4. For what value of x, the expression $x^2 - 5x - 14$ is positive.

Sol: Comparing $x^2 - 5x - 14$ with $ax^2 + bx + c = 0$ we get $a = 1, b = -5, c = -14$.

Now, $\Delta = b^2 - 4ac = (-5)^2 - 4(1)(-14) = 25 + 56 = 81 > 0$

Here, Δ is positive. $\qquad \therefore$ The roots are real

Now, $x^2 - 5x - 14 = 0 \Rightarrow x^2 - 7x + 2x - 14 = 0$

$\Rightarrow x(x-7) + 2(x-7) = 0 \Rightarrow (x + 2)(x - 7) = 0 \Rightarrow x = -2$ or 7

Also, $a = 1 > 0$

$\therefore\ 1, x^2 - 5x - 14$ have the same sign for $x < \alpha$ or $x > \beta$

$\Rightarrow x^2 - 5x - 14$ is positive for $x < -2$ or $x > 7$

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5. If α, β, γ are the roots of $4x^3 - 6x^2 + 7x + 3 = 0$ then find $\alpha\beta + \beta\gamma + \gamma\alpha$

Sol: Comparing $4x^3 - 6x^2 + 7x + 3 = 0$ with $a_0x^3 + a_1x^2 + a_2x + a_3 = 0$

We get, $a_0 = 4, a_1 = -6, a_2 = 7, a_3 = 3$ $\qquad \therefore \alpha\beta + \beta\gamma + \gamma\alpha = S_2 = a_2/a_0 = 7/4$

6. If ${}^{(n+1)}P_5 : {}^{n}P_5 = 3:2$ then find n.

Sol : Given that ${}^{(n+1)}P_5 : {}^{n}P_5 = 3:2 \Rightarrow \frac{{}^{(n+1)}P_5}{{}^{n}P_5} = \frac{3}{2}$

$$\Rightarrow \frac{(n+1)\,n(n-1)(n-2)(n-3)}{n(n-1)(n-2)(n-3)\,(n-4)} = \frac{3}{2}$$

$$\Rightarrow 2(n+1) = 3(n-4)$$

$$\Rightarrow 2n+2 = 3n-12 \Rightarrow 3n-2n = 2+12 \Rightarrow n = 14$$

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7. Find the number of permutations that can be made by using all the letters of the word INTERMEDIATE

Sol: The given word INTERMEDIATE contains 12 letters in which 3 'E's are alike,2 'I's are alike, 2'T's are alike and rest are different.

$\therefore$ The required number of arrangements $= \frac{n!}{p!q!r!} = \frac{12!}{3!2!2!}$

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8. Find the set E of x for which the binomial expansion $(3-4x)^{3/4}$ is valid.

Sol: $G.E = (3-4x)^{3/4} = 3^{3/4}\left(1-\frac{4x}{3}\right)^{3/4}$. This is valid when $\left|\frac{4x}{3}\right| < 1$,

$$\Rightarrow |x| < \frac{3}{4} \Rightarrow x \in \left(-\frac{3}{4}, \frac{3}{4}\right) \qquad \therefore E = \left(-\frac{3}{4}, \frac{3}{4}\right)$$

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9. Find the mean deviation about median for the data 6,7,10,12,13,4,12,16

Sol: The ascending order of the observation in the given data is 4,6,7,10,12,12,13,16

The number of observations = 8 (even)

$\therefore$ Median of the given data is $M = \left(\frac{4^{th}\text{ item} + 5^{th}\text{ item}}{2}\right) = \frac{10+12}{2} = 11$

The deviations of observations from the median are

11–4=7; 11–6=5; 11–7=4; 11–10=1; 11–12=–1;11–12=–1; 11–13=–2; 11–16=–5

Hence, the absolute values of the deviations are 7,5,4,1,1,1,2,5

$\therefore$ The mean deviation from the median is

$$M.D = \frac{\Sigma|x_i - M|}{8} = \frac{7+5+4+1+1+1+2+5}{8} = \frac{26}{8} = 3.25$$

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10. The probability that a person chosen at random is left-handed (in hand writing) is 0.1.What is the probability that in a group of 10 people, there is one who is left-handed?

Sol: Probability of getting a left-handed person = Probability of success p = 0.1 = 1/10

$\Rightarrow q = 1 - \frac{1}{10} = \frac{9}{10}$. Here, n = 10

$$\therefore P(X=r) = {}^{n}C_r q^{n-r}.p^r$$

$$\Rightarrow P(X=1) = {}^{10}C_1.\left(\frac{9}{10}\right)^{10-1}.\left(\frac{1}{10}\right)^1 = 10.\left(\frac{9}{10}\right)^9.\left(\frac{1}{10}\right) = \left(\frac{9}{10}\right)^9$$

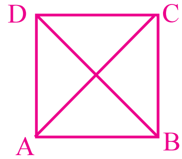
SECTION-B

11. Show that the four points in the Argand plane represented by the complex numbers 2 + i, 4 + 3i, 2 + 5i, 3i are the vertices of a square.

Sol: Let the given points are taken as A(2, 1), B(4, 3), C(2, 5), D(0, 3)

$$AB=\sqrt{(2-4)^2+(1-3)^2}=\sqrt{8}\ ;\quad BC=\sqrt{(4-2)^2+(3-5)^2}=\sqrt{8}$$

$$CD=\sqrt{(2-0)^2+(5-3)^2}=\sqrt{8}\ ;\quad DA=\sqrt{(2-0)^2+(1-3)^2}=\sqrt{8}$$



Hence, the four sides AB, BC,CD, DA are equal.

$$AC=\sqrt{(2-2)^2+(1-5)^2}=\sqrt{16}=4\ ;\ BD=\sqrt{(4-0)^2+(3-3)^2}=\sqrt{16}=4$$

The two diagonals AC, BD are equal. $\therefore$ A,B,C,D form a square.

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12. Determine the range of the expression $\dfrac{x+2}{2x^2+3x+6}$.

Sol: Let '**y**' be a real value of the given expression

$$\Rightarrow y=\frac{x+2}{2x^2+3x+6}\Rightarrow y(2x^2+3x+6)=x+2\Rightarrow 2yx^2+3yx+6y-x-2=0$$

$$\Rightarrow 2yx^2+(3y-1)x+(6y-2)=0\ \ ....(1)$$

But, x is real and (1) is a quadratic equation in x

$\therefore\ \Delta = b^2 - 4ac \geq 0$

$\Rightarrow (3y-1)^2 - 4(2y)(6y-2)\geq 0 \quad \Rightarrow (9y^2-6y+1)-48y^2+16y\geq 0$

$\Rightarrow -39y^2+10y+1\geq 0 \Rightarrow 39y^2-10y-1\leq 0 \quad \Rightarrow 39y^2-13y+3y-1\leq 0$

$\Rightarrow 13y(3y-1)+1(3y-1)\leq 0 \Rightarrow (13y+1)(3y-1)\leq 0 \quad \Rightarrow \dfrac{-1}{13}\leq y\leq \dfrac{1}{3}\Rightarrow y\in\left[-\dfrac{1}{13},\dfrac{1}{3}\right]$

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13. If the letters of the word 'PRISON' are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "PRISON"

Sol : The dictionary order of the letters of the word PRISON is

I, N, O, P, R, S

The number of words that begin with I – – – – – = 5! = 120

The number of words that begin with N – – – – – = 5! = 120

The number of words that begin with O – – – – – = 5! = 120

The number of words that begin with P I – – – – = 4! = 24

The number of words that begin with P N – – – – = 4! = 24

The number of words that begin with P O – – – – = 4! = 24

The number of words that begin with PRIN – – = 2! = 2

The number of words that begin with PRIO – – = 2! = 2

The number of words that begin with PRISN – = 1! = 1

The number of words that begin with PRISON = 0! = 1. This is the required word.

$\therefore$ Rank of the word PRISON = 3(120) + 3(24)+2(2)+1 + 1= 360 + 72 + 4 + 1+1 = 438.

14. Find the number of ways of forming a committee of 5 members out of 6 Indians and 5 Americans so that always the Indians will be in majority in the committee.

Sol: A 5 men committee out of 6 Indians, 5 Americans with majority indians can be selected in the following compositions:.

Indians (6)	Americans(5)	No. of selections
5	0	${}^6C_5 \times {}^5C_0 = 6\times 1 = 6$
4	1	${}^6C_4 \times {}^5C_1 = 15\times 5 = 75$
3	2	${}^6C_3 \times {}^5C_2 = 20\times 10 = 200$

$\because {}^6C_4 = {}^6C_2 = \frac{6\times5}{2\times1} = 15$

${}^6C_3 = \frac{6\times5\times4}{3\times2\times1} = 20$

${}^5C_2 = \frac{5\times4}{2\times1} = 10$

$\therefore$ the total number of selections = 6 + 75 + 200 = 281

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15. Resolve $\frac{x^3}{(x-a)(x-b)(x-c)}$ into partial fractions.

Sol : Let $\frac{x^3}{(x-a)(x-b)(x-c)} = 1 + \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$ Here, deg(Nr) = deg(Dr)

$$= \frac{(x-a)(x-b)(x-c) + A(x-b)(x-c) + B(x-a)(x-c) + c(x-a)(x-b)}{(x-a)(x-b)(x-c)}$$

$\Rightarrow (x-a)(x-b)(x-c) + A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b) = x^3$ **.....(1)**

Putting x=a in (1), we get 0+A(a–b)(a–c)+0+0 = $a^3 \Rightarrow A = \frac{a^3}{(a-b)(a-c)}$

Similarly by putting x=b and x=c we get, $B = \frac{b^3}{(b-a)(b-c)}$, $C = \frac{c^3}{(c-a)(c-b)}$

$$\therefore \frac{x^3}{(x-a)(x-b)(x-c)} = 1 + \frac{a^3}{(a-b)(a-c)(x-a)} + \frac{b^3}{(b-c)(b-a)(x-b)} + \frac{c^3}{(c-a)(c-b)(x-c)}$$

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16. A,B,C are three horses in a race. The probability of A to win the race is twice that of B, and probability of B is twice that of C. What are the probabilities of A,B, and C to win the race?

Sol: Let A, B, C be the events of winning the race by the horses A, B, C respectively.

Given that P(A) = 2P(B) and P(B) = 2P(C)

Hence, P(A) = 2P(B) = 2[2P(C)] = 4P(C)

Now, $P(A\cup B\cup C) = P(A) + P(B) + P(C)$ [$\because$ A,B,C are disjoint]

$\Rightarrow P(S) = 4P(C) + 2P(C) + P(C)$ [$\because$ only A,B,C run the race $\Rightarrow A\cup B\cup C = S$]

$\Rightarrow 1 = 7P(C)$ [$\because P(S) = 1$]

$\Rightarrow P(C) = \frac{1}{7}$

Hence, $P(A) = 4P(C) = 4x\frac{1}{7} = \frac{4}{7}$ and $P(B) = 2P(C) = 2x\frac{1}{7} = \frac{2}{7}$

$\therefore P(A) = \frac{4}{7}, P(B) = \frac{2}{7}, P(C) = \frac{1}{7}$

17. A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.

Sol: Let A, B denote the events of speaking truth by A, B respectively

$$P(A)=\frac{75}{100}=\frac{3}{4};\ P(B)=\frac{80}{100}=\frac{4}{5}$$

$$\therefore P(\bar{A})=1-P(A)=1-\frac{3}{4}=\frac{1}{4};\ \ P(\bar{B})=1-P(B)=1-\frac{4}{5}=\frac{1}{5}$$

Let E be the event that A and B contradict to each other

$$\Rightarrow P(E)=P\left[(A\cap\bar{B})\cup(\bar{A}\cap B)\right]=P(A\cap\bar{B})+P(\bar{A}\cap B)$$

$$=P(A)P(\bar{B})+P(\bar{A})P(B)\quad [\because A,B \text{ are independent}]$$

$$=\frac{3}{4}\times\frac{1}{5}+\frac{1}{4}\times\frac{4}{5}=\frac{7}{20}$$

SECTION-C

18. If n is a positive integer then show that $(1 + i)^{2n} + (1 - i)^{2n} = 2^{n+1}\cos(n\pi/2)$

Sol: First we find the mod-amp form of 1 + i .

Let x + iy = 1 + i ⇒ x = 1, y = 1

$\therefore\ r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$ Also $\theta = \text{Tan}^{-1}\left(\frac{y}{x}\right) = \text{Tan}^{-1}\left(\frac{1}{1}\right) = \text{Tan}^{-1}(1) = \frac{\pi}{4}$

$\therefore$ mod– Amp form of $1 + i$ is $r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$$\Rightarrow (1+i)^{2n} = \left(\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)\right)^{2n} = (\sqrt{2})^{2n}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)^{2n}$$

$$= 2^n\left(\cos 2n\frac{\pi}{4} + i\sin 2n\frac{\pi}{4}\right) = 2^n\left(\cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2}\right) \quad \text{..... (1) (by Demoivre's theorem)}$$

Similarly, $(1-i)^{2n} = 2^n\left(\cos\frac{n\pi}{2} - i\sin\frac{n\pi}{2}\right)$ ..... (2)

Adding (1) & (2), we get $(1+i)^{2n} + (1-i)^{2n}$

$$= 2^n\left(\left(\cos\frac{n\pi}{2} + i\sin\frac{n\pi}{2}\right) + \left(\cos\frac{n\pi}{2} - i\sin\frac{n\pi}{2}\right)\right) = 2^n.2\cos\frac{n\pi}{2} = 2^{n+1}.\cos\frac{n\pi}{2}$$

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19. Find the polynomial equation whose roots are the translates of the roots of the equation $x^5 - 4x^4 + 3x^2 - 4x + 6 = 0$ by –3.

Sol: The equation whose roots are translates of the roots of the given equation f(x) = 0 by –3 is f(x + 3) = 0. Here, the roots of the required equation are diminished by 3.

3	1	–4	0	3	–4	6	
	0	3	–3	–9	–18	–66	
	1	–1	–3	–6	–22	–60	$\to a_5$
	0	3	6	9	9		
	1	2	3	3	–13	$\to a_4$	
	0	3	15	54			
	1	5	18	57	$\to a_3$		
	0	3	24				
	1	8	42	$\to a_2$			
	0	3					
	1	11	$\to a_1$				
	1	$\to a_0$					

$\therefore$ the required equation is given by $a_0x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5 = 0$

$\Rightarrow x^5 + 11x^4 + 42x^3 + 57x^2 - 13x - 60 = 0$

20. If the coefficients of 4 consecutive terms in the expansion of $(1+x)^n$ are a_1,a_2,a_3,a_4 respectively, then show that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$

Sol: Let the coefficients of 4 consecutive terms of $(1+x)^n$ be $a_1={}^nC_r$, $a_2={}^nC_{r+1}$, $a_3={}^nC_{r+2}$, $a_4={}^nC_{r+3}$.

$$\text{L.H.S}=\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}+\frac{{}^nC_{r+2}}{{}^nC_{r+2}+{}^nC_{r+3}}$$

$$=\frac{{}^nC_r}{{}^{(n+1)}C_{r+1}}+\frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}}\quad\left(\because {}^nC_r+{}^nC_{r+1}={}^{(n+1)}C_{r+1}\right)$$

$$=\frac{{}^nC_r}{\left(\frac{n+1}{r+1}\right){}^nC_r}+\frac{{}^nC_{r+2}}{\left(\frac{n+1}{r+3}\right){}^nC_{r+2}}\quad\left(\because {}^nC_r=\left(\frac{n}{r}\right){}^{n-1}C_{r-1}\right)$$

$$=\frac{r+1}{n+1}+\frac{r+3}{n+1}=\frac{r+1+r+3}{n+1}=\frac{2r+4}{n+1}=\frac{2(r+2)}{n+1}\quad ....(1)$$

$$\text{R.H.S}=\frac{2a_2}{a_2+a_3}=\frac{2({}^nC_{r+1})}{{}^nC_{r+1}+{}^nC_{r+2}}=\frac{2({}^nC_{r+1})}{{}^{(n+1)}C_{r+2}}=\frac{2({}^nC_{r+1})}{\left(\frac{n+1}{r+2}\right)({}^nC_{r+1})}=\frac{2}{\frac{n+1}{r+2}}=\frac{2(r+2)}{n+1}\quad ...(2)$$

From (1) & (2), L.H.S = R.H.S

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21. If $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+....$, then prove that $9x^2+24x=11$

Sol: Given that $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+......=\frac{1.3}{2!}\left(\frac{1}{3}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3+\frac{1.3.5.7}{4!}\left(\frac{1}{3}\right)^4+..$

Adding $1+\frac{1}{3}$ on both sides, we have $1+\frac{1}{3}+x=1+\frac{1}{1!}\left(\frac{1}{3}\right)+\frac{1.3}{2!}\left(\frac{1}{3}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3+......$

Comparing the above series with $1+\frac{p}{1!}\left(\frac{y}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{y}{q}\right)^2+.....=(1-y)^{-p/q}$

We get, $p=1, p+q=3\Rightarrow 1+q=3\Rightarrow q=2$

Also, $\frac{y}{q}=\frac{1}{3}\Rightarrow y=\frac{q}{3}=\frac{2}{3}$

$$\therefore 1+\frac{1}{3}+x=(1-y)^{-p/q}=\left(1-\frac{2}{3}\right)^{-1/2}=\left(\frac{1}{3}\right)^{-1/2}=(3)^{1/2}=\sqrt{3}$$

$$\Rightarrow\frac{4}{3}+x=\sqrt{3}\Rightarrow x=\sqrt{3}-\frac{4}{3}=\frac{3\sqrt{3}-4}{3}=\frac{3\sqrt{3}-4}{3}\Rightarrow 3x=3\sqrt{3}-4\Rightarrow 3x+4=3\sqrt{3}$$

$$\Rightarrow(3x+4)^2=(3\sqrt{3})^2\Rightarrow 9x^2+24x+16=27\Rightarrow 9x^2+24x=11$$

22. Find the mean deviation about mean for the following data:

x_i	2	5	7	8	10	35
f_i	6	8	10	6	8	2

Sol: We form the following table from the given data.

x_i	f_i	f_ix_i	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
2	6	12	6	36
5	8	40	3	24
7	10	70	1	10
8	6	48	0	0
10	8	80	2	16
35	2	70	27	54
	40	**320**		**140**

Here, $N = \Sigma f_i = 40$; $\Sigma f_i x_i = 320$ $\Rightarrow$ Mean $\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{320}{40} = 8$

From the table $\Sigma f_i \lvert x_i - \bar{x} \rvert = 140$

$\therefore$ Mean deviation about mean is $M.D = \frac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{N} = \frac{140}{40} = 3.5$

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23. Three boxes numbered I, II, III contains balls as follows

	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

One box is randomly selected and a ball is drawn from it. If the ball is red, then find the probability that it is from box II.

Sol: Let B_1, B_2, B_3 be the events of selecting boxes B_1, B_2, B_3 and R be the event of drawing a red ball

$\therefore P(B_1) = 1/3, P(B_2) = 1/3, P(B_3) = 1/3$, $P\left(\frac{R}{B_1}\right) = \frac{3}{6} = \frac{1}{2}$, $P\left(\frac{R}{B_2}\right) = \frac{1}{4}$ and $P\left(\frac{R}{B_3}\right) = \frac{3}{12} = \frac{1}{4}$

From Baye's theorem, the required probability is

$$P\left(\frac{B_2}{R}\right) = \frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right) + P(B_2)P\left(\frac{R}{B_2}\right) + P(B_3)P\left(\frac{R}{B_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{4}}{\frac{1}{3}\times\frac{1}{2} + \frac{1}{3}\times\frac{1}{4} + \frac{1}{3}\times\frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{2} + \frac{1}{4} + \frac{1}{4}} = \frac{\frac{1}{\cancel{4}}}{\frac{2+1+1}{\cancel{4}}} = \frac{1}{4}$$

24.

$X = x_i$	-3	-2	-1	0	1	2	3
$P(X = x_i)$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$

is the probability distribution of a random variable X. Find the variance of X.

Sol:

$$\text{Mean}(\mu) = -3\left(\frac{1}{9}\right) - 2\left(\frac{1}{9}\right) - 1\left(\frac{1}{9}\right) + 0\left(\frac{1}{3}\right) + 1\left(\frac{1}{9}\right) + 2\left(\frac{1}{9}\right) + 3\left(\frac{1}{9}\right)$$

$$= -\frac{3}{9} - \frac{2}{9} - \frac{1}{9} + 0 + \frac{1}{9} + \frac{2}{9} + \frac{3}{9} = 0 \Rightarrow \mu = 0$$

$$\text{Variance}(\sigma) = (-3)^2\left(\frac{1}{9}\right) + (-2)^2\left(\frac{1}{9}\right) + (-1)^2\left(\frac{1}{9}\right) + (0)^2\left(\frac{1}{3}\right) + (1)^2\left(\frac{1}{9}\right) + (2)^2\left(\frac{1}{9}\right) + (3)^2\left(\frac{1}{9}\right) - \mu^2$$

$$= \frac{9}{9} + \frac{4}{9} + \frac{1}{9} + 0 + \frac{1}{9} + \frac{4}{9} + \frac{9}{9} - 0 = \frac{28}{9}$$