Previous IPE

SOLVED PAPERS

MARCH -2025(AP)

PREVIOUS PAPERS

IPE: MARCH-2025(AP)

Time : 3 Hours

MATHS-2A

Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: $10 \times 2 = 20$

1. Write the complex number $\frac{4+3i}{(2+3i)(4-3i)}$ in the form a + ib.
2. Write $z = -\sqrt{7} + i\sqrt{21}$ in the polar form.
3. If 1, ω, ω^2 are the cube roots of unity, find $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$.
4. Form quadratic equation whose roots are $\frac{p-q}{p+q}, -\frac{p+q}{p-q}$ $(p \neq \pm q)$.
5. Find the algebraic equation whose roots are 2 times the roots of $x^5 - 2x^4 + 3x^3 - 2x^2 + 4x + 3 = 0$
6. Find the number of (i) 6 (ii) 7 letter Palindromes that can be formed using the letters of the word EQUATION.
7. If ${}^{n}P_{r} = 5040$ and ${}^{n}C_{r} = 210$, find n and r.
8. Find the number of terms with non zero coefficients in $(4x-7y)^{49} + (4x+7y)^{49}$
9. Find the mean deviation about median for the data 4, 6, 9, 3, 10, 13, 2
10. A poisson variable satisfies P(X = 1) = P(X = 2). Find P(X = 5)

SECTION-B

II. Answer any FIVE of the following SAQs: $5 \times 4 = 20$

11. If $x + iy = \frac{1}{1+\cos\theta + i\sin\theta}$, then show that $4x^2 - 1 = 0$
12. Determine the range of the expression $\frac{x+2}{2x^2+3x+6}$.
13. If the letters of the word 'PRISON' are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "PRISON"
14. Prove that $\frac{{}^{4n}C_{2n}}{{}^{2n}C_{n}} = \frac{1.3.5.....(4n-1)}{[1.3.5.....(2n-1)]^2}$
15. Resolve $\frac{x^2}{(x-1)(x-2)}$ into partial fractions
16. If two numbers are selected randomly from 20 consecutive natural numbers, find the probability that the sum of the two numbers is (i) an even number (ii) an odd number.
17. If A and B are independent events with P(A) = 0.2, P(B) = 0.5, find
    (i) P(A/B) (ii) P(B/A) (iii) $P(A\cap B)$ (iv) $P(A\cup B)$

SECTION-C

III. Answer any FIVE of the following LAQs: **$5 \times 7 = 35$**

18. Show that one value of $\left(\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^{8/3} = -1$

19. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, given that $1 + i$ is one of its roots.

20. If the 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(a+x)^n$ are respectively 240, 720 and 1080, then find the value of a, x and n.

21. If $x = \dfrac{1.3}{3.6} + \dfrac{1.3.5}{3.6.9} + \dfrac{1.3.5.7}{3.6.9.12} + \ldots$, then prove that $9x^2 + 24x = 11$

22. Find the mean deviation about the mean for the following continuous distribution:

Height in cms	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	13	26	30	12	10

23. Three boxes numbered I, II, III contain the balls as follows :

	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

One box is randomly selected and a ball is drawn from it.

If the ball is red, then find the probability that it is from BOX - II

24. A random variable X has its range $\{0, 1, 2\}$ and the probabilities are

$P(X = 0) = 3c^3$, $P(X = 1) = 4c - 10c^2$, $P(X = 2) = 5c - 1$ where 'c' is a constant,

find (i) c (ii) $P(0 < x < 3)$ (iii) $P(1 < x \leq 2)$ (iv) $P(x < 1)$

IPE AP MARCH-2025
SOLUTIONS

SECTION-A

1. Write the complex number $\frac{4+3i}{(2+3i)(4-3i)}$ in the form a + ib.

Sol: $G.E = \frac{4+3i}{(2+3i)(4-3i)} = \frac{4+3i}{[2(4)-3(-3)]+i[2(-3)+3(4)]} = \frac{4+3i}{17+6i}$

$= \frac{4(17)+3(6)}{(17)^2+(6)^2} + i\left(\frac{3(17)-6(4)}{17^2+6^2}\right) = \frac{86}{289+36} + i\left(\frac{51-24}{289+36}\right) = \left(\frac{86}{325}\right) + i\left(\frac{27}{325}\right)$

Which is in $a+ib$ form.

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2. Write $z = -\sqrt{7}+i\sqrt{21}$ in the polar form.

Sol: Let $-\sqrt{7}+i\sqrt{21} = x+iy \Rightarrow x = -\sqrt{7}, y = \sqrt{21}$

$\therefore r = \sqrt{x^2+y^2} = \sqrt{(-\sqrt{7})^2+(\sqrt{21})^2} = \sqrt{7+21} = \sqrt{28} = \sqrt{4\times 7} = 2\sqrt{7}$

Now, $\theta = Tan^{-1}\left(\frac{y}{x}\right) = Tan^{-1}\left(\frac{\sqrt{21}}{-\sqrt{7}}\right) = Tan^{-1}(-\sqrt{3}) = \frac{2\pi}{3}$ $[\because (-\sqrt{7}, \sqrt{21})$ lies in $Q_2]$

$\therefore$ the polar form of $-\sqrt{7}+i\sqrt{21}$ is $r(\cos\theta + i\sin\theta) = 2\sqrt{7}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

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3. If 1, ω, ω^2 are the cube roots of unity, find $(1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5$.

Sol: $G.E = (1-\omega+\omega^2)^5 + (1+\omega-\omega^2)^5$

$= (1+\omega^2-\omega)^5 + (1+\omega-\omega^2)^5 = (-\omega-\omega)^5 + (-\omega^2-\omega^2)^5$

$= (-2\omega)^5 + (-2\omega^2)^5 = -2^5[\omega^2+\omega] = -32(-1) = 32$

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4. Form quadratic equation whose roots are $\frac{p-q}{p+q}, -\frac{p+q}{p-q}$ $(p \neq \pm q)$.

Sol: Here, $\alpha+\beta = \frac{p-q}{p+q} - \frac{p+q}{p-q} = \frac{(p-q)^2-(p+q)^2}{(p+q)(p-q)} = \frac{-4pq}{p^2-q^2}$;

Also $\alpha.\beta = -\left(\frac{p-q}{p+q}\right)\left(\frac{p+q}{p-q}\right) = -1$

$\therefore$ The required quadratic equation is $x^2-(\alpha+\beta)x+\alpha\beta = 0$

$\Rightarrow x^2 + \left(\frac{4pq}{p^2-q^2}\right)x - 1 = 0$

$\Rightarrow (p^2-q^2)x^2 + 4pqx - (p^2-q^2) = 0$

5. Find the algebraic equation whose roots are 2 times the roots of $x^5 - 2x^4 + 3x^3 - 2x^2 + 4x + 3 = 0$

Sol : Let $f(x) = x^5 - 2x^4 + 3x^3 - 2x^2 + 4x + 3$.

Required equation is $f\left(\frac{x}{2}\right) = 0$

$$\Rightarrow \left(\frac{x}{2}\right)^5 - 2\left(\frac{x}{2}\right)^4 + 3\left(\frac{x}{2}\right)^3 - 2\left(\frac{x}{2}\right)^2 + 4\left(\frac{x}{2}\right) + 3 = 0$$

$$\Rightarrow \frac{1}{2^5}\left[x^5 - 4x^4 + 12x^3 - 16x^2 + 64x + 96\right] = 0$$

$$\Rightarrow x^5 - 4x^4 + 12x^3 - 16x^2 + 64x + 96 = 0$$

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6. Find the number of (i) 6 (ii) 7 letter Palindromes that can be formed using the letters of the word EQUATION.

Sol: The number of letters in the word EQUATION is n = 8. Also r = 6 & r = 7.

(i) Number of 6 letter palindromes = $n^{r/2} = 8^{6/2} = 8^3$

□□□ | □□□
8 8 8

(ii) Number of 7 letter palindromes = $n^{\frac{r+1}{2}} = 8^{\frac{7+1}{2}} = 8^4$

□□□□ | □□□
8 8 8 8

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7. If $^nP_r = 5040$, $^nC_r = 210$ then find n and r.

Sol: We know that $\frac{^nP_r}{^nC_r} = r!$

$$\Rightarrow \frac{^nP_r}{^nC_r} = \frac{5040}{210} = 24 = 4 \times 3 \times 2 \times 1 = 4! = r!$$

$\therefore r = 4$

Now, $^nP_4 = 5040 = 10 \times 504 = 10 \times 9 \times 56 = 10 \times 9 \times 8 \times 7 = {}^{10}P_4$ $\therefore$ n = 10

$\therefore$ n = 10, r = 4.

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8. Find the number of terms with non zero coefficients in $(4x-7y)^{49} + (4x+7y)^{49}$

Sol : As n = 49 is odd the number of non-zero terms $= \frac{n+1}{2} = \frac{49+1}{2} = \frac{50}{2} = 25$

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9. Find the mean deviation about median for the data 4, 6, 9, 3, 10, 13, 2

Sol: The ascending order of the observation in the given data is 2,3,4,<u>6</u>,9,10,13.

The number of observations = 7 (odd)

$\therefore$ Median of the given data is $M = \left(\frac{7+1}{2}\right)^{th}$ item = 4^{th} item = 6

The deviations of observations from the median are

2–6= –4; 3–6= –3; 4–6= –2; 6–6=0; 9–6=3; 10–6=4; 13–6=7

Hence, the absolute values of the deviations are 4, 3, 2, 0, 3, 4, 7

$\therefore$ The mean deviation from the median is

$$M.D = \frac{\Sigma|x_i - M|}{7} = \frac{4+3+2+0+3+4+7}{7} = \frac{23}{7} = 3.29$$

$$M.D = \frac{\Sigma|x_i - M|}{n}$$

10. A poisson variable satisfies P(X = 1) = P(X = 2). Find P(X = 5)

Sol: We have $P(X=r)=\frac{e^{-\lambda}\lambda^r}{r!}, \lambda > 0$

Given that $P(X = 1) = P(X = 2)$

$$\Rightarrow \frac{\lambda e^{-\lambda}}{1!} = \frac{\lambda^2 e^{-\lambda}}{2!} \Rightarrow \frac{\lambda}{1} = \frac{\lambda^2}{2} \Rightarrow \lambda^2 = 2\lambda \Rightarrow \lambda(\lambda-2)=0 \Rightarrow \lambda = 2 \, (\because \lambda > 0)$$

$$\therefore P(X=5) = \frac{e^{-2}2^5}{5!}$$

SECTION-B

11. If $x + iy = \dfrac{1}{1+\cos\theta + i\sin\theta}$, then show that $4x^2 - 1 = 0$

Sol: Given that $x+iy=\dfrac{1}{(1+\cos\theta)+i\sin\theta}=\dfrac{1}{\left(2\cos^2\frac{\theta}{2}\right)+i\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)}$

$$\Rightarrow x+iy=\frac{1}{\left(2\cos\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)}=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\left(2\cos\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)\left(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}\right)}$$

$$=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\left(2\cos\frac{\theta}{2}\right)\left(\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\right)}=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\left(2\cos\frac{\theta}{2}\right)(1)}=\frac{\cos\frac{\theta}{2}}{2\cos\frac{\theta}{2}}-\frac{i\sin\frac{\theta}{2}}{2\cos\frac{\theta}{2}}=\frac{1}{2}-\frac{1}{2}i\tan\frac{\theta}{2}$$

Equating the real parts, we get $x=\frac{1}{2}\Rightarrow 2x=1\Rightarrow(2x)^2=1^2\Rightarrow 4x^2=1\Rightarrow 4x^2-1=0$

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12. Determine the range of the expression $\dfrac{x+2}{2x^2+3x+6}$.

Sol: Let '**y**' be a real value of the given expression

$\Rightarrow y=\dfrac{x+2}{2x^2+3x+6}\Rightarrow y(2x^2+3x+6)=x+2\Rightarrow 2yx^2+3yx+6y-x-2=0$

$\Rightarrow 2yx^2+(3y-1)x+(6y-2)=0$ .... (1)

But, x is real and (1) is a quadratic equation in x

$\therefore \Delta = b^2 - 4ac \ge 0$

$\Rightarrow (3y-1)^2 - 4(2y)(6y-2) \ge 0$

$\Rightarrow (9y^2 - 6y + 1) - 48y^2 + 16y \ge 0$

$\Rightarrow -39y^2 + 10y + 1 \ge 0 \Rightarrow 39y^2 - 10y - 1 \le 0$

$\Rightarrow 39y^2 - 13y + 3y - 1 \le 0$

$\Rightarrow 13y(3y-1) + 1(3y-1) \le 0 \Rightarrow (13y+1)(3y-1) \le 0$

$\Rightarrow \dfrac{-1}{13} \le y \le \dfrac{1}{3} \Rightarrow y \in \left[-\dfrac{1}{13}, \dfrac{1}{3}\right]$

13. If the letters of the word 'PRISON' are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "PRISON"

Sol : The dictionary order of the letters of the word PRISON is

I, N, O, P, R, S

The number of words that begin with I – – – – – = 5! = 120

The number of words that begin with N – – – – – = 5! = 120

The number of words that begin with O – – – – – = 5! = 120

The number of words that begin with P I – – – – = 4! = 24

The number of words that begin with P N – – – – = 4! = 24

The number of words that begin with P O – – – – = 4! = 24

The number of words that begin with PRIN – – = 2! = 2

The number of words that begin with PRIO – – = 2! = 2

The number of words that begin with PRISN – = 1! = 1

The number of words that begin with PRISON = 0! = 1. This is the required word.

∴ Rank of the word PRISON = 3(120) + 3(24)+2(2)+1 + 1= 360 + 72 + 4 + 1+1 = 438.

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14. Prove that $\dfrac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \dfrac{1.3.5.....(4n-1)}{[1.3.5.....(2n-1)]^2}$

Sol: $\text{L.H.S} = \dfrac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \dfrac{\dfrac{4n!}{2n!.2n!}}{\dfrac{2n!}{n!.n!}} = \dfrac{(4n)!}{(2n!)^2} \text{x} \dfrac{(n!)^2}{(2n)!}$ [Since ${}^nC_r = \dfrac{n!}{r!(n-r)!}$]

$$= \frac{(4n)(4n-1)(4n-2)(4n-3)(4n-4)..........6.5.4.3.2.1}{[(2n)(2n-1)(2n-2)(2n-3).........5.4.3.2.1]^2} \times \frac{(n!)^2}{(2n)!}$$

$$= \frac{[(4n)(4n-2)(4n-4)..........(6)(4)(2)][(4n-1)(4n-3).....5.3.1]}{[(2n)(2n-2)......4.2]^2[(2n-1)(2n-3).......(5)(3)(1)]^2} \times \frac{(n!)^2}{(2n)!}$$

$$= \frac{[2(2n)2(2n-1)2(2n-2)..........2(3)2(2)2(1)][(4n-1)(4n-3).....5.3.1]}{[2(n)2(n-1)......2(2).2(1)]^2[(2n-1)(2n-3).......(5)(3)(1)]^2} \times \frac{(n!)^2}{(2n)!}$$

$$= \frac{[2^{2n}(2n)(2n-1)(2n-2).......(3)(2)(1)][(4n-1)(4n-3).....5.3.1]}{[2^n(n)(n-1)........(2)(1)]^2[(2n-1)(2n-3).......(5)(3)(1)]^2} \times \frac{(n!)^2}{(2n)!}$$

$$= \frac{[2^{2n}(2n!)][(4n-1)(4n-3).....5.3.1]}{2^{2n}(n!)^2[(2n-1)(2n-3).......(5)(3)(1)]^2} \times \frac{(n!)^2}{(2n)!}$$

$$= \frac{1.3.5......(4n-3)(4n-1)}{[1.3.5......(2n-3)(2n-1)]^2} = \text{R.H.S}$$

15. Resolve $\frac{x^2}{(x-1)(x-2)}$ into partial fractions

Sol: Let $\frac{x^2}{(x-1)(x-2)} = 1 + \frac{A}{x-1} + \frac{B}{x-2} = \frac{(x-1)(x-2) + A(x-2) + B(x-1)}{(x-1)(x-2)}$

$\Rightarrow (x-1)(x-2) + A(x-2) + B(x-1) = x^2$.......**(1)**

Putting x =1 in (1), we get $0 + A(1-2) + B(0) = 1 \Rightarrow -A = 1 \Rightarrow A = -1$

Putting x =2 in (1), we get $0 + A(0) + B(2-1) = 2^2 \Rightarrow B = 4$

$\therefore \frac{x^2}{(x-1)(x-2)} = 1 + \frac{A}{x-1} + \frac{B}{x-2} = 1 - \frac{1}{x-1} + \frac{4}{x-2}$

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16. If two numbers are selected randomly from 20 consecutive natural numbers, find the probability that the sum of the two numbers is (i) an even number (ii) an odd number.

Sol: Let E be the event of choosing two numbers such that their sum is even.
The total number of ways of choosing any 2 out of 20 is $n(S) = {}^{20}C_2 = 190$
Any 20 consecutive natural numbers contain 10 even numbers and 10 odd numbers.
(i) To get an even sum, either both must be odd or both must be even.
So the number of favourable cases to E is $n(E) = {}^{10}C_2 + {}^{10}C_2 = 45 + 45 = 90$

$$\therefore P(\text{even sum}) = P(E) = \frac{n(E)}{n(S)} = \frac{90}{190} = \frac{9}{19}$$

(ii) Getting an 'even sum' and an 'odd sum' are both complementary events

$$\therefore P(\text{odd sum}) = P(\bar{E}) = 1 - P(E) = 1 - \frac{9}{19} = \frac{19-9}{19} = \frac{10}{19}$$

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17. If A and B are independent events with P(A) = 0.2, P(B) = 0.5, find (i) P(A/B) (ii) P(B/A) (iii) $P(A \cap B)$ (iv) $P(A \cup B)$

Sol: Given that A,B are independent, hence

(i) P(A/B) = P(A) = 0.2

(ii) P(B/A) = P(B) = 0.5

(iii) $P(A \cap B) = P(A).P(B) = 0.2 \times 0.5 = 0.1$

(iv) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.5 - 0.1 = 0.6$

SECTION-C

18. Show that one value of $\left(\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^{8/3} = -1$

Sol:

$$\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}} = \frac{1+\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+i\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)}{1+\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)-i\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)} = \frac{\left(1+\cos\frac{3\pi}{8}\right)+i\left(\sin\frac{3\pi}{8}\right)}{\left(1+\cos\frac{3\pi}{8}\right)-i\left(\sin\frac{3\pi}{8}\right)}$$

$$= \frac{2\cos^2\frac{3\pi}{16}+i\left(2\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}\right)}{2\cos^2\frac{3\pi}{16}-i\left(2\sin\frac{3\pi}{16}\cos\frac{3\pi}{16}\right)} = \frac{\cancel{2\cos\frac{3\pi}{16}}\left(\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)}{\cancel{2\cos\frac{3\pi}{16}}\left(\cos\frac{3\pi}{16}-i\sin\frac{3\pi}{16}\right)} \times \left(\frac{\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}}{\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}}\right)$$

$$= \frac{\left(\cos\frac{3\pi}{16}+i\sin\frac{3\pi}{16}\right)^2}{\cos^2\frac{3\pi}{16}-i^2\sin^2\frac{3\pi}{16}} = \frac{\cos2\left(\frac{3\pi}{16}\right)+i\sin2\left(\frac{3\pi}{16}\right)}{\cos^2\left(\frac{3\pi}{16}\right)+\sin^2\left(\frac{3\pi}{16}\right)} = \frac{\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}}{1} = \cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}$$

$$\therefore \left(\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^{8/3} = \left(\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}\right)^{8/3} = \left(\cos\frac{8}{3}\left(\frac{3\pi}{8}\right)+i\sin\frac{8}{3}\left(\frac{3\pi}{8}\right)\right)$$

$$= \cos\pi + i\sin\pi = (-1)+i(0) = -1$$

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19. Solve the equation $x^4 + 2x^3 - 5x^2 + 6x + 2 = 0$, given that $1 + i$ is one of its roots.

Sol: For the polynomial equation with real coefficients imaginary roots occur in conjugate pairs.

So $1 + i$ and $1 - i$ are two roots of the given equation.

Sum of roots = $(1 + i) + (1 - i) = 2$; Product of roots = $(1 + i) + (1 - i) = 1 + 1 = 2$

Now, the quadratic factor corresponding to these roots is

$x^2 -$ (sum of roots)x + Product of roots = $x^2 - 2x + 2$

On dividing $x^4+2x^3-5x^2+6x+2$ by quadratic x^2-2x+2 using Synthetic Division, we have

	1	2	–5	6	2
2	0	2	8	2	0
–2	0	0	–2	–8	–2
	1	4	1	0	0

Now, $x^2 + 4x + 1 = 0 \Rightarrow x = \dfrac{-4\pm\sqrt{16-4}}{2} = \dfrac{-4\pm\sqrt{12}}{2} = \dfrac{-4\pm2\sqrt{3}}{2} = -2\pm\sqrt{3}$

$\therefore$ The roots of the given equation are $1+i, 1-i, -2+\sqrt{3}, -2-\sqrt{3}$

20. If the 2^{nd}, 3^{rd} and 4^{th} terms in the expansion of $(a+x)^n$ are respectively 240, 720 and 1080, then find the value of a, x and n.

Sol : The second term of $(a+x)^n$ is $T_2 = T_{1+1} = {}^nC_1 a^{n-1}x^1 = 240$ ....(1)

The third term of $(a+x)^n$ is $T_3 = T_{2+1} = {}^nC_2 a^{n-2}x^2 = 720$ ....(2)

The fourth term of $(a+x)^n$ is $T_4 = T_{3+1} = {}^nC_3 a^{n-3}x^3 = 1080$ ....(3)

$\frac{(2)}{(1)} \Rightarrow \frac{{}^nC_2 a^{n-2}x^2}{{}^nC_1 a^{n-1}x} = \frac{720}{240} \Rightarrow \left(\frac{{}^nC_2}{{}^nC_1}\right)(a^{-1})(x) = 3 \Rightarrow \left(\frac{n-1}{2}\right)\left(\frac{x}{a}\right) = 3 \quad \left(\because \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{n-r}{r+1}\right)$

$\Rightarrow (n-1)(x) = 6a$ ....(4)

$\frac{(3)}{(2)} \Rightarrow \frac{{}^nC_3 a^{n-3}x^3}{{}^nC_2 a^{n-2}x^2} = \frac{1080}{720} = \frac{9}{6} = \frac{3}{2} \Rightarrow \left(\frac{{}^nC_3}{{}^nC_2}\right)(a^{-1})(x) = \frac{3}{2} \Rightarrow \left(\frac{n-2}{3}\right)\left(\frac{x}{a}\right) = \frac{3}{2}$

$\Rightarrow 2(n-2)(x) = 9a$ ....(5)

$\frac{(5)}{(4)} \Rightarrow \frac{2(n-2)(x)}{(n-1)(x)} = \frac{9a}{6a} = \frac{3}{2} \Rightarrow 4(n-2) = 3(n-1) \Rightarrow 4n-8 = 3n-3 \Rightarrow n = 5$

Now (4) $\Rightarrow (5-1)x = 6a \Rightarrow 4x = 6a \Rightarrow 2x = 3a \Rightarrow x = 3a/2$ .....(6)

Also, (1) $\Rightarrow {}^nC_1 a^{n-1}x^1 = 240 \Rightarrow na^{n-1}x = 240$

$\Rightarrow 5a^{5-1}\left(\frac{3a}{2}\right) = (24)(10) \Rightarrow a^4.a = \frac{(24)(10)(2)}{(5)(3)} = 32 \Rightarrow a^5 = 32 = 2^5 \Rightarrow a = 2$

$\therefore$ from (6), $x = \frac{3a}{2} = \frac{3(2)}{2} = 3$ $\quad\therefore a = 2, x = 3, n = 5$

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21. If $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + ...$, then prove that $9x^2 + 24x = 11$

Sol: Given that $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + ... = \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \frac{1.3.5.7}{4!}\left(\frac{1}{3}\right)^4 + ..$

Adding $1 + \frac{1}{3}$ on both sides, we have $1 + \frac{1}{3} + x = 1 + \frac{1}{1!}\left(\frac{1}{3}\right) + \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + ...$

Comparing the above series with $1 + \frac{p}{1!}\left(\frac{y}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{y}{q}\right)^2 + ... = (1-y)^{-p/q}$

We get, $p = 1, p+q = 3 \Rightarrow 1+q = 3 \Rightarrow q = 2$

Also, $\frac{y}{q} = \frac{1}{3} \Rightarrow y = \frac{q}{3} = \frac{2}{3}$

$\therefore 1 + \frac{1}{3} + x = (1-y)^{-p/q} = \left(1 - \frac{2}{3}\right)^{-1/2} = \left(\frac{1}{3}\right)^{-1/2} = (3)^{1/2} = \sqrt{3}$

$\Rightarrow \frac{4}{3} + x = \sqrt{3} \Rightarrow x = \sqrt{3} - \frac{4}{3} = \frac{3\sqrt{3}-4}{3} = \frac{3\sqrt{3}-4}{3} \Rightarrow 3x = 3\sqrt{3} - 4 \Rightarrow 3x + 4 = 3\sqrt{3}$

$\Rightarrow (3x+4)^2 = (3\sqrt{3})^2 \Rightarrow 9x^2 + 24x + 16 = 27 \Rightarrow 9x^2 + 24x = 11$

22. Find the mean deviation about the mean for the following continuous distribution:

Height in cms	95-105	105-115	115-125	125-135	135-145	145-155
Number of boys	9	13	26	30	12	10

Sol: We form the following table from the given data

Height	No. of boys (f_i)	M.P(x_i)	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$f_1 \lvert x_i - \bar{x}\rvert$
95-105	9	100	900	25.3	227.7
105-115	13	110	1430	15.3	198.9
115-125	26	120	3120	5.3	137.8
125-135	30	130	3900	4.7	141.0
135-145	12	140	1680	14.7	176.4
145-155	10	150	1500	24.7	247.0
	$\Sigma f_i = 100 = N$		$\Sigma f_ix_i = 12530$		$\Sigma f_i \lvert x_i - \bar{x}\rvert = 1128.8$

Here, $N = \Sigma f_i = 100$ and Mean, $\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{12530}{100} = 125.3$

$\therefore$ Mean deviation about the mean M.D $= \frac{\Sigma f_i \lvert x_i - \bar{x}\rvert}{N} = \frac{1128.8}{100} = 11.28$

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23. Three boxes numbered I, II, III contains balls as follows

	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

One box is randomly selected and a ball is drawn from it. If the ball is red, then find the probability that it is from box II.

Sol: Let B_1, B_2, B_3 be the events of selecting boxes B_1, B_2, B_3 and R be the event of drawing a red ball

$\therefore P(B_1) = 1/3, P(B_2) = 1/3, P(B_3) = 1/3,\ P\left(\frac{R}{B_1}\right) = \frac{3}{6} = \frac{1}{2},\ P\left(\frac{R}{B_2}\right) = \frac{1}{4}$ and $P\left(\frac{R}{B_3}\right) = \frac{3}{12} = \frac{1}{4}$

From Baye's theorem, the required probability is

$$P\left(\frac{B_2}{R}\right) = \frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right) + P(B_2)P\left(\frac{R}{B_2}\right) + P(B_3)P\left(\frac{R}{B_3}\right)}$$

$$= \frac{\frac{1}{3} \times \frac{1}{4}}{\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{1}{2} + \frac{1}{4} + \frac{1}{4}} = \frac{\frac{1}{\cancel{4}}}{\frac{2+1+1}{\cancel{4}}} = \frac{1}{4}$$

24. A random variable X has its range{0, 1, 2} and the probabilities are $P(X = 0) = 3c^3$, $P(X = 1) = 4c-10c^2$, $P(X = 2) = 5c - 1$ where 'c' is a constant, find (i) c (ii) $P(0 < x < 3)$ (iii) $P(1 < x \le 2)$ (iv) $P(x < 1)$

Sol: (i) We know that $\Sigma P(X = x_i) = 1$

$$\Rightarrow 3c^3 + 4c - 10c^2 + 5c - 1 = 1 \Rightarrow 3c^3 - 10c^2 + 9c - 1 = 1 \Rightarrow 3c^3 - 10c^2 + 9c - 2 = 0$$

Here, the sum of the coefficients is 3–10+9–2=0. Hence 1 is a root of the above equation.

∴ By synthetic division, we have

	1 3	−10	9	−2
0	3	−7	2	
3	−7	2	0	

$$\therefore 3c^3 - 10c^2 + 9c - 2 = (c-1)(3c^2 - 7c + 2) = (c-1)[3c^2 - 6c - c + 2]$$

$$= (c-1)[3c(c-2) - 1(c-2)] = (c-1)(c-2)(3c-1)$$

Now, $3c^3 - 10c^2 + 9c - 2 = 0 \Rightarrow (c-1)(c-2)(3c-1) = 0 \Rightarrow c = 1, 2, \frac{1}{3}$

If, c = 1 then, $P(X = 0) = 3c^3 = 3.1^3 = 3 > 1$, which is impossible

If, c = 2 then, $3c^3 = 3(2)^3 = 24 > 1$, which is also impossible.

∴ c=1/3 is the only possible value.

(ii) $P(0 < X < 3) = P(X = 1) + P(X = 2) = (4c - 10c^2) + (5c - 1) = 9c - 10c^2 - 1$

$$= 9\left(\frac{1}{3}\right) - 10\left(\frac{1}{3}\right)^2 - 1 = \frac{9}{3} - \frac{10}{9} - 1 = 3 - \frac{10}{9} - 1 = 2 - \frac{10}{9} = \frac{8}{9}$$

(iii) $P(1 < x \le 2) = P(X = 2) = 5c - 1 = 5\left(\frac{1}{3}\right) - 1 = \frac{5}{3} - 1 = \frac{2}{3}$

(iv) $P(X < 1) = P(X = 0) = 3c^3 = 3\left(\frac{1}{3}\right)^3 = 3.\frac{1}{27} = \frac{1}{9}$