Previous IPE

SOLVED PAPERS

MARCH -2025 (TS)

PREVIOUS PAPERS

IPE: MARCH-2025 (TS)

Time : 3 Hours **MATHS-1B** Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **$10 \times 2 = 20$**

1. Find the slope of the line passing through the points$(-3, 8), (10, 5)$
2. Transform the equation $3x + 4y = 5$ into (i) slope intercept form (ii)intercept form
3. Find the distance of $P(3, -2, 4)$ from the origin
4. Find the equation of the plane which makes intercepts 1, 2, 4 on the x,y,z-axes respectively.
5. Compute $\underset{x\to 1}{Lt}(x^2+2x+3)$ **6.** Find $\underset{x\to\infty}{Lt}\dfrac{8|x|+3x}{3|x|-2x}$
7. Find the derivative of $(4+x^2)e^{2x}$ **8.** Find the derivative of $\sin^{-1}(3x-4x^3)$ w.r.to x.
9. If $y = x^2 + 3x + 6$ then find Δy and dy when $x = 10, \Delta x = 0.01$.
10. Verify Rolle's theorem for the function $x^2 - 1$ on $[-1,1]$

SECTION-B

II. Answer any FIVE of the following SAQs: **$5 \times 4 = 20$**

11. The ends of the hypotenuse of a right angled triangle are (0, 6) and (6, 0). Find the equation of locus of its third vertex.
12. When the origin is shifted to the point (2, 3), the transformed equation of a curve is $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$. Find the original equation of the curve.
13. Find the foot of the perpendicular drawn from (4, 1) on the line $3x - 4y + 12 = 0$.
14. Check the continuity of $f(x)=\begin{cases}\frac{1}{2}(x^2-4) & \text{if } 0<x<2\\ 0 & \text{if } x=2\\ 2-8x^{-3} & \text{if } x>2\end{cases}$ at 2.
15. Find the derivative of $\sin 2x$ from the first principle.
16. Find the equations of tangent and normal to the curve $y = x^3 + 4x^2$ at $(-1, 3)$
17. The displacement s of a particle travelling in a straight line in t seconds is given by $s = 45t + 11t^2 - t^3$. Find the time when the particle comes to rest.

SECTION-C

III. Answer any FIVE of the following LAQs: **$5 \times 7 = 35$**

18. Find the circumcentre of the triangle whose vertices are (1, 3), (–3, 5) and (5, –1).
19. If θ is the angle between the pair of lines $ax^2 + 2hxy + by^2 = 0$ then $\cos\theta=\dfrac{a+b}{\sqrt{(a-b)^2+4h^2}}$
20. Show that the lines joining the origin to the points of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$ are mutually perpendicular.
21. Find the angle between the lines whose d.c's are related by $l + m + n = 0$ & $l^2 + m^2 - n^2 = 0$
22. If $x^{\log y} = \log x$ then $\dfrac{dy}{dx}=\dfrac{y}{x}\left(\dfrac{1-\log x\log y}{(\log x)^2}\right)$
23. At any point t on the curve $x = a(t + \sin t), y = a(1 - \cos t)$, find the lengths of tangent, normal, subtangent and subnormal.
24. Find two positive integers whose sum is 15 and the sum of whose squares is minimum.

IPE TS MARCH-2025
SOLUTIONS

SECTION-A

1. Find the slope of the line passing through the points (–3, 8), (10, 5)

Sol: Slope of the line passing through

$A(x_1, y_1) = (-3, 8)$ and $B(x_2, y_2) = (10, 5)$ is

$$m = \frac{y_1 - y_2}{x_1 - x_2} = \frac{8-5}{-3-10} = \frac{-3}{13}$$

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2. Transform the equation 3x + 4y = 5 into (i) slope intercept form (ii)intercept form

Sol: (i) Slope intercept form is y = mx + c

$$\therefore 3x + 4y = 5 \Rightarrow 4y = -3x + 5 \Rightarrow y = \left(-\frac{3}{4}\right)x + \frac{5}{4}$$

(ii) Intercept form is $\frac{x}{a} + \frac{y}{b} = 1$

$$\therefore 3x + 4y = 5 \Rightarrow \frac{3x}{5} + \frac{4y}{5} = 1 \Rightarrow \frac{x}{5/3} + \frac{y}{5/4} = 1$$

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3. Find the distance of P(3, –2, 4) from the origin

Sol: The distance between O(0, 0, 0) and P(x, y, z) = (3, –2, 4) is

$$OP = \sqrt{x^2 + y^2 + z^2} = \sqrt{9+4+16} = \sqrt{29} \text{ units}$$

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4. Find the equation of the plane which makes intercepts 1,2,4 on the x,y,z-axes respectively.

Sol: The equation of the plane with intercepts a, b, c is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$$\therefore \text{The required equation of the plane is } \frac{x}{1} + \frac{y}{2} + \frac{z}{4} = 1 \Rightarrow \frac{4x+2y+z}{4} = 1 \Rightarrow 4x + 2y + z = 4$$

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5. Compute $\underset{x\to 1}{Lt}(x^2 + 2x + 3)$

Sol: $\underset{x\to 1}{Lt}(x^2 + 2x + 3) = 1^2 + 2(1) + 3 = 1 + 2 + 3 = 6$

6. Find $\underset{x\to\infty}{Lt}\dfrac{8|x|+3x}{3|x|-2x}$

Sol: If $x \to \infty$ then, $x > 0$. Hence, $|x| = x$

$$\therefore \underset{x\to\infty}{Lt}\frac{8|x|+3x}{3|x|-2x} = \underset{x\to\infty}{Lt}\frac{8x+3x}{3x-2x} = \underset{x\to\infty}{Lt}\frac{11x}{x} = \underset{x\to\infty}{Lt}\ 11 = 11$$

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7. Find the derivative of $(4 + x^2)e^{2x}$

Sol: $\dfrac{d}{dx}(4+x^2)e^{2x} = (4+x^2)\dfrac{d}{dx}(e^{2x}) + e^{2x}\dfrac{d}{dx}(4+x^2)$

$= (4 + x^2)\, 2.e^{2x} + e^{2x}(0 + 2x) = 2e^{2x}\,[4 + x^2 + x] = 2e^{2x}\,(x^2 + x + 4)$

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8. Find the derivative of $Sin^{-1}(3x-4x^3)$

Sol: We take $x = \sin\theta$, then $\theta = Sin^{-1}x$

$\therefore Sin^{-1}(3x-4x^3) = Sin^{-1}(3\sin\theta - 4\sin^3\theta)$

$= Sin^{-1}(\sin 3\theta) = 3\theta = 3(Sin^{-1}x)$

$\therefore \dfrac{d}{dx}(3Sin^{-1}x)$

$= 3\dfrac{d}{dx}Sin^{-1}x = 3\left(\dfrac{1}{\sqrt{1-x^2}}\right) = \dfrac{3}{\sqrt{1-x^2}}$

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9. If $y = x^2 + 3x + 6$ then find Δy and dy when $x = 10$, $\Delta x = 0.01$.

Sol: We take $y = f(x) = x^2 + 3x + 6$, $x = 10$, $\Delta x = 0.01$

(i) $\Delta y = f(x + \Delta x) - f(x)$

$= [(x+\Delta x)^2 + 3(x+\Delta x) + 6] - (x^2+3x+6)$

$= [\cancel{x^2} + (\Delta x)^2 + 2x\Delta x] + \cancel{3x} + 3\Delta x + \cancel{6} - \cancel{x^2} - \cancel{3x} - \cancel{6}$

$= (\Delta x)^2 + 2x\Delta x + 3\Delta x$

$= \Delta x[\Delta x + 2x + 3]$

$= (0.01)[0.01 + 2(10) + 3]$

$= (0.01)[0.01 + 23]$

$= 0.01(23.01) = 0.2301$

(ii) $dy = f'(x)\Delta x = (2x + 3)(\Delta x)$

$= [2(10) + 3](0.01)$

$= (23)(0.01) = 0.23$

10. Verify Rolle's theorem for the function x^2-1 on $[-1,1]$

Sol: Given $f(x) = x^2-1 \Rightarrow f'(x) = 2x$

$f(x)$ is (i) continuous on $[-1,1]$ and (ii) differentiable in $(-1,1)$

(iii) $f(-1) = (-1)^2-1 = 1-1 = 0$; $f(1) = 1^2-1 = 1-1 = 0$

$\Rightarrow f(-1) = f(1)$

So, from Rolle's theorem, $f'(c) = 0 \Rightarrow 2c = 0 \Rightarrow c = 0$

$\therefore c = 0 \in (-1,1)$ Hence, Rolle's theorem is verified.

SECTION-B

11. The ends of the hypotenuse of a right angled triangle are (0, 6) and (6, 0). Find the equation of locus of its third vertex.

Sol: We take A = (0, 6), B = (6, 0) and

P = (x,y) is a point on the locus.

Given condition: $\angle APB = 90^\circ$

$\Rightarrow PA^2 + PB^2 = AB^2$

$\Rightarrow [(x-0)^2 + (y-6)^2] + [(x-6)^2 + (y-0)^2] = (0-6)^2 + (6-0)^2$

$\Rightarrow x^2 + (y^2 + \cancel{36} - 12y) + (x^2 + \cancel{36} - 12x) + y^2 = \cancel{36} + \cancel{36}$

$\Rightarrow 2x^2 + 2y^2 - 12x - 12y = 0 \Rightarrow \cancel{2}(x^2 + y^2 - 6x - 6y) = 0$

$\Rightarrow x^2 + y^2 - 6x - 6y = 0$

Hence, locus of P is $x^2 + y^2 - 6x - 6y = 0$

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12. When the origin is shifted to the point (2, 3), the transformed equation of a curve is $x^2 + 3xy - 2y^2 + 17x - 7y - 11 = 0$, find the original equation of the curve.

Sol:
- ★ Given transformed(new) equation is taken as $X^2 + 3XY - 2Y^2 + 17X - 7Y - 11 = 0$.........(1)
- We take origin (h, k) = (2, 3), then
- ★ $X = x - h \Rightarrow X = x - 2$
- ★ $Y = y - k \Rightarrow Y = y - 3$
- From (1), original equation is $(x-2)^2 + 3(x-2)(y-3) - 2(y-3)^2 + 17(x-2) - 7(y-3) - 11 = 0$
- $\Rightarrow (x^2 + 4 - 4x) + 3(xy - 3x - 2y + 6) - 2(y^2 + 9 - 6y) + 17x - 34 - 7y + 21 - 11 = 0$
- $\Rightarrow x^2 + 4 - 4x + 3xy - 9x - 6y + 18 - 2y^2 - 18 + 12y + 17x - 34 - 7y + 21 - 11 = 0$
- $\Rightarrow x^2 + 3xy - 2y^2 + 4x - y - 20 = 0$
- Hence, required original equation is $x^2 + 3xy - 2y^2 + 4x - y - 20 = 0$.

13. Find the foot of the perpendicular drawn from (4, 1) on the line 3x – 4y + 12 = 0.

Sol: Let (h, k) be the foot of the perpendicular from (4, 1) on the line $3x - 4y + 12 = 0$

Here $(x_1, y_1) = (4, 1)$, $a = 3$, $b = -4$, $c = 12$.

$$\therefore \frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$$

$$\Rightarrow \frac{h-4}{3} = \frac{k-1}{-4} = \frac{-[3(4) - 4(1) + 12]}{3^2 + 4^2} = \frac{-(12-4+12)}{9+16} = \frac{-20}{25} = \frac{-4}{5}$$

Now, $\frac{h-4}{3} = \frac{-4}{5} \Rightarrow 5h - 20 = -12 \Rightarrow 5h = 20 - 12 = 8 \Rightarrow h = \frac{8}{5}$

Also $\frac{k-1}{-4} = \frac{-4}{5} \Rightarrow 5k - 5 = 16 \Rightarrow 5k = 16 + 5 = 21 \Rightarrow k = \frac{21}{5}$ $\quad \therefore (h,k) = \left(\frac{8}{5}, \frac{21}{5}\right)$

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14. Check the continuity of $f(x) = \begin{cases} \frac{1}{2}(x^2 - 4) & \text{if } 0 < x < 2 \\ 0 & \text{if } x = 2 \\ 2 - 8x^{-3} & \text{if } x > 2 \end{cases}$ **at 2.**

Sol: (a) When $x < 2$, L.H.L $= \underset{x\to 2-}{Lt} f(x) = \underset{x\to 2-}{Lt} \frac{1}{2}(x^2 - 4) = \frac{1}{2}(4-4) = 0$ ....(1)

(b) When $x > 2$,

R.H.L $= \underset{x\to 2+}{Lt} f(x) = \underset{x\to 2+}{Lt} (2 - 8x^{-3}) = \underset{x\to 2+}{Lt}\left(2 - \frac{8}{x^3}\right) = 2 - \frac{8}{8} = 2 - 1 = 1$ ....(2)

From (1) & (2), L.H.L≠R.H.L Hence, proved that f(x) is not continuous at 2.

15. Find the derivative of sin2x from the first principle.

Sol:
- We take f(x) = sin2x, then
- ★ f(x + h) = sin2(x + h) = sin (2x + 2h)
- From the first principle,
- ★ $f'(x) = \underset{h\to 0}{Lt} \frac{f(x+h)-f(x)}{h}$
- ★ $= \underset{h\to 0}{Lt} \frac{\sin(2x+2h)-\sin 2x}{h}$
- ★ $= \underset{h\to 0}{Lt} \frac{1}{h}\left(2\cos\left(\frac{(2x+2h)+2x}{2}\right)\sin\left(\frac{(\cancel{2x}+2h)-\cancel{2x}}{2}\right)\right)$ $\left[\because \sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\right]$
- ★ $=2\underset{h\to 0}{Lt}\frac{1}{h}\cos\left(\frac{4x+2h}{2}\right)\sin\left(\frac{\cancel{2}h}{\cancel{2}}\right)$
- ★ $=2\underset{h\to 0}{Lt}\frac{1}{h}\cos\left(\frac{\cancel{2}(2x+h)}{\cancel{2}}\right)\sin(h)$
- $=2\underset{h\to 0}{Lt}\frac{1}{h}\cos(2x+h)\sin(h)$
- ★ $=2\underset{h\to 0}{Lt}\cos(2x+h).\underset{h\to 0}{Lt}\frac{\sinh}{h}$
- ★ $=2\cos(2x+0)(1)=2\cos 2x$

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16. Find the equations of the tangent and the normal to the curve $y = x^3 + 4x^2$ at (–1, 3)

Sol:
- Given curve is $y = x^3 + 4x^2$,
- $\Rightarrow \frac{dy}{dx}=3x^2+8x$
- ∴ Slope of the tangent at P(–1, 3) is
- ★ $m = 3(-1)^2 + 8(-1) = 3-8 = -5$
- (i) Tangent at (–1, 3) with slope –5 is
  - ★ $y - y_1 = m(x - x_1)$
  - $\Rightarrow y - 3 = -5(x + 1) = -5x - 5 \Rightarrow 5x + y + 2 = 0$
- ★ (ii) Slope of the normal is $\frac{-1}{m}=\frac{-1}{-5}=\frac{1}{5}$
  - Normal at (–1,3) with slope $\frac{1}{5}$ is
  - ★ $y-y_1=-\frac{1}{m}(x-x_1)$
  - ★ $\Rightarrow y-3=\frac{1}{5}(x+1)$
  - $\Rightarrow 5y-15=x+1\Rightarrow x-5y+16=0$

17. The displacement s of a particle travelling in a straight line in t seconds is given by $s = 45t + 11t^2 - t^3$. Find the time when the particle comes to rest.

Sol : The distance-time relation is given by $s = 45t + 11t^2 - t^3 \Rightarrow v = \frac{ds}{dt} = 45 + 22t - 3t^2$

If the particle comes to rest then $v = 0$

$\Rightarrow 45 + 22t - 3t^2 = 0 \Rightarrow 3t^2 - 22t - 45 = 0$

$\Rightarrow 3t^2 - 27t + 5t - 45 = 0 \Rightarrow 3t(t-9) + 5(t-9) = 0 \Rightarrow (3t+5)(t-9) = 0$

$\Rightarrow t = 9$ or $t = -5/3$

$\therefore t = 9$ ($\because$ t cannot be negative)

$\therefore$ The particle becomes to rest at $t = 9$ sec

SECTION-C

18. Find the circumcentre of the triangle whose vertices are (1,3), (–3,5),(5,–1).

Sol:
- Take S(x, y) as circumcentre
- Vertices A = (1, 3), B = (–3,5), C = (5,–1).
- We know SA = SB = SC

Step-1: Take SA = SB

$$\Rightarrow \sqrt{(x-1)^2+(y-3)^2} = \sqrt{(x+3)^2+(y-5)^2}$$

Squaring on both sides, we get

$$(x-1)^2 + (y-3)^2 = (x+3)^2 + (y-5)^2$$

$$\Rightarrow (\cancel{x^2}+1-2x)+(\cancel{y^2}+9-6y) = (\cancel{x^2}+9+6x)+(\cancel{y^2}+25-10y)$$

$$\Rightarrow 6x+2x-10y+6y+25\cancel{+9}\cancel{-9}-1=0$$

$$\Rightarrow 8x-4y+24=0 \Rightarrow \cancel{4}(2x-y+6)=0$$

$$\Rightarrow 2x - y + 6 = 0 \text{ ....(1)}$$

Step-2: Take SB = SC

$$\Rightarrow \sqrt{(x+3)^2+(y-5)^2} = \sqrt{(x-5)^2+(y+1)^2}$$

Squaring on both sides, we get

$$(x+3)^2 + (y-5)^2 = (x-5)^2 + (y+1)^2$$

$$\Rightarrow (\cancel{x^2}+9+6x)+(\cancel{y^2}+25-10y) = (\cancel{x^2}+25-10x)+(\cancel{y^2}+1+2y)$$

$$\Rightarrow 6x+10x-10y-2y+9\cancel{+25}\cancel{-25}-1=0$$

$$\Rightarrow 16x-12y+8=0 \Rightarrow \cancel{4}(4x-3y+2)=0$$

$$\Rightarrow 4x - 3y + 2 = 0 \text{ ....(2)}$$

Step-3: Solving (1) and (2), we get S;

$(1) \Rightarrow 2x - y + 6 = 0;$

$(2) \Rightarrow 4x - 3y + 2 = 0$

$$\Rightarrow \frac{x}{(-1)2-(-3)6} = \frac{y}{6(4)-2(2)} = \frac{1}{2(-3)-4(-1)}$$

$$\Rightarrow \frac{x}{-2+18} = \frac{y}{24-4} = \frac{1}{-6+4}$$

$$\Rightarrow \frac{x}{16} = \frac{y}{20} = \frac{1}{-2}$$

$$\Rightarrow x = \frac{-16}{2} = -8,\ y = \frac{-20}{2} = -10$$

$\therefore$ Circumcentre S (x, y) = (–8, –10)

19. If θ is the angle between the pair of lines $ax^2 + 2hxy + by^2 = 0$, then $\cos\theta = \frac{a+b}{\sqrt{(a-b)^2+4h^2}}$

Sol: Let the separate equations of $ax^2 + 2hxy + by^2 = 0$ be $l_1x + m_1y = 0$ .....(1) and

$l_2x + m_2y = 0$ ......(2)

$\therefore ax^2 + 2hxy + by^2 \equiv (l_1x + m_1y)(l_2x + m_2y)$

Comparing both sides, we get $l_1l_2 = a$, $l_1m_2 + l_2m_1 = 2h$, $m_1m_2 = b$.

If θ is an angle between the lines (1) and (2), then

$$\cos\theta = \frac{l_1l_2 + m_1m_2}{\sqrt{(l_1^2+m_1^2).(l_2^2+m_2^2)}} = \frac{l_1l_2 + m_1m_2}{\sqrt{l_1^2l_2^2 + m_1^2m_2^2 + l_1^2m_2^2 + l_2^2m_1^2}}$$

$$= \frac{l_1l_2 + m_1m_2}{\sqrt{(l_1l_2 - m_1m_2)^2 + \cancel{2l_1l_2m_1m_2} + (l_1m_2 + l_2m_1)^2 - \cancel{2l_1l_2m_1m_2}}}$$

$$= \frac{a+b}{\sqrt{(a-b)^2 + (2h)^2}} = \frac{a+b}{\sqrt{(a-b)^2 + 4h^2}} \qquad \left[\begin{array}{l} \because a^2+b^2 = (a-b)^2 + 2ab \\ a^2+b^2 = (a+b)^2 - 2ab \end{array}\right]$$

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20. Show that the lines joining the origin to the points of intersection of the curve $x^2 - xy + y^2 + 3x + 3y - 2 = 0$ and the straight line $x - y - \sqrt{2} = 0$ are mutually perpendicular.

Sol:
- Given line is $x - y = \sqrt{2} \Rightarrow \frac{x-y}{\sqrt{2}} = 1$ ...(1)
- Given curve is $x^2 - xy + y^2 + 3x + 3y - 2 = 0$..........(2)
- Homogenising (1)&(2), we get

$x^2 - xy + y^2 + 3x(1) + 3y(1) - 2(1)^2 = 0$

$$\Rightarrow x^2 - xy + y^2 + 3x\left(\frac{x-y}{\sqrt{2}}\right) + 3y\left(\frac{x-y}{\sqrt{2}}\right) - \cancel{2}\frac{(x-y)^2}{\cancel{2}} = 0$$

$$\Rightarrow \frac{\sqrt{2}x^2 - \sqrt{2}xy + \sqrt{2}y^2 + 3x(x-y) + 3y(x-y) - \sqrt{2}(x-y)^2}{\sqrt{2}} = 0$$

$$\Rightarrow \sqrt{2}x^2 - \sqrt{2}xy + \sqrt{2}y^2 + 3x^2 \cancel{-3xy} \cancel{+3yx} - 3y^2 - \sqrt{2}(x^2 + y^2 - 2xy) = 0$$

$$\Rightarrow \sqrt{2}x^2 - \sqrt{2}xy + \sqrt{2}y^2 + 3x^2 - 3y^2 - \sqrt{2}x^2 - \sqrt{2}y^2 + 2\sqrt{2}xy = 0$$

$$\Rightarrow 3x^2 - 3y^2 + \sqrt{2}xy = 0$$

- Here, coeff. of x^2 + coeff. of y^2 is $3 - 3 = 0$

$\therefore$ The pair of lines are perpendicular

21. Find the angle between the lines whose d.c's are related by $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$

Sol:
- Given $l + m + n = 0 \Rightarrow l = -(m+n)$.....(1),
- $l^2 + m^2 - n^2 = 0$ ......(2)
- Solving (1) & (2) we get

$[-(m+n)]^2 + m^2 - n^2 = 0 \Rightarrow (m^2 + n^2 + 2mn) + m^2 - n^2 = 0$

$\Rightarrow 2m^2 + 2mn = 0 \Rightarrow 2(m^2 + mn) = 0$

$\Rightarrow m^2 + mn = 0 \Rightarrow m(m+n) = 0$

★ $\Rightarrow m = 0$ (or) $m + n = 0 \Rightarrow m = 0$ (or) $m = -n$

- **Case (i):** Put $m = 0$ in (1), then

$l = -(0 + n) = -n$

$\therefore l = -n$

Now, $l : m : n = -n : 0 : n = -1 : 0 : 1$

★ So, d.r's of $L_1 = (a_1, b_1, c_1) = (-1, 0, 1)$ .....(3)

Case (ii): Put $m = -n$ in (1), then

$l = -(-n + n) = 0$

$\therefore l = 0$

Now, $l : m : n = 0 : -n : n = 0 : -1 : 1$

So, d.r's of $L_2 = (a_2, b_2, c_2) = (0, -1, 1)$......(4)

- If θ is the angle between the lines, then from (3), (4), we get

★ $$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{(a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2)}} = \frac{|(-1)(0) + (0)(-1) + 1(1)|}{\sqrt{((-1)^2 + 0^2 + 1^2)(0^2 + (-1)^2 + 1^2)}}$$

★ $$= \frac{1}{\sqrt{(2)(2)}} = \frac{1}{\sqrt{4}} = \frac{1}{2} = \cos 60^\circ \Rightarrow \theta = 60^\circ$$

Hence, angle between the lines is 60°.

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22. If $x^{\log y} = \log x$ then find $\frac{dy}{dx}$

Sol : Given that $x^{\log y} = \log x \Rightarrow \log(x^{\log y}) = (\log y)(\log x) \Rightarrow \log y(\log x) = \log(\log x)$

Differentiating w.r.t x, we have

$$\log y\left(\frac{1}{x}\right) + \log x\left(\frac{1}{y}\frac{dy}{dx}\right) = \frac{1}{\log x}\left(\frac{1}{x}\right) \Rightarrow \frac{\log y}{x} + \left(\frac{\log x}{y}\frac{dy}{dx}\right) = \frac{1}{x\log x}$$

$$\Rightarrow \left(\frac{\log x}{y}\right)\frac{dy}{dx} = \frac{1}{x\log x} - \frac{\log y}{x} = \frac{1}{x}\left(\frac{1}{\log x} - \log y\right)$$

$$\Rightarrow \frac{x}{y}\frac{dy}{dx} = \frac{1}{\log x}\left(\frac{1 - \log x \log y}{\log x}\right) = \frac{1 - \log x \log y}{(\log x)^2} \Rightarrow \frac{dy}{dx} = \frac{y}{x}\left(\frac{1 - \log x \log y}{(\log x)^2}\right)$$

23. At any point t on the curve x = a(t + sint), y = a(1 – cost), find the lengths of tangent, normal, subtangent and subnormal.

Sol:
- Given x = a(t + sin t), on diff. w.r.to t, we get
- $\frac{dx}{dt} = a\frac{d}{dt}(t + \sin t) = a(1 + \cos t)$
- Also given y = a(1–cost), on diff. w.r.to t, we get
- $\frac{dy}{dt} = a\frac{d}{dt}(1 - \cos t) = a(0 + \sin t) = a(\sin t)$

★ $\therefore m = \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{a(\sin t)}{a(1+\cos t)} = \frac{\sin t}{1+\cos t} = \frac{2\sin\frac{t}{2}.\cos\frac{t}{2}}{2\cos^2\frac{t}{2}} = \frac{\sin\frac{t}{2}}{\cos\frac{t}{2}} = \tan\frac{t}{2}$

★ So, $m = \tan\frac{t}{2}$ and given y = a(1–cos t)

★ **(i) Length of tangent** $\left|\frac{y\sqrt{1+m^2}}{m}\right|$

★ $= \left|\frac{a(1-\cos t)\sqrt{1+\tan^2\frac{t}{2}}}{\tan\frac{t}{2}}\right| = \left|\frac{a.\left(2\sin^2\frac{t}{2}\right)\sec\frac{t}{2}}{\tan\frac{t}{2}}\right|$

★ $= \left|2a.\sin^2\frac{t}{2}\sec\frac{t}{2}.\cot\frac{t}{2}\right| = \left|2a.\sin^2\frac{t}{2}.\frac{1}{\cos\frac{t}{2}}.\frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}\right| = \left|2a\sin\frac{t}{2}\right|$

★ **(ii) Length of normal** $= \left|y\sqrt{1+m^2}\right|$

★ $= \left|a(1-\cos t)\sqrt{1+\tan^2\frac{t}{2}}\right| = \left|a.\left(2\sin^2\frac{t}{2}\right).\sec\frac{t}{2}\right|$

★ $= \left|2a\sin^2\frac{t}{2}.\frac{1}{\cos\frac{t}{2}}\right| = \left|2a\sin\frac{t}{2}.\frac{\sin\frac{t}{2}}{\cos\frac{t}{2}}\right| = \left|2a\sin\frac{t}{2}.\tan\frac{t}{2}\right|$

★ **(iii) Length of subtangent** $\left|\frac{y}{m}\right|$

★ $= \left|\frac{a(1-\cos t)}{\tan\frac{t}{2}}\right| = \left|a.\left(2\sin^2\frac{t}{2}\right)\cot\frac{t}{2}\right| = \left|a.2\sin^2\frac{t}{2}\frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}\right| = \left|a.2\sin\frac{t}{2}.\cos\frac{t}{2}\right| = |a\sin t|$

★ **(iv) Length of subnormal** $= |y.m| = \left|a(1-\cos t)\tan\frac{t}{2}\right| = \left|a\left(2\sin^2\frac{t}{2}\right)\tan\frac{t}{2}\right| = \left|2a\sin^2\frac{t}{2}\tan\frac{t}{2}\right|$

24. Find two positive integers whose sum is 15 and the sum of whose squares is minimum.

Sol: • Let the two positive numbers be x,y

1) Given that $x + y = 15$

$\Rightarrow y = 15 - x$...........(1)

2) Let $f(x) = x^2 + y^2 = x^2 + (15 - x)^2$

$\therefore f(x) = x^2 + (15 - x)^2$......(2)

3) Diff. (2) w.r.t x, we get

$f'(x) = 2x + 2(15 - x)(-1) = 2x - 30 + 2x$

$= 4x - 30 = 2(2x - 15)$........(3)

4) At max. or min. we have $f'(x) = 0$

$\Rightarrow 2x - 15 = 0 \quad \Rightarrow x = 15/2$

5) Now, Diff. (3) w.r.t x,we get $f''(x)=4$ ...(4)

6) At $x = 15/2$, from (4), $f''(15/2) = 4 > 0$

7) $\therefore$ f(x) is minimum when $x = 15/2$ and

$$y = 15 - \frac{15}{2} = \frac{15}{2}$$

$\therefore$ Required numbers are $x = \frac{15}{2}, y = \frac{15}{2}$