Previous IPE

SOLVED PAPERS

MARCH-2025 (TS)

PREVIOUS PAPERS

IPE: MARCH-2025(TS)

Time: 3 Hours

MATHS-1A

Max. Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: $10 \times 2 = 20$

1. If $f: R \to R$ is $f(x) = \frac{1-x^2}{1+x^2}$, then show that $f(\tan\theta) = \cos 2\theta$.
2. If $f: R \to R$, $g: R \to R$ are defined by $f(x) = 2x^2 + 3$ and $g(x) = 3x - 2$ then find $(fog)(x)$
3. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = 0$, then find the value of k.
4. Find the rank of the matrix $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$
5. If vectors $-3\bar{i} + 4\bar{j} + \lambda\bar{k}$, $\mu\bar{i} + 8\bar{j} + 6\bar{k}$ are collinear vectors then find λ & μ.
6. Find the vector equation of the plane passing through the points $\bar{i} - 2\bar{j} + 5\bar{k}$, $-5\bar{j} - \bar{k}$, $-3\bar{i} + 5\bar{j}$
7. For what values of λ the vectors $\bar{i} - \lambda\bar{j} + 2\bar{k}$, $8\bar{i} + 6\bar{j} - \bar{k}$ are at right angles.
8. Eliminate 'θ' from $x = a\cos^3\theta$, $y = b\sin^3\theta$.
9. Find a sine function whose period is 2/3
10. Prove that $(\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$

SECTION-B

II. Answer any FIVE of the following SAQs: $5 \times 4 = 20$

11. If $A = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 0 \\ 3 & -1 & -5 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & 0 \\ 0 & -2 & 5 \\ 1 & 2 & 0 \end{bmatrix}$ then find $3A - 4B'$.
12. Show that $A(2\bar{i} - \bar{j} + \bar{k})$, $B(\bar{i} - 3\bar{j} - 5\bar{k})$, $C(3\bar{i} - 4\bar{j} - 4\bar{k})$ are the vertices of a right angled triangle.
13. Find unit vector perpendicular to the plane passing through the points (1,2,3),(2,–1,1) and (1,2,–4)
14. Show that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$
15. Solve $2\cos^2\theta + 11\sin\theta = 7$
16. Prove that $\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$
17. If $\sin\theta = \frac{a}{(b+c)}$, then show that $\cos\theta = \frac{2\sqrt{bc}}{b+c}\cos\left(\frac{A}{2}\right)$

SECTION-C

III. Answer any FIVE of the following LAQs: $5 \times 7 = 35$

18. If $f: A \to B$, $g: B \to C$ are two bijective functions then prove that $(gof)^{-1} = f^{-1}og^{-1}$
19. Using the Principle of Mathematical Induction, $\forall n \in N$, prove that $1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$
20. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$
21. Solve the following system of equations $x - y + 3z = 5$, $4x + 2y - z = 0$, $-x + 3y + z = 5$ by using Gauss-Jordan method.
22. If $\bar{a} = \bar{i} - 2\bar{j} + \bar{k}$, $\bar{b} = 2\bar{i} + \bar{j} + \bar{k}$, $\bar{c} = \bar{i} + 2\bar{j} - \bar{k}$ then find $\bar{a} \times (\bar{b} \times \bar{c})$ and $|(\bar{a} \times \bar{b}) \times \bar{c}|$
23. If A, B, C are angles of a triangle, then S.T $\sin 2A - \sin 2B + \sin 2C = 4\cos A \sin B \cos C$
24. In ΔABC, prove that $r(r_1 + r_2 + r_3) = ab + bc + ca - s^2$.

IPE TS MARCH-2025

SOLUTIONS

SECTION-A

1. If $f : R \to R$ is $f(x) = \frac{1-x^2}{1+x^2}$ then show that $f(\tan\theta) = \cos 2\theta$.

Sol: Given $f(x) = \frac{1-x^2}{1+x^2}$ $\therefore f(\tan\theta) = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$. [Formula from Trigonometry]

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2. If $f : R \to R$, $g : R \to R$ are defined by $f(x) = 2x^2 + 3$ and $g(x) = 3x - 2$ then find (fog)(x)

Sol: Given that $f(x) = 2x^2 + 3$; $g(x) = 3x - 2$

$(fog)(x) = f[g(x)] = f(3x-2)$ $[\because g(x) = 3x - 2]$

$= 2(3x-2)^2 + 3$ $[\because f(x) = 2x^2 + 3]$

$= 2(9x^2 - 12x + 4) + 3 = 18x^2 - 24x + 8 + 3 = 18x^2 - 24x + 11$

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3. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, then find the value of k

Sol: Given $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$

$\therefore A^2 = A \times A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}\begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix} = \begin{bmatrix} 2(2)+4(-1) & 2(4)+4(k) \\ -1(2)+k(-1) & -1(4)+k(k) \end{bmatrix}$

$= \begin{bmatrix} 4-4 & 8+4k \\ -2-k & -4+k^2 \end{bmatrix} = \begin{bmatrix} 0 & 8+4k \\ -2-k & -4+k^2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 & 8+4k \\ -2-k & -4+k^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $[\because A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}]$

$\Rightarrow 8 + 4k = 0 \Rightarrow \not{4}k = -\not{8} \Rightarrow k = -2$

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4. Find the rank of the matrix $\begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

Sol: Let $A = \begin{bmatrix} 1 & 4 & -1 \\ 2 & 3 & 0 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow |A| = 1(6-0)-4(4-0)-1(2-0) = 6-16-2 = -12 \neq 0$ $\therefore$ Rank (A) = 3

5. If $-3\bar{i}+4\bar{j}+\lambda\bar{k}$, $\mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear vectors then find λ & μ.

Sol: Given that the vectors $\bar{a}=-3\bar{i}+4\bar{j}+\lambda\bar{k}$, $\bar{b}=\mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear.

$$\therefore \frac{-3}{\mu}=\frac{4}{8}=\frac{\lambda}{6}$$

$$\Rightarrow \frac{-3}{\mu}=\frac{1}{2}\Rightarrow \mu=2\times -3=-6 \text{ and } \frac{\lambda}{6}=\frac{1}{2}\Rightarrow \lambda=\frac{6}{2}=3 \qquad \therefore \lambda=3,\ \mu=-6$$

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6. Find the vector equation of the plane passing through the points $\bar{i}-2\bar{j}+5\bar{k}$, $-5\bar{j}-\bar{k}$, $-3\bar{i}+5\bar{j}$

Sol: Given $A(\bar{a})=\bar{i}-2\bar{j}+5\bar{k}$, $B(\bar{b})=-5\bar{j}-\bar{k}$, $C(\bar{c})=-3\bar{i}+5\bar{j}$

Vector equation of the plane is $\bar{r}=(1-s-t)\bar{a}+s\bar{b}+t\bar{c}$, s, t $\in$ R

$\therefore\ \bar{r}=(1-s-t)(\bar{i}-2\bar{j}+5\bar{k})+s(-5\bar{j}-\bar{k})+t(-3\bar{i}+5\bar{j})$, s, t $\in$ R

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7. For what values of λ the vectors $\bar{i}-\lambda\bar{j}+2\bar{k}$, $8\bar{i}+6\bar{j}-\bar{k}$ are at right angles.

Sol: Let $\bar{a}=\bar{i}-\lambda\bar{j}+2\bar{k}$, $\bar{b}=8\bar{i}+6\bar{j}-\bar{k}$

If $\bar{a}, \bar{b}$ are at right angle then $\bar{a}.\bar{b}=0$

$\Rightarrow(\bar{i}-\lambda\bar{j}+2\bar{k}).(8\bar{i}+6\bar{j}-\bar{k})=0 \Rightarrow 8-6\lambda-2=0 \Rightarrow 6\lambda=6\Rightarrow \lambda=1$

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8. Eliminate 'θ' from x = acos³θ, y = bsin³θ.

Sol: Given x = acos³θ, y = bsin³θ $\Rightarrow \cos^3\theta=\frac{x}{a}, \sin^3\theta=\frac{y}{b} \Rightarrow \cos\theta=\left(\frac{x}{a}\right)^{1/3}, \sin\theta=\left(\frac{y}{b}\right)^{1/3}$

Now, $\cos^2\theta+\sin^2\theta=1 \Rightarrow \left(\left(\frac{x}{a}\right)^{1/3}\right)^2+\left(\left(\frac{y}{b}\right)^{1/3}\right)^2=1 \Rightarrow \left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$

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9. Find a sine function whose period is 2/3

Sol: Let the required sine function be sinkx.

Period of sinkx $=\frac{2}{3} \Rightarrow \frac{2\pi}{|k|}=\frac{2}{3}\Rightarrow k=\pm 3\pi$ $\qquad\therefore$ Required sine function is $\sin 3\pi x$

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10. Prove that $(\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$

Sol: L.H.S $= (\cosh x - \sinh x)^n = \left[\frac{e^x + e^{-x}}{2} - \frac{e^x - e^{-x}}{2}\right]^n$

$$= \left(\frac{\cancel{e^x} + e^{-x} - \cancel{e^x} + e^{-x}}{2}\right)^n = \left(\frac{\cancel{2}e^{-x}}{\cancel{2}}\right)^n = e^{-nx} \text{.............}(1)$$

R.H.S $= \cosh nx - \sinh nx = \left(\frac{e^{nx} + e^{-nx}}{2}\right) - \left(\frac{e^{nx} - e^{-nx}}{2}\right)$

$$= \frac{\cancel{e^{nx}} + e^{-nx} \cancel{-e^{nx}} + e^{-nx}}{2} = \frac{\cancel{2}e^{-nx}}{\cancel{2}} = e^{-nx} \text{.............}(2)$$

From (1) & (2), $(\cosh x - \sinh x)^n = \cosh nx - \sinh nx$

SECTION-B

11. If $A=\begin{bmatrix}1 & 5 & 3\\ 2 & 4 & 0\\ 3 & -1 & -5\end{bmatrix}, B=\begin{bmatrix}2 & -1 & 0\\ 0 & -2 & 5\\ 1 & 2 & 0\end{bmatrix}$ **then find 3A–4B'.**

Sol: $B=\begin{bmatrix}2 & -1 & 0\\ 0 & -2 & 5\\ 1 & 2 & 0\end{bmatrix} \Rightarrow B'=\begin{bmatrix}2 & -1 & 0\\ 0 & -2 & 5\\ 1 & 2 & 0\end{bmatrix}^T=\begin{bmatrix}2 & 0 & 1\\ -1 & -2 & 2\\ 0 & 5 & 0\end{bmatrix}$

$$3A-4B'=3\begin{bmatrix}1 & 5 & 3\\ 2 & 4 & 0\\ 3 & -1 & -5\end{bmatrix}-4\begin{bmatrix}2 & 0 & 1\\ -1 & -2 & 2\\ 0 & 5 & 0\end{bmatrix}=\begin{bmatrix}3 & 15 & 9\\ 6 & 12 & 0\\ 9 & -3 & -15\end{bmatrix}-\begin{bmatrix}8 & 0 & 4\\ -4 & -8 & 8\\ 0 & 20 & 0\end{bmatrix}$$

$$=\begin{bmatrix}3-8 & 15-0 & 9-4\\ 6+4 & 12+8 & 0-8\\ 9-0 & -3-20 & -15-0\end{bmatrix}=\begin{bmatrix}-5 & 15 & 5\\ 10 & 20 & -8\\ 9 & -23 & -15\end{bmatrix}$$

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12. Show that the points $A(2\bar{i}-\bar{j}+\bar{k})$, $B(\bar{i}-3\bar{j}-5\bar{k})$, $C(3\bar{i}-4\bar{j}-4\bar{k})$ are the vertices of a right angled triangle.

Sol: We take $\overline{OA}=(2\bar{i}-\bar{j}+\bar{k})$, $\overline{OB}=(\bar{i}-3\bar{j}-5\bar{k})$, $\overline{OC}=(3\bar{i}-4\bar{j}-4\bar{k})$, where 'O' is the origin.

$\overline{AB}=\overline{OB}-\overline{OA}=(\bar{i}-3\bar{j}-5\bar{k})-(2\bar{i}-\bar{j}+\bar{k})=-\bar{i}-2\bar{j}-6\bar{k}$

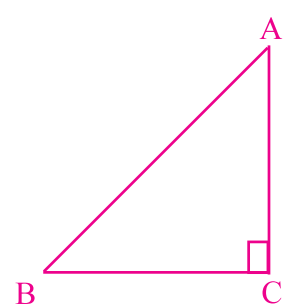
$\Rightarrow |\overline{AB}|=\sqrt{(-1)^2+(-2)^2+(-6)^2}=\sqrt{1+4+36}=\sqrt{41}$

$\overline{BC}=\overline{OC}-\overline{OB}=(3\bar{i}-4\bar{j}-4\bar{k})-(\bar{i}-3\bar{j}-5\bar{k})=2\bar{i}-\bar{j}+\bar{k}$

$\Rightarrow |\overline{BC}|=\sqrt{2^2+(-1)^2+1^2}=\sqrt{4+1+1}=\sqrt{6}$

$\overline{CA}=\overline{OA}-\overline{OC}=(2\bar{i}-\bar{j}+\bar{k})-(3\bar{i}-4\bar{j}-4\bar{k})=-\bar{i}+3\bar{j}+5\bar{k}$

$\Rightarrow |\overline{CA}|=\sqrt{(-1)^2+3^2+5^2}=\sqrt{1+9+25}=\sqrt{35}$



Here, $|\overline{AB}|^2=41; |\overline{BC}|^2+|\overline{CA}|^2=6+35=41 \Rightarrow |\overline{AB}|^2=|\overline{BC}|^2+|\overline{CA}|^2$

$\therefore$ A, B, C are the vertices of a right angled triangle

13. Find unit vector perpendicular to the plane passing through the points (1, 2, 3), (2, –1, 1) and (1, 2, –4)

Sol: We take $\overline{OA} = \bar{i} + 2\bar{j} + 3\bar{k}, \overline{OB} = 2\bar{i} - \bar{j} + \bar{k}, \overline{OC} = \bar{i} + 2\bar{j} - 4\bar{k}$ where 'O' is the origin.

Now, $\overline{AB} = \overline{OB} - \overline{OA} = (2\bar{i} - \bar{j} + \bar{k}) - (\bar{i} + 2\bar{j} + 3\bar{k}) = \bar{i} - 3\bar{j} - 2\bar{k}$

$\overline{AC} = \overline{OC} - \overline{OA} = (\bar{i} + 2\bar{j} - 4\bar{k}) - (\bar{i} + 2\bar{j} + 3\bar{k}) = -7\bar{k}$

$$\Rightarrow \overline{AB} \times \overline{AC} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & -3 & -2 \\ 0 & 0 & -7 \end{vmatrix} = \bar{i}[(-3)(-7) - 0(-2)] - \bar{j}[1(-7) - (0)(-2)] + \bar{k}[1(0) - (0)(-3)]$$

$$= \bar{i}(21) - j(-7) = 21\bar{i} + 7\bar{j} = 7(3\bar{i} + \bar{j})$$

$$|\overline{AB} \times \overline{AC}| = 7\sqrt{3^2 + 1^2} = 7\sqrt{9+1} = 7\sqrt{10}$$

$$\therefore \text{ Required Unit vector} = \pm\frac{\overline{AB} \times \overline{AC}}{|\overline{AB} \times \overline{AC}|} = \pm\frac{\cancel{7}(3\bar{i} + \bar{j})}{\cancel{7}\sqrt{10}} = \pm\frac{1}{\sqrt{10}}(3\bar{i} + \bar{j})$$

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14. Show that $\frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ} = 4$

Sol: L.H.S $= \frac{1}{\sin 10^\circ} - \frac{\sqrt{3}}{\cos 10^\circ}$

$$= \frac{\cos 10^\circ - \sqrt{3}\sin 10^\circ}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{2(\frac{1}{2}\cos 10^\circ - \frac{\sqrt{3}}{2}\sin 10^\circ)}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{2(\sin 30^\circ \cos 10^\circ - \cos 30^\circ \sin 10^\circ)}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{2\sin(30^\circ - 10^\circ)}{\sin 10^\circ \cos 10^\circ} = \frac{2\sin 20^\circ}{\sin 10^\circ \cos 10^\circ}$$

$$= \frac{2.2\cancel{\sin 10^\circ \cos 10^\circ}}{\cancel{\sin 10^\circ \cos 10^\circ}}$$

$= 2.2 = 4 =$ R.H.S

15. Solve $2\cos^2\theta+11\sin\theta=7$

Sol: Given that $2\cos^2\theta + 11\sin\theta = 7$

$\Rightarrow 2(1 - \sin^2\theta) + 11\sin\theta - 7 = 0 \Rightarrow 2 - 2\sin^2\theta + 11\sin\theta - 7 = 0$

$\Rightarrow 2\sin^2\theta - 11\sin\theta + 5=0 \Rightarrow 2\sin^2\theta - 10\sin\theta - \sin\theta + 5 = 0$

$\Rightarrow 2\sin\theta(\sin\theta - 5) -1(\sin\theta - 5) = 0 \Rightarrow (2\sin\theta - 1)(\sin\theta - 5) = 0$

$\Rightarrow 2\sin\theta - 1 = 0$ or $\sin\theta - 5 = 0$

$\Rightarrow \sin\theta = 1/2$ or $\sin\theta = 5 > 1$, and hence this has no solution

$\therefore \sin\theta = 1/2 = \sin \pi/6$. Here P.V is $\alpha = \pi/6$.

G.S is given by $\theta = n\pi + (-1)^n \alpha, n\in Z$

$\Rightarrow \theta = \{n\pi + (-1)^n \pi/6, n\in Z\}$

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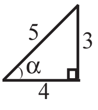
16. P.T. $\mathrm{Sin}^{-1}\frac{3}{5}+\mathrm{Cos}^{-1}\frac{12}{13}=\mathrm{Cos}^{-1}\frac{33}{65}$

Sol: Take $\mathrm{Sin}^{-1}\frac{3}{5}=\alpha$ and $\mathrm{Cos}^{-1}\frac{12}{13}=\beta$

Required To Prove (RTP):

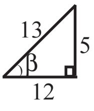
$\alpha+\beta=\mathrm{Cos}^{-1}\frac{33}{65}\Rightarrow \cos(\alpha+\beta)=\frac{33}{65}$

$\mathrm{Sin}^{-1}\frac{3}{5}=\alpha\Rightarrow \sin\alpha=\frac{3}{5}$



$\Rightarrow \cos\alpha=\frac{4}{5}$

$\mathrm{Cos}^{-1}\frac{12}{13}=\beta\Rightarrow \cos\beta=\frac{12}{13}$



$\Rightarrow \sin\beta=\frac{5}{13}$

$\therefore \cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$=\frac{4}{5}\times\frac{12}{13}-\frac{3}{5}\times\frac{5}{13}=\frac{48-15}{65}=\frac{33}{65}$

Hence, proved.

17. If $\sin\theta = \dfrac{a}{(b+c)}$ then show that $\cos\theta = \dfrac{2\sqrt{bc}}{b+c}\cos\left(\dfrac{A}{2}\right)$

Sol: Given $\sin\theta = \dfrac{a}{b+c}$

$$\Rightarrow \sin^2\theta = \frac{a^2}{(b+c)^2}$$

$\therefore \cos^2\theta = 1 - \sin^2\theta \; [\because \sin^2\theta + \cos^2\theta = 1]$

$$= 1 - \frac{a^2}{(b+c)^2} = \frac{(b+c)^2 - a^2}{(b+c)^2} = \frac{(b^2+c^2+2bc) - a^2}{(b+c)^2}$$

$$= \frac{2bc + (b^2+c^2-a^2)}{(b+c)^2} = \frac{2bc + 2bc\cos A}{(b+c)^2} \qquad \left(\because \frac{b^2+c^2-a^2}{2bc} = \cos A\right)$$

$$= \frac{2bc(1+\cos A)}{(b+c)^2} = \frac{2bc.2\cos^2\frac{A}{2}}{(b+c)^2}$$

$$= \frac{4bc\cos^2\frac{A}{2}}{(b+c)^2}$$

$$\therefore \cos\theta = \frac{2\sqrt{bc}}{b+c}\cos\left(\frac{A}{2}\right)$$

SECTION-C

18. If $f:A\to B$, $g:B\to C$ are two bijective functions then prove that $(gof)^{-1}=f^{-1}og^{-1}$

Sol: **Part -1:** Given that $f:A\to B$, $g:B\to C$ are two bijective functions, then

(i) $gof:A\to C$ is bijection $\Rightarrow (gof)^{-1}:C\to A$ is also a bijection

(ii) $f^{-1}:B\to A$, $g^{-1}:C\to B$ are both bijections $\Rightarrow (f^{-1}og^{-1}): C\to A$ is also a bijection.

So, $(gof)^{-1}$ and $f^{-1}og^{-1}$, both have same domain 'C'

Part−2: Given $f:A\to B$ is bijection, then $f(a) = b \Rightarrow a = f^{-1}(b)$........(1), [Here $a\in A$, $b\in B$]

$g:B\to C$ is bijection, then $g(b) = c \Rightarrow b = g^{-1}(c)$........(2), [Here $b\in B$, $c\in C$]

$gof:A\to C$ is bijection, then $gof(a) = c \Rightarrow a= (gof)^{-1}(c)$.........(3)

Now, $(f^{-1}og^{-1})(c) = f^{-1}[g^{-1}(c)] = f^{-1}(b) = a$ ....(4), [From (1) & (2)]

$\therefore (gof)^{-1}(c)=(f^{-1}og^{-1})(c), \forall c\in C$, [from (3) & (4)]

Hence, we proved that $(gof)^{-1}= f^{-1}og^{-1}$

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19. Using the Principle of Mathematical Induction, $\forall n\in N$, prove that

$$1^2+2^2+3^2+....+n^2=\frac{n(n+1)(2n+1)}{6}$$

Sol: Let $S(n): 1^2+2^2+3^2+....+n^2=\frac{n(n+1)(2n+1)}{6}$

Step-1: L.H.S of $S(1)=1^2=1$ and R.H.S of $S(1)=\frac{1(1+1)(2x1+1)}{6}=\frac{2x3}{6}=1$

$\therefore$ L.H.S of S(1) = R.H.S of S(1) $\Rightarrow$ S(1) is true

Step-2: Assume that S(k) is true for $k\in N$.

$$S(k): 1^2+2^2+....+k^2=\frac{k(k+1)(2k+1)}{6} \quad .....(1)$$

Step-3: We show that S(k+1) is true

S(k+1): $[1^2+2^2+....+k^2]+(k+1)^2=\frac{(k+1)(k+2)[2(k+1)+1]}{6}$

Now, L.H.S of S(k+1)$=[1^2+2^2+....+k^2]+(k+1)^2$

$=\frac{k(k+1)(2k+1)}{6}+(k+1)^2$, [From (1)]

$=\frac{k(k+1)(2k+1)+6(k+1)^2}{6}$

$$=\frac{(k+1)\left[k(2k+1)+6(k+1)\right]}{6}$$

$$=\frac{(k+1)[2k^2+k+6k+6]}{6}$$

$$=\frac{(k+1)[2k^2+7k+6]}{6}$$

$$=\frac{(k+1)(k+2)(2k+3)}{6}$$

$$=\frac{(k+1)(k+2)[2(k+1)+1]}{6} = \text{R.H.S of } S(k+1)$$

∴ L.H.S of S(k + 1) = R.H.S of S(k + 1)

⇒ S(k + 1) is true whenever S(k) is true

Hence, by the principle of finite Mathematical Induction, S(n) is true $\forall\, n \in N$

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20. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

Sol: L.H.S = $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$

$$=\begin{vmatrix} 2a+2b+2c & a & b \\ 2a+2b+2c & b+c+2a & b \\ 2a+2b+2c & a & c+a+2b \end{vmatrix} (\because C_1 \to C_1+C_2+C_3)$$

$$=2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$$=2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix} \begin{pmatrix}\because R_2 \to R_2-R_1 \\ R_3 \to R_3-R_1\end{pmatrix}$$

$$=2(a+b+c)1[(a+b+c)^2-0]=2(a+b+c)^3=\text{R.H.S}$$

21. Solve the equations x – y + 3z = 3, 4x + 2y – z = 0, –x + 3y + z = 5, by Gauss-Jordan method

Sol: Augumented Matrix of the given system of equations is $[AD]=\begin{bmatrix}1 & -1 & 3 & 5\\ 4 & 2 & -1 & 0\\ -1 & 3 & 1 & 5\end{bmatrix}$

$$\sim\begin{bmatrix}1 & -1 & 3 & 5\\ 0 & 6 & -13 & -20\\ 0 & 2 & 4 & 10\end{bmatrix}\begin{matrix}R_2\to R_2-4R_1\\ R_3\to R_3+R_1\end{matrix}\sim\begin{bmatrix}1 & -1 & 3 & 5\\ 0 & 6 & -13 & -20\\ 0 & 1 & 2 & 5\end{bmatrix}R_3\to R_3/2$$

$$\sim\begin{bmatrix}6 & 0 & 5 & 10\\ 0 & 6 & -13 & -20\\ 0 & 0 & 25 & 50\end{bmatrix}\begin{matrix}R_1\to 6R_1+R_2\\ \\ R_3\to 6R_3-R_2\end{matrix}\sim\begin{bmatrix}6 & 0 & 5 & 10\\ 0 & 6 & -13 & 20\\ 0 & 0 & 1 & 2\end{bmatrix}R_3\to R_3/25$$

$$\sim\begin{bmatrix}6 & 0 & 0 & 0\\ 0 & 6 & 0 & 6\\ 0 & 0 & 1 & 2\end{bmatrix}\begin{matrix}R_1\to R_1-5R_3\\ R_2\to R_2+13R_3\end{matrix}\sim\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 1\\ 0 & 0 & 1 & 2\end{bmatrix}\begin{matrix}R_1\to R_1/6\\ R_2\to R_2/6\end{matrix}\quad\text{................(I)}$$

Thus [AD] is reduced to Form I of Gauss Jordan Solution.
Hence, there exists a Unique solution.
$\therefore$ From (I), the solution is x = 0, y = 1, z = 2.

22. If $\bar{a}=\bar{i}-2\bar{j}+\bar{k}, \bar{b}=2\bar{i}+\bar{j}+\bar{k}, \bar{c}=\bar{i}+2\bar{j}-\bar{k}$ then find $\bar{a}\times(\bar{b}\times\bar{c})$ and $|(\bar{a}\times\bar{b})\times\bar{c}|$.

Sol: Given $\bar{a}=\bar{i}-2\bar{j}+\bar{k}, \bar{b}=2\bar{i}+\bar{j}+\bar{k}, \bar{c}=\bar{i}+2\bar{j}-\bar{k}$

1) To find $\bar{a}\times(\bar{b}\times\bar{c})$, first we have to find $\bar{b}\times\bar{c}$ (term in the bracket)

$$\bar{b}\times\bar{c}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 2 & 1 & 1\\ 1 & 2 & -1\end{vmatrix}$$

$$=\bar{i}(-1-2)-\bar{j}(-2-1)+\bar{k}(4-1)$$

$$=-3\bar{i}+3\bar{j}+3\bar{k}$$

$$\therefore\ \bar{a}\times(\bar{b}\times\bar{c})=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & -2 & 1\\ -3 & 3 & 3\end{vmatrix}$$

$$=\bar{i}(-6-3)-\bar{j}(3+3)+\bar{k}(3-6)$$

$$=-9\bar{i}-6\bar{j}-3\bar{k}$$

2) To find $(\bar{a}\times\bar{b})\times\bar{c}$, we have to find $\bar{a}\times\bar{b}$ (term in the bracket)

$$\bar{a}\times\bar{b}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & -2 & 1\\ 2 & 1 & 1\end{vmatrix}$$

$$=\bar{i}(-2-1)-\bar{j}(1-2)+\bar{k}(1+4)=-3\bar{i}+\bar{j}+5\bar{k}$$

$$\therefore(\bar{a}\times\bar{b})\times\bar{c}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ -3 & 1 & 5\\ 1 & 2 & -1\end{vmatrix}$$

$$=\bar{i}(-1-10)-\bar{j}(3-5)+\bar{k}(-6-1)$$

$$=-11\bar{i}+2\bar{j}-7\bar{k}$$

$$\therefore|(\bar{a}\times\bar{b})\times\bar{c}|=|-11\bar{i}+2\bar{j}-7\bar{k}|$$

$$=\sqrt{(-11)^2+(2)^2+(-7)^2}=\sqrt{121+4+49}=\sqrt{174}$$

23. If A,B,C are angles of a triangle, then S.T sin2A−sin2B+sin2C=4cosAsinBcosC

Sol: Given A,B,C are angles of a triangle, then A+B+C=180º

L.H.S = sin2A−sin2B+sin2C

$= 2\cos\left(\frac{2A+2B}{2}\right)\sin\left(\frac{2A-2B}{2}\right) + \sin 2C \quad \left(\because \sin C - \sin D = 2\cos\frac{C+D}{2}\sin\frac{C-D}{2}\right)$

= 2cos(A+B)sin(A−B)+sin2C

= 2cos(180º–C)sin(A−B)+sin2C

= −2cosC sin(A−B)+2sinCcosC [∵ sin2θ=2sinθcosθ]

= 2cosC[sinC−sin(A−B)]

= 2cosC[sin(180º–(A+B))−sin(A−B)]

= 2cosC[sin(A+B)−sin(A−B)]

= 2cosC (2cosA sinB) [∵ sin(A+B)−sin(A−B)=2cosAsinB]

= 4cosA sinB cosC=R.H.S

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24. Show that $r(r_1 + r_2 + r_3) = ab + bc + ca - s^2$.

Sol: $L.H.S = r(r_1 + r_2 + r_3) = \frac{\Delta}{s}\left(\frac{\Delta}{s-a} + \frac{\Delta}{s-b} + \frac{\Delta}{s-c}\right) = \frac{\Delta^2}{s}\left(\frac{(s-b)(s-c)+(s-a)(s-c)+(s-a)(s-b)}{(s-a)(s-b)(s-c)}\right)$

$= \frac{\Delta^2(s^2 - s(b+c) + bc + s^2 - s(a+c) + ac + s^2 - s(a+b) + ab)}{\Delta^2}$

$= 3s^2 - s(b+c+a+c+a+b) + ab + bc + ca = 3s^2 - s(2(a+b+c)) + ab + bc + ca$

$= 3s^2 - 2s(2s) + ab + bc + ca = 3s^2 - 4s^2 + ab + bc + ca = ab + bc + ca - s^2 = R.H.S$