10. RANDOM VARIABLES AND DISTRIBUTION

Sections	No. of periods (10)	Weightage in IPE [1x2 + 1x7 =9]
1. Random Variables	4	7 marks
2. Binomial Distribution, Poisson Distribution	6	2 or 7 marks

Random variable is a Real Function defined on the sample space of a random experiment i.e., Random variable is a function whose values are determined by chance. The concept of Random variable converts the discriptive language of elementary events into correspondingly "a unique 'number line'language". The Random variables are usually denoted by capital letter such as X,Y,Z. The real valued function defined on a discrete sample space is called discrete random variable and it takes only a discrete set of real values. The information given in the table with the values of the random variable X and its functional values P(X), is called probability distribution of X.

There is resemblance between probability distribution and frequency distribution. The probability distribution spells out how a total probability of 1 is distributed over several values of the random variable; while a frequency distribution spells out, how a total frequency of n is distributed over several classes.

The 'mean' of a random variable X is, the mean of the probability distribution of a random variable. Mean of a random variable is also called as average or expected value.

The Binomial distribution also known as Bernoulli distribution, is associated with the name of Swiss Mathematician Jacob Bernoulli (1654-1705). A trail of a random experiment having only 2 possible out comes viz success(p) and failure(q), is called a Bernoulli Experiment. The random variable X of a Bernoulli experiment takes only 2 values viz 1 and 0 with the probabilities p,q of a Binomial variate.

Poisson Distribution is a discrete probability distribution and is very widely used in statistical work. It was developed by a French Mathematician Simon Denis Poisson in 1837. If the number of trails of an experiment is sufficiently large with very small probability of success then we prefer the Poisson distribution. Poisson distribution is a limiting case of Binomial distribution.

SYNOPSIS POINTS

1. A real value function $X : S \to R$ is called a random variable (r.v) defined by $X(\omega)=x$ for $\omega \in S$ and $x \in R$.

2. **Mean**: Let $X: S \to R$ be a discrete random variable, with range $\{x_1, x_2 ...\}$ then the mean of X if exists, denoted by μ or $\overline{x}$ is given by $\mu = \Sigma xi\, P(X=x_i)$ i.e., $\mu = \Sigma x_i.P(x_i)$.

3. **Variance**: The variance, if exists, is given by $\sigma^2 = \Sigma x_i^2 P(X=x_i) - \mu^2 = \Sigma (x_i - \mu)^2.P(X=x_i)$

4. **Standard Deviation**: The Standard Deviation (S.D) of the random variable X is the non negative value of the square root of the variance.

5. Let n be a positive integer and p be a real number such that $0 \le p \le 1$. A random variable X with range {0,1,2...n} is said to follow binomial distribution or Bernoulli distribution with parameters n and p if $P(X=r) = {}^nC_r p^r q^{n-r}$ for r=0,1,2....n where $q=1-p$.
   Here, the Binomial Distribution is given by $(q+p)^n$.

6. If the random variable X follows a binomial distribution with parameters n and p then (i) the mean of X is np (ii) the variance is npq where $q=1-p$.

7. Let $\lambda > 0$ be a real number. A random variable X with range {0,1,2,....} is said to follow Poisson Distribution with parameter λ if $P(X=r) = \dfrac{e^{-\lambda}\lambda^r}{r!}$ for r=0,1,2.....

8. If a random variable X follows Poisson distribution with parameter λ, then the mean of X is λ and the variance of X is λ.

STAR-Q

1.

$X = x_i$	−3	−2	−1	0	1	2	3
$P(X = x_i)$	1/9	1/9	1/9	1/3	1/9	1/9	1/9

is the probability distribution of a random variable X. Find the variance of X.

Sol: $\text{Mean}(\mu) = -3\left(\frac{1}{9}\right) - 2\left(\frac{1}{9}\right) - 1\left(\frac{1}{9}\right) + 0\left(\frac{1}{3}\right) + 1\left(\frac{1}{9}\right) + 2\left(\frac{1}{9}\right) + 3\left(\frac{1}{9}\right)$

$$= -\frac{3}{9} - \frac{2}{9} - \frac{1}{9} + 0 + \frac{1}{9} + \frac{2}{9} + \frac{3}{9} = 0 \Rightarrow \mu = 0$$

$$\text{Variance}(\sigma) = (-3)^2\left(\frac{1}{9}\right) + (-2)^2\left(\frac{1}{9}\right) + (-1)^2\left(\frac{1}{9}\right) + (0)^2\left(\frac{1}{3}\right) + (1)^2\left(\frac{1}{9}\right) + (2)^2\left(\frac{1}{9}\right) + (3)^2\left(\frac{1}{9}\right) - \mu^2$$

$$= \frac{9}{9} + \frac{4}{9} + \frac{1}{9} + 0 + \frac{1}{9} + \frac{4}{9} + \frac{9}{9} - 0 = \frac{28}{9}$$

STAR-Q