Previous IPE

SOLVED PAPERS

MARCH -2024(TS)

PREVIOUS PAPERS

IPE: MARCH-2024(TS)

Time : 3 Hours

MATHS-2A

Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: $10 \times 2 = 20$

1. Find a square root of $-5+12i$.

2. If $z_1=-1$, $z_2=-i$ find Arg $z_1.z_2$

3. Simplify $\dfrac{(\cos\alpha+i\sin\alpha)^4}{(\sin\beta+i\cos\beta)^8}$

4. If the equations $x^2-6x+5=0$, $x^2-3ax+35=0$ have a common root then find a.

5. If $1,1,\alpha$ are the roots of $x^3-6x^2+9x-4=0$ then find α.

6. If ${}^{12}P_5+5.\,{}^{12}P_4={}^{13}P_r$ find r.

7. If ${}^{n}C_5={}^{n}C_6$ then find ${}^{13}C_n$

8. Write down and simplify the 7^{th} term in $(3x-4y)^{10}$.

9. Find the mean deviation about mean for the data 6,7,10,12,13,4,12,16.

10. If the mean and variance of a binomial variable X are 2.4 & 1.44 respectively, then find $P(1<x\le 4)$.

SECTION-B

II. Answer any FIVE of the following SAQs: $5 \times 4 = 20$

11. Show that $\dfrac{2-i}{(1-2i)^2}, \dfrac{-2-11i}{25}$ are conjugate to each other.

12. Determine the range of the $\dfrac{x^2+x+1}{x^2-x+1}$

13. Find the sum of all 4 digit numbers that can be formed using digits 1,3,5,7,9.

14. Find the number of ways of selecting a cricket team of 11 players from 7 batsmen and 6 bowlers such that there are atleast 5 bowlers in the team.

15. Resolve $\dfrac{5x+6}{(2+x)(1-x)}$ into Partial Fractions

16. If A,B are 2 events with $P(A\cup B)=0.65$ & $P(A\cap B)=0.15$, then find the value of $P(A^c)+P(B^c)$.

17. A problem in calculus is given to two students A and B whose chances of solving it are 1/3 and 1/4. What is the probability that the problem will be solved if both of them try independently?

SECTION-C

III. Answer any FIVE of the following LAQs: $5 \times 7 = 35$

18. Show that one value of $\left(\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^{8/3}=-1$

19. Solve $x^4-10x^3+26x^2-10x+1=0$.

20. If P and Q are the sum of odd terms and the sum of even terms respectively, in the expansion of $(x+a)^n$ then prove that (i) $P^2-Q^2=(x^2-a^2)^n$ (ii) $4PQ=(x+a)^{2n}-(x-a)^{2n}$

21. If $x=\dfrac{1.3}{3.6}+\dfrac{1.3.5}{3.6.9}+\dfrac{1.3.5.7}{3.6.9.12}+......$ then prove that $9x^2+24x=11$

22. Find the mean deviation about the mean for the following continuous distribution:

Sales (in Rs. thousand)	40-50	50-60	60-70	70-80	80-90	90-100
Number of companies	5	15	25	30	20	5

23. State and Prove "Addition theorem on Probability.

24. The range of a random variable X is {0,1,2} and the probabilities are $P(X=0)=3c^3$, $P(X=1)=4c-10c^2$, $P(X=2)=5c-1$ where 'c' is a constant, find (i) c (ii) $P(0<X<3)$ (iii) $P(1<X\le 2)$ (iv) $P(X<1)$

IPE TS MARCH-2024
SOLUTIONS

SECTION-A

1. Find the square root of –5 +12i

Sol: Let –5+12i = a+bi ⇒ a= –5, b=12 $\therefore\ r=\sqrt{a^2+b^2}=\sqrt{(-5)^2+12^2}$

$=\sqrt{25+144}=\sqrt{169}=13$

Formula: $\sqrt{a+ib}=\pm\left(\sqrt{\frac{r+a}{2}}+i\sqrt{\frac{r-a}{2}}\right)$

$\therefore\ \sqrt{-5+12i}=\pm\left(\sqrt{\frac{13-5}{2}}+i\sqrt{\frac{13+5}{2}}\right)=\pm\left(\sqrt{\frac{8}{2}}+i\sqrt{\frac{18}{2}}\right)=\pm(\sqrt{4}+i\sqrt{9})=\pm(2+3i)$

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2. If $z_1 = -1$, $z_2 = -i$, then find $Arg(z_1\, z_2)$

Sol: We know Arg(–1)=π and Arg(–i)=–π/2

$\therefore\ Arg(z_1 z_2)=Arg(z_1)+Arg(z_2)=\pi-\frac{\pi}{2}=\frac{\pi}{2}$

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3. Simplify $\frac{(\cos\alpha+i\sin\alpha)^4}{(\sin\beta+i\cos\beta)^8}$

Sol: $G.E=\frac{(\cos\alpha+i\sin\alpha)^4}{(\sin\beta+i\cos\beta)^8}=\frac{(\cos\alpha+i\sin\alpha)^4}{(i\cos\beta+\sin\beta)^8}=\frac{(\cos\alpha+i\sin\alpha)^4}{(i\cos\beta-i^2\sin\beta)^8}=\frac{(\cos\alpha+i\sin\alpha)^4}{[i(\cos\beta-i\sin\beta)]^8}$

$=\frac{(\cos\alpha+i\sin\alpha)^4}{(i)^8(\cos\beta-i\sin\beta)^8}=(\cos\alpha+i\sin\alpha)^4(\cos\beta-i\sin\beta)^{-8}\quad[\because i^8=(i^2)^4=(-1)^4=1]$

$=(\cos 4\alpha+i\sin 4\alpha)(\cos 8\beta+i\sin 8\beta)=(cis4\alpha)(cis8\beta)=cis(4\alpha+8\beta)$

$=\cos(4\alpha+8\beta)+i\sin(4\alpha+8\beta)$

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4. If the equations x^2–6x+5=0, x^2–3ax+35=0 have a common root then find a.

Sol: Let α be the common root.

Then $\alpha^2-6\alpha+5=0$ and $\alpha^2-3a\alpha+35=0 \Rightarrow (\alpha-1)(\alpha-5)=0 \Rightarrow \alpha=1$ or $\alpha=5$

Put α =1 in $\alpha^2-3a\alpha+35=0 \Rightarrow 1^2-3a(1)+35=0 \Rightarrow 3a=36 \Rightarrow a=12$

Put α =5 in $\alpha^2-3a\alpha+35=0 \Rightarrow 5^2-3a(5)+35=0 \Rightarrow 25-15a+35=0$

$\Rightarrow 15a=60 \Rightarrow a=4$

5. If 1,1,α are the roots of $x^3-6x^2+9x-4=0$ then find α.

Sol: From the given equation we get, $a_0=1$, $a_1=-6$, $a_2=9$, $a_3=-4$

Product of roots $1.1.\alpha = S_3 = \frac{-a_3}{a_0} = \frac{4}{1}$ $\therefore \alpha = 4$.

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6. If $^{12}P_5 + 5.\,^{12}P_4 = {}^{13}P_r$ find r.

Sol: We use the formula

$^{(n-1)}P_r + r.\,^{(n-1)}P_{(r-1)} = {}^nP_r$

$\therefore {}^{12}P_5 + 5.\,^{12}P_4 = {}^{(13-1)}P_5 + 5.\,^{(13-1)}P_{(5-1)} = {}^{13}P_5 = {}^{13}P_r$

$\therefore r = 5.$

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7. If $^nC_5 = {}^nC_6$, then find $^{13}C_n$

Sol : Formula: $^nC_r = {}^nC_s \Rightarrow r+s=n$ (or) $r=s$

$\therefore {}^nC_5 = {}^nC_6 \Rightarrow n = 5+6 = 11$

$\therefore {}^{13}C_n = {}^{13}C_{11} = {}^{13}C_{13-11}$

$= {}^{13}C_2 = \frac{13\times 12}{1\times 2} = 13\times 6 = 78$

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8. Write down and simplify the 7^{th} term in $(3x-4y)^{10}$.

Sol: General term of $(x-a)^n$ is $T_{r+1} = (-1)^r\,{}^nC_r x^{n-r} a^r$.

$\therefore T_7 = T_{6+1} = (-1)^6\,{}^{10}C_6\,(3x)^{10-6}.\,(4y)^6 = \frac{10!}{(10-6)!6!}\times 3^4.x^4.4^6.y^6 = \frac{10!}{4!6!}\times 3^4.4^6.x^4.y^6$

$= \frac{10\times \overset{3}{9}\times 8\times 7\times 6!}{4\times 3\times 2\times 1\times 6!}\times 3^4.4^6.x^4.y^6 = 210.3^4.4^6.x^4\,y^6$

9. Find the mean deviation about mean for the data 6,7,10,12,13,4,12,16.

Sol: **Given data:** 6,7,10,12,13,4,12,16. Here n=8

Mean $\bar{x}=\frac{6+7+10+12+13+4+12+16}{8}=\frac{80}{8}=10$

Deviations from the mean:

6–10= –4; 7–10= –3; 10–10=0; 12–10=2; 13–10= 3; 4 –10= –6;12–10=2;16–10= 6

Absolute values of these deviations:

4,3,0,2,3,6,2,6

$\therefore$ M.D. from Mean is $M.D.=\frac{\Sigma|x_i-\bar{x}|}{8}$

$=\frac{4+3+0+2+3+6+2+6}{8}=\frac{26}{8}=3.25$

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10. If the mean and variance of a binomial variable X are 2.4 & 1.44 respectively, then find P(1<x≤4)

Sol: Given Mean = np = 2.4 ....(1)
Variance = npq = 1.44 ......(2)

Dividing (2) by (1), $\frac{npq}{np}=\frac{1.44}{2.4}=\frac{3}{5}$

$\therefore q=\frac{3}{5}\Rightarrow p=1-q=1-\frac{3}{5}=\frac{2}{5}$

Take np=2.4$\Rightarrow n\left(\frac{2}{5}\right)=2.4\Rightarrow n=2.4\left(\frac{5}{2}\right)=6 \quad \therefore n=6, q=\frac{3}{5}$ and $p=\frac{2}{5}$

$P(1<X\le4) = P(X=2)+P(X=3)+P(X=4)={}^6C_2.q^4.p^2+{}^6C_3q^3.p^3+{}^6C_4q^2.p^4$

$={}^6C_2\left(\frac{3}{5}\right)^4\left(\frac{2}{5}\right)^2+{}^6C_3\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)^3+{}^6C_4\left(\frac{3}{5}\right)^2.\left(\frac{2}{5}\right)^4=15\left(\frac{3^4.2^2}{5^6}\right)+20\left(\frac{3^3.2^3}{5^6}\right)+15\left(\frac{3^2.2^4}{5^6}\right)$

$=\frac{3\times5(3^4\times2^2)}{5^6}+\frac{2^2\times5(3^3\times2^3)}{5^6}+\frac{3\times5(3^2\times2^4)}{5^6}$

$=\frac{36}{15625}(135+120+60)=\frac{2268}{3125}$

SECTION-B

11. Show that $\frac{2-i}{(1-2i)^2}$ and $\left(\frac{-2-11i}{25}\right)$ are conjugate to each other.

Sol: Let $z=\frac{2-i}{(1-2i)^2}=\frac{2-i}{1+4i^2-4i}=\frac{2-i}{1-4-4i}=\frac{2-i}{-3-4i}$

$$=\frac{(2-i)}{-(3+4i)}\frac{(3-4i)}{(3-4i)}=\frac{6-8i-3i+4i^2}{-25}=\frac{6-11i+4(-1)}{-25}=\frac{6-4-11i}{-25}=\frac{2-11i}{-25}=\frac{-2+11i}{25}.$$

The conjugate of $z=\frac{-2+11i}{25}$ is $\bar{z}=\frac{-2-11i}{25}$

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12. Determine the range of the $\frac{x^2+x+1}{x^2-x+1}$

Sol: Let $y=\frac{x^2+x+1}{x^2-x+1}$

$\Rightarrow y(x^2-x+1)=x^2+x+1$

$\Rightarrow yx^2-yx+y=x^2+x+1$

$\Rightarrow yx^2-x^2-yx-x+y-1=0$

$\Rightarrow x^2(y-1)-x(y+1)+(y-1)=0$

$\Rightarrow (y-1)x^2-(y+1)x+(y-1)=0$ ....(1)

(1) is a quadratic in x and its roots are reals.

$\therefore \Delta=b^2-4ac\geq 0$

$\Rightarrow (y+1)^2-4(y-1)^2\geq 0$

$\Rightarrow (y+1)^2-(2y-2)^2\geq 0$

$\Rightarrow (y+1+2y-2)(y+1)-(2y-2)\geq 0$ $\quad [\because a^2-b^2=(a+b)(a-b)]$

$\Rightarrow (3y-1)(3-y)\geq 0 \Rightarrow (3y-1)(y-3)\leq 0$

$\Rightarrow y\in\left[\frac{1}{3},3\right]$ $\quad \therefore$ Range $=\left[\frac{1}{3},3\right]$

13. Find the sum of all 4 digit numbers that can be formed using digits 1,3,5,7,9.

Sol: First we find the sum contributed by 9 in the total sum

The sum contributed by 9 when it is in units place is $\square\,\square\,\square\,\boxed{9} = {}^4P_3 \times 9$

The sum contributed by 9 when it is in tens place is $\square\,\square\,\boxed{9}\,\square = {}^4P_3 \times 90$

The sum contributed by 9 when it is in hundreds place is $\square\,\boxed{9}\,\square\,\square = {}^4P_3 \times 900$

The sum contributed by 9 when it is in thousands place is $\boxed{9}\,\square\,\square\,\square = {}^4P_3 \times 9000$

$\therefore$ The sum contributed by 9 in the total sum is ${}^4P_3 \times (9+90+900+9000)$

$= {}^4P_3 \times 9(1+10+100+1000) = {}^4P_3 \times 9(1111)$ ....(1)

Similarly, the sum contributed by 7 is ${}^4P_3 \times 7(1111)$ ....(2)

the sum contributed by 5 is ${}^4P_3 \times 5(1111)$ ....(3)

the sum contributed by 3 is ${}^4P_3 \times 3(1111)$ ....(4)

the sum contributed by 1 is ${}^4P_3 \times 1(1111)$ ....(5)

Hence from (1), (2), (3), (4), (5), the sum of all 4 digited numbers is ${}^4P_3 \times 1111(9+7+5+3+1)$

$= (4 \times 3 \times 2) \times 1111\ (25) = 24 \times 25 \times 1111 = 6,66,600$

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14. Find the number of ways of selecting a cricket team of 11 players from batsmen and 6 bowlers such that there will be atleast 5 bowlers in the team.

Sol: A Team of 11 players with atleast 5 bowlers can be selected in the following compositions:

Bowlers(6)	Batsmen(7)	No. of selections
5	6	${}^6C_5 \times {}^7C_6 = 6 \times 7 = 42$
6	5	${}^6C_6 \times {}^7C_5 = 1 \times 21 = 21$

$\because {}^7C_5 = {}^7C_2 = \dfrac{7 \times 6}{2 \times 1} = 21$

$\therefore$ the total number of selections=42+21= 63

15. Resolve $\frac{5x+6}{(2+x)(1-x)}$ into partial fractions.

Sol: Let $\frac{5x+6}{(2+x)(1-x)}=\frac{A}{2+x}+\frac{B}{1-x}=\frac{A(1-x)+B(2+x)}{(2+x)(1-x)}$

$\Rightarrow A(1-x)+B(2+x)=5x+6$............**(1)**

Putting $x=-2$ in (1), we get $A(1-(-2))+B(2-2)=5(-2)+6 \Rightarrow 3A=-4 \Rightarrow A=-\frac{4}{3}$

Putting $x=1$ in (1), we get $B(2+1)=5+6 \Rightarrow 3B=11 \Rightarrow B=\frac{11}{3}$

$\therefore \frac{5x+6}{(2+x)(1-x)}=\frac{A}{2+x}+\frac{B}{1-x}=-\frac{4}{3(2+x)}+\frac{11}{3(1-x)}$

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16. If A,B are two events with $P(A\cup B)=0.65$ and $P(A\cap B)=0.15$, then find the value of $P(A^c)+P(B^c)$.

Sol: We know $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ [From Addition theorem]

$\Rightarrow P(A)+P(B)=P(A\cup B)+P(A\cap B)=0.65+0.15=0.8$

$\therefore P(A^c)+P(B^c)=[1-P(A)]+[1-P(B)]=2-[P(A)+P(B)]=2-0.8=1.2$

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17. A problem in calculus is given to two students A and B whose chances of solving it are 1/3, 1/4 respectively. Find the probability of the problem being solved if both of them try independently.

Sol: Let A,B denote the events of solving the problem by A, B respectively $\Rightarrow P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$

$\therefore P(\bar{A})=1-P(A)=1-\frac{1}{3}=\frac{2}{3};\quad P(\bar{B})=1-P(B)=1-\frac{1}{4}=\frac{3}{4}$

$\therefore P(A\cup B)=1-P(\bar{A}\cap\bar{B})=1-P(\bar{A}).P(\bar{B})=1-\left(\frac{2}{\cancel{3}}\right)\left(\frac{\cancel{3}}{4}\right)=1-\frac{1}{2}=\frac{1}{2}$

SECTION-C

18. Show that one value of $\left(\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^{8/3}=-1$

Sol: Let $z=\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}$.

Then $\frac{1}{z}=\dfrac{1}{\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}=\dfrac{1\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)}{\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)}=\dfrac{\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)}{\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}}=\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)$

$$\therefore \text{G.E}=\left(\dfrac{1+\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)}{1+\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)}\right)^{8/3}=\left(\dfrac{1+z}{1+\frac{1}{z}}\right)^{8/3}=\left(\dfrac{\cancel{1+z}}{\frac{\cancel{z+1}}{z}}\right)^{8/3}=(z)^{8/3}$$

$$=\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)^{8/3}=\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+i\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)\right]^{8/3}$$

$$=\left(\cos\frac{4\pi-\pi}{8}+i\sin\frac{4\pi-\pi}{8}\right)^{8/3}=\left(\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}\right)^{8/3}=\left(\cos\frac{\cancel{8}}{\cancel{3}}\left(\frac{\cancel{3}\pi}{\cancel{8}}\right)+i\sin\frac{\cancel{8}}{\cancel{3}}\left(\frac{\cancel{3}\pi}{\cancel{8}}\right)\right)$$

$$=\cos\pi+i\sin\pi=\cos180^\circ+i\sin180^\circ=-1+i(0)=-1$$

19. Solve $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$.

Sol: The degree of the given equation is n=4, which is even. Also $a_k = a_{n-k}\ \forall\ k = 0,1,2,3,4$

So, the given equation is a Standard Reciprocal Equation.

Now, dividing the equation by x^2, we get $x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 26 = 0 \quad ....(1)$$

If $x + \frac{1}{x} = y$ then $x^2 + \frac{1}{x^2} = y^2 - 2$ $\left[\because x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 = y^2 - 2\right]$

$\therefore$ (1) $\Rightarrow (y^2 - 2) - 10y + 26 = 0 \Rightarrow y^2 - 10y + 24 = 0$

$\Rightarrow y^2 - 6y - 4y + 24 = 0$

$\Rightarrow y(y-6) - 4(y-6) = 0$

$\Rightarrow (y-4)(y-6) = 0$

$\Rightarrow y - 4 = 0$ (or) $y - 6 = 0 \Rightarrow y = 4$ (or) $y = 6$

If y=4 then $x + \frac{1}{x} = 4 \Rightarrow \frac{x^2+1}{x} = 4 \Rightarrow x^2 + 1 = 4x \Rightarrow x^2 - 4x + 1 = 0$

$$\Rightarrow x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2.1} = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

If y=6 then $x + \frac{1}{x} = 6 \Rightarrow \frac{x^2+1}{x} = 6 \Rightarrow x^2 + 1 = 6x \Rightarrow x^2 - 6x + 1 = 0$

$$\Rightarrow x = \frac{6 \pm \sqrt{(-6)^2 - 4(1)(1)}}{2} = \frac{6 \pm \sqrt{36-4}}{2} = \frac{6 \pm \sqrt{32}}{2} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

$\therefore$ the roots of the given equation are $2 \pm \sqrt{3}, 3 \pm 2\sqrt{2}$

20. If P and Q are the sum of odd terms and the sum of even terms respectively, in the expansion of $(x+a)^n$ then prove that (i) $P^2-Q^2=(x^2-a^2)^n$ (ii) $4PQ=(x+a)^{2n}-(x-a)^{2n}$

Sol: We know that $(x+a)^n = {}^nC_0x^n + {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 + {}^nC_3x^{n-3}a^3 + \dots\dots\dots + {}^nC_na^n$

$$= \left({}^nC_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4 + \dots\dots\right) + \left({}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + {}^nC_5x^{n-5}a^5 + \dots\dots\right) = P + Q$$

Also, $(x-a)^n = {}^nC_0x^n - {}^nC_1x^{n-1}a + {}^nC_2x^{n-2}a^2 - {}^nC_3x^{n-3}a^3 + \dots\dots\dots + {}^nC_n(-1)^na^n$

$$= \left({}^nC_0x^n + {}^nC_2x^{n-2}a^2 + {}^nC_4x^{n-4}a^4 + \dots\dots\right) - \left({}^nC_1x^{n-1}a + {}^nC_3x^{n-3}a^3 + {}^nC_5x^{n-5}a^5 + \dots\dots\right) = P - Q$$

i) $P^2-Q^2 = (P+Q)(P-Q) = (x+a)^n(x-a)^n = [(x+a)(x-a)]^n = (x^2-a^2)^n$

ii) $4PQ = (P+Q)^2 - (P-Q)^2 = [(x+a)^n]^2 - [(x-a)^n]^2 = (x+a)^{2n} - (x-a)^{2n}$

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21. If $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \dots\dots$ then prove that $9x^2+24x = 11$

Sol: Given that $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \dots\dots = \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \frac{1.3.5.7}{4!}\left(\frac{1}{3}\right)^4 + ..$

Adding $1+\frac{1}{3}$ on both sides, we get $1+\frac{1}{3}+x = 1+\frac{1}{1!}\left(\frac{1}{3}\right)+\frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \dots\dots$

Comparing the above series with $1+\frac{p}{1!}\left(\frac{y}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{y}{q}\right)^2 + \dots\dots = (1-y)^{-p/q}$

we get, $p = 1, p+q = 3 \Rightarrow 1+q = 3 \Rightarrow q = 2$

Also, $\frac{y}{q} = \frac{1}{3} \Rightarrow y = \frac{q}{3} = \frac{2}{3}$

$$\therefore 1+\frac{1}{3}+x = (1-y)^{-p/q} = \left(1-\frac{2}{3}\right)^{-1/2} = \left(\frac{1}{3}\right)^{-1/2} = (3)^{1/2} = \sqrt{3}$$

$$\Rightarrow \frac{4}{3}+x = \sqrt{3} \Rightarrow x = \sqrt{3}-\frac{4}{3} = \frac{3\sqrt{3}-4}{3} = \frac{3\sqrt{3}-4}{3} \Rightarrow 3x = 3\sqrt{3}-4 \Rightarrow 3x+4 = 3\sqrt{3}$$

$$\Rightarrow (3x+4)^2 = (3\sqrt{3})^2 \Rightarrow 9x^2+24x+16 = 27 \Rightarrow 9x^2+24x = 11$$

22. Find the mean deviation about the mean for the following continuous distribution:

Sales (in Rs. thousand)	40-50	50-60	60-70	70-80	80-90	90-100
Number of companies	5	15	25	30	20	5

Sol: We form the following table from the given data

Sales	Number of companies(f_i)	Midpoints of class interval(x_i)	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$f_i \lvert x_i - \bar{x}\rvert$
40-50	5	45	225	26	130
50-60	15	55	825	16	240
60-70	25	65	1625	6	150
70-80	30	75	2250	4	120
80-90	20	85	1700	14	280
90-100	5	95	475	24	120
	$\Sigma f_i = 100 = N$		$\Sigma f_ix_i = 7100$		$\Sigma f_i \lvert x_i - \bar{x}\rvert = 1040$

Here $N = \Sigma f_i = 100$ and Mean $\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{7100}{100} = 71$

$\therefore$ Mean deviation about the mean $M.D = \frac{\Sigma f_i \lvert x_i - \bar{x}\rvert}{N} = \frac{1040}{100} = 10.4$

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23. State and Prove Addition theorem on Probability.

Sol: ***Statement*** **: If E_1, E_2 are the 2 events of a sample space S then $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$**

Proof: Case (i): When $E_1 \cap E_2 = \phi$

$E_1 \cap E_2 = \phi \Rightarrow P(E_1 \cap E_2) = 0$

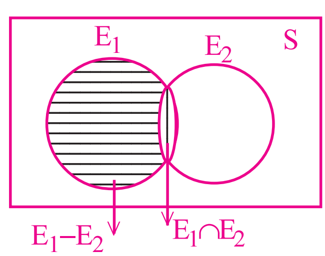
$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2)$ [From the Union axiom]

$= P(E_1) + P(E_2) - 0 = P(E_1) + P(E_2) - P(E_1 \cap E_2)$

Case (ii) : When $E_1 \cap E_2 \neq \phi$

$E_1 \cup E_2$ is the union of disjoint sets $(E_1 - E_2), E_2$

$\therefore P(E_1 \cup E_2) = P[(E_1 - E_2) \cup E_2] = P(E_1 - E_2) + P(E_2)$ ....(1)



E_1 is the union of disjoint sets $(E_1 - E_2), (E_1 \cap E_2)$.

$\therefore P(E_1) = P[(E_1 - E_2) \cup (E_1 \cap E_2)] = P(E_1 - E_2) + P(E_1 \cap E_2)$

$\Rightarrow P(E_1 - E_2) = P(E_1) - P(E_1 \cap E_2)$

$\therefore$ from (1), $P(E_1 \cup E_2) = [P(E_1) - P(E_1 \cap E_2)] + P(E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$. Hence proved.

24. A random variable X has its range{0,1,2} and the probabilities are $P(X=0)=3c^3$, $P(X=1)=4c-10c^2$, $P(X=2)=5c-1$ where 'c' is a constant, find (i) c (ii) $P(0<X<3)$ (iii) $P(1<X\leq 2)$ (iv) $P(X<1)$

Sol: (i) We know $\Sigma P(X=xi)=1$

$$\Rightarrow 3c^3+4c-10c^2+5c-1=1 \Rightarrow 3c^3-10c^2+9c-1=1 \Rightarrow 3c^3-10c^2+9c-2=0$$

Here, the sum of the coefficients is 3–10+9–2=0. Hence 1 is a root of the above equation.

∴ By synthetic division, we have

1	3	–10	9	–2
	0	3	–7	2
	3	–7	2	0

$$\therefore 3c^3-10c^2+9c-2=(c-1)(3c^2-7c+2)=(c-1)(c-2)(3c-1)$$

Now, $(c-1)(c-2)(3c-1)=0 \Rightarrow c=1, 2, \frac{1}{3}$

∴ c=1/3 is the only possible value. $[\because 0\leq p\leq 1]$

(ii) $P(0<X<3)=P(X=1)+P(X=2)=(4c-10c^2)+(5c-1)=9c-10c^2-1$

$$=9\left(\frac{1}{3}\right)-10\left(\frac{1}{3}\right)^2-1=\frac{9}{3}-\frac{10}{9}-1=3-\frac{10}{9}-1=2-\frac{10}{9}=\frac{8}{9}$$

(iii) $P(1<X\leq 2)=P(X=2)=5c-1=5\left(\frac{1}{3}\right)-1=\frac{5}{3}-1=\frac{2}{3}$

(iv) $P(X<1)=P(X=0)=3c^3=3\left(\frac{1}{3}\right)^3=3.\frac{1}{27}=\frac{1}{9}$