Previous IPE

SOLVED PAPERS

MARCH-2024 (AP)

PREVIOUS PAPERS

IPE: MARCH-2024(AP)

Time : 3 Hours **MATHS-1A** Max Marks: 75

SECTION-A

I. Answer ALL the following VSAQ: **$10 \times 2 = 20$**

1. Find the domain of $f(x) = \frac{1}{\sqrt{|x| - x}}$
2. If A={1,2,3,4} and f:A→R is a function defined by $f(x) = \frac{x^2 - x + 1}{x + 1}$ then find the range of f.
3. If $A = \begin{bmatrix} 2 & 4 \\ -1 & k \end{bmatrix}$ and $A^2=0$, then find the value of k.
4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and 2X+A=B then find X.
5. OABC is a parallelogram. If $\overline{OA} = \bar{a}, \overline{OC} = \bar{c}$ find the vector equation of the side $\overline{BC}$.
6. Find a vector in the direction of vector $\bar{a} = \bar{i} - 2\bar{j}$ that has magnitude 7 units.
7. If $4\bar{i} + \frac{2p}{3}\bar{j} + p\bar{k}$ is parallel to the vector $\bar{i} + 2\bar{j} + 3\bar{k}$, find p.
8. P.T $\frac{\cos 9^\circ + \sin 9^\circ}{\cos 9^\circ - \sin 9^\circ} = \cot 36^\circ$
9. Find a sine function whose period is 2/3.
10. For any $x \in R$, prove that $\cosh^4 x - \sinh^4 x = \cosh(2x)$

SECTION-B

II. Answer any FIVE of the following SAQs: **5 x 4=20**

11. Find the adjoint and the inverse of the matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$
12. S.T $A(2\bar{i} - \bar{j} + \bar{k}),\ B(\bar{i} - 3\bar{j} - 5\bar{k}),\ C(3\bar{i} - 4\bar{j} - 4\bar{k})$ are the vertices of a right angled triangle.
13. Let $\bar{e}_1$ and $\bar{e}_2$ be unit vectors making angle θ. If $\frac{1}{2}|\bar{e}_1 - \bar{e}_2| = \sin(\lambda\theta)$ then find λ.
14. Show that $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$
15. Solve $1 + \sin^2\theta = 3\sin\theta\cos\theta$
16. Prove that $\cos\left(2\mathrm{Tan}^{-1}\frac{1}{7}\right) = \sin\left(2\mathrm{Tan}^{-1}\frac{3}{4}\right)$
17. If $a=(b-c)\sec\theta$, then prove that $\tan\theta = \frac{2\sqrt{bc}}{b-c}\sin\left(\frac{A}{2}\right)$

SECTION-C

III. Answer any FIVE of the following LAQs: **5 x 7=35**

18. If f:A→B, g:B→C are two bijective functions then prove that $(gof)^{-1} = f^{-1}og^{-1}$
19. Using P.M.I, prove that $1^2 + (1^2 + 2^2) + \ldots n \text{ terms} = \frac{n(n+1)^2(n+2)}{12}, \forall n \in N$
20. Solve x+y+z=9, 2x+5y+7z=52, 2x+y–z=0 by using Cramer's rule.
21. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$
22. If $\bar{a} = 2\bar{i} + 3\bar{j} + 4\bar{k}, \bar{b} = \bar{i} + \bar{j} - \bar{k}, \bar{c} = \bar{i} - \bar{j} + \bar{k}$, compute $\bar{a} \times (\bar{b} \times \bar{c})$ and verify that it is perpendicular to $\bar{a}$
23. If A,B,C are angles in a triangle, then prove that $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2\sin A \sin B \cos C$.
24. In a ΔABC if a=13, b=14, c=15 then show that $R = \frac{65}{8}, r = 4, r_1 = \frac{21}{2}, r_2 = 12, r_3 = 14$

IPE AP MARCH-2024

SOLUTIONS

SECTION-A

1. Find the domain of $f(x)=\frac{1}{\sqrt{|x|-x}}$

Sol: $|x|-x>0 \Rightarrow |x|>x......(1)$ It is valid only when $x<0$

$\therefore$ Domain $=(-\infty,0)$ [For verification of (1), take x=2.5; 2; 0; –2.5]

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2. If A={1,2,3,4} and f:A→R is a function defined by $f(x)=\frac{x^2-x+1}{x+1}$ then find the range of f.

Sol: Given A={1,2,3,4} and $f(x)=\frac{x^2-x+1}{x+1}$

Now, $f(1)=\frac{1^2-1+1}{1+1}=\frac{1}{2}$; $f(2)=\frac{2^2-2+1}{2+1}=1$; $f(3)=\frac{3^2-3+1}{3+1}=\frac{7}{4}$;

$f(4)=\frac{4^2-4+1}{4+1}=\frac{13}{5}$

$\therefore$ Range of f is $f(A)=\left\{\frac{1}{2},1,\frac{7}{4},\frac{13}{5}\right\}$

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3. If $A=\begin{bmatrix}2 & 4\\ -1 & k\end{bmatrix}$ and $A^2=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ then find the value of k

Sol: Given $A=\begin{bmatrix}2 & 4\\ -1 & k\end{bmatrix}$

$$\therefore A^2=A\times A=\begin{bmatrix}2 & 4\\ -1 & k\end{bmatrix}\begin{bmatrix}2 & 4\\ -1 & k\end{bmatrix}=\begin{bmatrix}2(2)+4(-1) & 2(4)+4(k)\\ -1(2)+k(-1) & -1(4)+k(k)\end{bmatrix}$$

$$=\begin{bmatrix}4-4 & 8+4k\\ -2-k & -4+k^2\end{bmatrix}=\begin{bmatrix}0 & 8+4k\\ -2-k & -4+k^2\end{bmatrix}$$

$$\Rightarrow \begin{bmatrix}0 & 8+4k\\ -2-k & -4+k^2\end{bmatrix}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix} \quad [\because A^2=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}]$$

$\Rightarrow 8+4k=0 \Rightarrow 4k=-8 \Rightarrow k=-2$

4. If $A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}, B=\begin{bmatrix}3 & 8\\7 & 2\end{bmatrix}$ and 2X+A=B then find X.

Sol: Given $A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}, B=\begin{bmatrix}3 & 8\\7 & 2\end{bmatrix}$

$$2X + A = B \Rightarrow 2X = B - A = \begin{bmatrix}3 & 8\\7 & 2\end{bmatrix} - \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} = \begin{bmatrix}3-1 & 8-2\\7-3 & 2-4\end{bmatrix} = \begin{bmatrix}2 & 6\\4 & -2\end{bmatrix} = 2\begin{bmatrix}1 & 3\\2 & -1\end{bmatrix}$$

$$\therefore \not{2}X = \not{2}\begin{bmatrix}1 & 3\\2 & -1\end{bmatrix} \Rightarrow X = \begin{bmatrix}1 & 3\\2 & -1\end{bmatrix}$$

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5. OABC is a parallelogram. If $\overline{OA} = \bar{a}, \overline{OC} = \bar{c}$ find the vector equation of the side $\overline{BC}$.

Sol: Given that $\overline{OA} = \bar{a}, \overline{OC} = \bar{c}$.

From Parallelogram OABC, we have $\overline{CB} = \overline{OA} = \bar{a}$.

$\therefore$ Vector equation of the side $\overline{BC}$ passing through the point

$C(\bar{c})$ and parallel to the vector $\overline{OA} = \bar{a}$ is $\bar{r} = \bar{c} + t\bar{a}, t \in R$

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6. Find a vector in the direction of vector $\bar{a} = \bar{i} - 2\bar{j}$ that has magnitude 7 units.

Sol: Given $\bar{a} = \bar{i} - 2\bar{j}$, then $|\bar{a}| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

$\therefore$ Unit vector $\dfrac{\bar{a}}{|\bar{a}|} = \dfrac{\bar{i} - 2j}{\sqrt{5}}$

$\therefore$ Vector of magnitude 7 $= 7\left(\dfrac{\bar{i} - 2\bar{j}}{\sqrt{5}}\right)$

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7. If $4\bar{i} + \frac{2p}{3}\bar{j} + p\bar{k}$ is parallel to the vector $\bar{i} + 2\bar{j} + 3\bar{k}$, find p.

Sol: Let $\bar{a} = 4\bar{i} + \frac{2p}{3}\bar{j} + p\bar{k}$, $\bar{b} = \bar{i} + 2\bar{j} + 3\bar{k} \Rightarrow 4\bar{b} = 4\bar{i} + 8\bar{j} + 12\bar{k}$ $\bar{a}$ is parallel to $4\bar{b}$

Hence, equating the coefficients of $\bar{k}$, then p=12

8. Prove that $\frac{\cos 9^\circ+\sin 9^\circ}{\cos 9^\circ-\sin 9^\circ}=\cot 36^\circ$

Sol: $\text{L.H.S}=\frac{\cos 9^\circ+\sin 9^\circ}{\cos 9^\circ-\sin 9^\circ}$

$$=\frac{\frac{\cos 9^\circ}{\cos 9^\circ}+\frac{\sin 9^\circ}{\cos 9^\circ}}{\frac{\cos 9^\circ}{\cos 9^\circ}-\frac{\sin 9^\circ}{\cos 9^\circ}}=\frac{1+\tan 9^\circ}{1-\tan 9^\circ}=\frac{\tan 45^\circ+\tan 9^\circ}{1-\tan 45^\circ\tan 9^\circ}=\tan(45^\circ+9^\circ)=\tan 54^\circ$$

$=\tan(90^\circ-36^\circ)=\cot 36^\circ=\text{R.H.S}$

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9. Find a sine function whose period is2/3

Sol: Let sinkx be the required sine function.

Period of sinkx$=\frac{2\pi}{k}$

$\therefore$ Period $\frac{2\pi}{k}=\frac{2}{3}\Rightarrow 2k=6\pi\Rightarrow k=3\pi$

$\therefore$ $\sin(3\pi)x$ is the required sine function.

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10. Prove that $\cosh^4x-\sinh^4x=\cosh 2x$

Sol: $\text{L.H.S}=\cosh^4x-\sinh^4x=(\cosh^2x-\sinh^2x)(\cosh^2x+\sinh^2x)$

$=(1)(\cosh 2x)=\cosh 2x=\text{R.H.S}$

SECTION-B

11. Find the adjoint and the inverse of the matrix $A=\begin{bmatrix}1&3&3\\1&4&3\\1&3&4\end{bmatrix}$

Sol: Given $A=\begin{bmatrix}1&3&3\\1&4&3\\1&3&4\end{bmatrix}\Rightarrow \det A = 1(16-9)-3(4-3)+3(3-4)=7-3-3=1\neq 0$

$\therefore$ A is non-singular

$$\text{Now, Adj } A=\begin{bmatrix}\begin{vmatrix}4&3\\3&4\end{vmatrix} & -\begin{vmatrix}1&3\\1&4\end{vmatrix} & \begin{vmatrix}1&4\\1&3\end{vmatrix}\\ -\begin{vmatrix}3&3\\3&4\end{vmatrix} & \begin{vmatrix}1&3\\1&4\end{vmatrix} & -\begin{vmatrix}1&3\\1&3\end{vmatrix}\\ \begin{vmatrix}3&3\\4&3\end{vmatrix} & -\begin{vmatrix}1&3\\1&3\end{vmatrix} & \begin{vmatrix}1&3\\1&4\end{vmatrix}\end{bmatrix}^T=\begin{bmatrix}7&-1&-1\\-3&1&0\\-3&0&1\end{bmatrix}^T$$

$$\therefore \text{Adj } A=\begin{bmatrix}7&-3&-3\\-1&1&0\\-1&0&1\end{bmatrix}$$

$$\therefore A^{-1}=\frac{1}{\det A}(\text{Adj } A)=\frac{1}{1}\begin{bmatrix}7&-3&-3\\-1&1&0\\-1&0&1\end{bmatrix}=\begin{bmatrix}7&-3&-3\\-1&1&0\\-1&0&1\end{bmatrix}$$

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12. Show that the points $A(2\bar{i}-\bar{j}+\bar{k})$, $B(\bar{i}-3\bar{j}-5\bar{k})$, $C(3\bar{i}-4\bar{j}-4\bar{k})$ are the vertices of a right angled triangle.

Sol: We take $\overline{OA}=(2\bar{i}-\bar{j}+\bar{k})$, $\overline{OB}=(\bar{i}-3\bar{j}-5\bar{k})$, $\overline{OC}=(3\bar{i}-4\bar{j}-4\bar{k})$, where 'O' is the origin.

$\overline{AB}=\overline{OB}-\overline{OA}=(\bar{i}-3\bar{j}-5\bar{k})-(2\bar{i}-\bar{j}+\bar{k})=-\bar{i}-2\bar{j}-6\bar{k}$

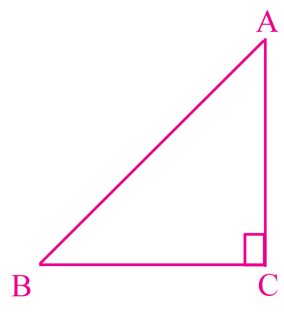
$\Rightarrow|\overline{AB}|=\sqrt{(-1)^2+(-2)^2+(-6)^2}=\sqrt{1+4+36}=\sqrt{41}$

$\overline{BC}=\overline{OC}-\overline{OB}=(3\bar{i}-4\bar{j}-4\bar{k})-(\bar{i}-3\bar{j}-5\bar{k})=2\bar{i}-\bar{j}+\bar{k}$

$\Rightarrow|\overline{BC}|=\sqrt{2^2+(-1)^2+1^2}=\sqrt{4+1+1}=\sqrt{6}$

$\overline{CA}=\overline{OA}-\overline{OC}=(2\bar{i}-\bar{j}+\bar{k})-(3\bar{i}-4\bar{j}-4\bar{k})=-\bar{i}+3\bar{j}+5\bar{k}$

$\Rightarrow|\overline{CA}|=\sqrt{(-1)^2+3^2+5^2}=\sqrt{1+9+25}=\sqrt{35}$



Here, $|\overline{AB}|^2=41; |\overline{BC}|^2+|\overline{CA}|^2=6+35=41\Rightarrow|\overline{AB}|^2=|\overline{BC}|^2+|\overline{CA}|^2$

$\therefore$ A, B, C are the vertices of a right angled triangle

13. Let $\bar{e}_1$ and $\bar{e}_2$ be unit vectors including angle θ. If $\frac{1}{2}|\bar{e}_1 - \bar{e}_2| = \sin\lambda\theta$ then find λ.

Sol: Given $\bar{e}_1, \bar{e}_2$ are unit vectors and $(\bar{e}_1, \bar{e}_2) = \theta$ Also, $\frac{1}{2}|\bar{e}_1 - \bar{e}_2| = \sin\lambda\theta \Rightarrow |\bar{e}_1 - \bar{e}_2| = 2\sin\lambda\theta$

$$\Rightarrow |\bar{e}_1 - \bar{e}_2|^2 = (2\sin\lambda\theta)^2 \Rightarrow (|\bar{e}_1|^2 + |\bar{e}_2|^2 - 2\bar{e}_1.\bar{e}_2) = 4\sin^2(\lambda\theta)$$

$$\Rightarrow (1+1-2|\bar{e}_1||\bar{e}_2|\cos\theta) = 4\sin^2\lambda\theta$$

$$\Rightarrow (2-2(1)(1)\cos\theta) = 4\sin^2\lambda\theta \Rightarrow 2(1-\cos\theta) = 4\sin^2\lambda\theta$$

$$\Rightarrow \cancel{2}\left(\cancel{2}\sin^2\frac{\theta}{2}\right) = \cancel{4}\sin^2\lambda\theta \Rightarrow \sin^2\frac{\theta}{2} = \sin^2\lambda\theta \qquad \therefore \lambda = \frac{1}{2}$$

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14. Show that $\cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8} = \frac{3}{2}$

Sol: $$\cos\frac{3\pi}{8} = \cos\left(\frac{4\pi-\pi}{8}\right) = \cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right) = \sin\frac{\pi}{8}$$

$$\cos\frac{5\pi}{8} = \cos\left(\frac{4\pi+\pi}{8}\right) = \cos\left(\frac{\pi}{2}+\frac{\pi}{8}\right) = -\sin\frac{\pi}{8}$$

$$\cos\frac{7\pi}{8} = \cos\left(\frac{8\pi-\pi}{8}\right) = \cos\left(\pi-\frac{\pi}{8}\right) = -\cos\frac{\pi}{8}$$

L.H.S $= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

$$= \cos^4\frac{\pi}{8} + \left(\sin\frac{\pi}{8}\right)^4 + \left(-\sin\frac{\pi}{8}\right)^4 + \left(-\cos\frac{\pi}{8}\right)^4 = \cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8} + \sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}$$

$$= 2\left[\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}\right]$$

$$= 2\left[\left(\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right)^2 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right], \qquad [\because a^2+b^2 = (a+b)^2 - 2ab]$$

$$= 2\left[1 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right] = 2 - 4\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8} = 2 - \left[2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right]^2 = 2 - \left[\sin\frac{2\pi}{8}\right]^2$$

$$= 2 - \left[\sin\frac{\pi}{4}\right]^2 = 2 - \left(\frac{1}{\sqrt{2}}\right)^2 = 2 - \frac{1}{2} = \frac{3}{2} = \text{R.H.S}$$

15. Solve $1 + \sin^2\theta = 3\sin\theta\cos\theta$

Sol: Given equation is $1+\sin^2\theta=3\sin\theta\cos\theta$

Dividing the given equation by $\cos^2\theta$, we get $\frac{1}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}=\frac{3\sin\theta\cos\theta}{\cos^2\theta}$

$\Rightarrow \sec^2\theta+\tan^2\theta=3\tan\theta$

$\Rightarrow (1+\tan^2\theta)+\tan^2\theta=3\tan\theta$ $[\because \sec^2\theta-\tan^2\theta=1]$

$\Rightarrow 2\tan^2\theta-3\tan\theta+1=0 \Rightarrow 2\tan^2\theta-2\tan\theta-\tan\theta+1=0$

$\Rightarrow 2\tan\theta(\tan\theta-1)-(\tan\theta-1)=0 \Rightarrow (2\tan\theta-1)(\tan\theta-1)=0$

$\Rightarrow \tan\theta=1$ (or) $\tan\theta=1/2$

Now, $\tan\theta=1=\tan\pi/4$.

Here P.V is $\alpha=\pi/4$

$\therefore$ General solution is $\theta=n\pi+\frac{\pi}{4}, n\in Z$

Also, $\tan\theta=1/2\Rightarrow\theta=\mathrm{Tan}^{-1}(1/2)$

$\therefore$ P.V is $\alpha=\mathrm{Tan}^{-1}(1/2)$

$\therefore$ GS is $\theta=n\pi+\mathrm{Tan}^{-1}(1/2)$ $n\in Z$

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16. Prove that $\cos\left(2\mathrm{Tan}^{-1}\frac{1}{7}\right)=\sin\left(2\mathrm{Tan}^{-1}\frac{3}{4}\right)$

Sol: We take $\mathrm{Tan}^{-1}\frac{1}{7}=\alpha\Rightarrow\tan\alpha=\frac{1}{7}$ and $\mathrm{Tan}^{-1}\frac{3}{4}=\beta\Rightarrow\tan\beta=\frac{3}{4}$, then

$$\text{L.H.S}=\cos\left(2\mathrm{Tan}^{-1}\frac{1}{7}\right)=\cos2\alpha=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}=\frac{1-\frac{1}{49}}{1+\frac{1}{49}}=\frac{48}{50}=\frac{24}{25}\text{ ......(1)}$$

$$\text{R.H.S}=\sin\left(2\mathrm{Tan}^{-1}\frac{3}{4}\right)=\sin2\beta=\frac{2\tan\beta}{1+\tan^2\beta}=\frac{2.\frac{3}{4}}{1+\frac{9}{16}}=\frac{3}{2}\times\frac{16}{25}=\frac{24}{25}\text{ ....(2)}$$

$\therefore$ from (1) and (2), $\cos\left(2\mathrm{Tan}^{-1}\frac{1}{7}\right)=\sin\left(2\mathrm{Tan}^{-1}\frac{3}{4}\right)$

17. If a = (b – c) secθ, then prove that $\tan\theta = \frac{2\sqrt{bc}}{b-c}\sin\left(\frac{A}{2}\right)$

Sol: Given a = (b – c) secθ $\Rightarrow \sec\theta = \frac{a}{b-c} \Rightarrow \sec^2\theta = \frac{a^2}{(b-c)^2}$

$\therefore \tan^2\theta = \sec^2\theta - 1 \; [\because \sec^2\theta - \tan^2\theta = 1]$

$= \frac{a^2}{(b-c)^2} - 1 = \frac{a^2-(b-c)^2}{(b-c)^2} = \frac{a^2-(b^2+c^2-2bc)}{(b-c)^2}$

$= \frac{2bc-(b^2+c^2-a^2)}{(b-c)^2} = \frac{2bc-(2bc\cos A)}{(b-c)^2} \left(\because \frac{b^2+c^2-a^2}{2bc} = \cos A\right)$

$= \frac{2bc(1-\cos A)}{(b-c)^2} = \frac{2bc.\left(2\sin^2\frac{A}{2}\right)}{(b-c)^2} = \frac{4bc.\sin^2\frac{A}{2}}{(b-c)^2}$

$\therefore \tan\theta = \frac{2\sqrt{bc}}{b-c}\sin\left(\frac{A}{2}\right)$

SECTION-C

18. If $f : A \to B$, $g : B \to C$ are two bijective functions then prove that $(gof)^{-1} = f^{-1}og^{-1}$

A: Part -1: Given that $f : A \to B$, $g : B \to C$ are two bijective functions, then

(i) $gof : A \to C$ is bijection $\Rightarrow (gof)^{-1} : C \to A$ is also a bijection

(ii) $f^{-1} : B \to A$, $g^{-1} : C \to B$ are both bijections $\Rightarrow (f^{-1}og^{-1}) : C \to A$ is also a bijection.

So, $(gof)^{-1}$ and $f^{-1}og^{-1}$, both have same domain 'C'

Part–2: Given $f: A \to B$ is bijection, then $f(a) = b \Rightarrow a = f^{-1}(b)$........(1), [Here $a \in A$, $b \in B$]

$g: B \to C$ is bijection, then $g(b) = c \Rightarrow b = g^{-1}(c)$........(2), [Here $b \in B$, $c \in C$]

$gof: A \to C$ is bijection, then $gof(a) = c \Rightarrow a = (gof)^{-1}(c)$.........(3)

Now, $(f^{-1}og^{-1})(c) = f^{-1}[g^{-1}(c)] = f^{-1}(b) = a$ ....(4), [From (1) & (2)]

$\therefore (gof)^{-1}(c) = (f^{-1}og^{-1})(c), \forall c \in C$, [from (3) & (4)]

Hence, we proved that $(gof)^{-1} = f^{-1}og^{-1}$

19. Using the principle of finite Mathematical Induction prove that

$$1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \text{....n terms} = \frac{n(n+1)^2(n+2)}{12}, \forall n \in N$$

Sol: To find n^{th} term:

Clearly, n^{th} term is $T_n = 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$

Let $S(n): 1^2 + (1^2 + 2^2) + ... + \frac{n(n+1)(2n+1)}{6} = \frac{n(n+1)^2(n+2)}{12}$

Step 1: L.H.S of S(1) = $1^2 = 1$

R.H.S of S(1) $= \frac{1(1+1)^2(1+2)}{12} = \frac{1(2^2)3}{12} = \frac{1(4)3}{12} = \frac{12}{12} = 1$

$\therefore$ L.H.S = R.H.S.

So, S(1) is true

Step 2: Assume that S(k) is true for $k \in N$

$$S(k): 1^2 + (1^2 + 2^2) + ... + \frac{k(k+1)(2k+1)}{6} = \frac{k(k+1)^2(k+2)}{12} \qquad .....(1)$$

Step 3: We show that S(k+1) is true

$(k+1)^{th}$ term $= \frac{(k+1)(k+2)(2(k+1)+1)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$

On adding $(k+1)^{th}$ term to both sides of (1), we have

$$\text{L.H.S} = \left[1^2 + (1^2 + 2^2) + ... + \frac{k(k+1)(2k+1)}{6}\right] + \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{k(k+1)^2(k+2)}{12} + \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{k(k+1)^2(k+2) + 2(k+1)(k+2).(2k+3)}{12}$$

$$= \frac{(k+1)(k+2)[k(k+1) + 2(2k+3)]}{12}$$

$$= \frac{(k+1)(k+2)(k^2 + 5k + 6)}{12}$$

$$= \frac{(k+1)(k+2)(k+2)(k+3)}{12} = \frac{(k+1)(k+2)^2(k+3)}{12} = \text{R.H.S}$$

$\therefore$ L.H.S = R.H.S.

So, S(k+1) is true whenever S(k) is true

Hence, by P.M.I the given statement is true, for all $n \in N$

20. By using Cramer's solve x+y+z=9, 2x+5y+7z=52, 2x+y–z=0

Sol: Given equations can be written as AX = D, where

$$A=\begin{bmatrix}1&1&1\\2&5&7\\2&1&-1\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, D=\begin{bmatrix}9\\52\\0\end{bmatrix}$$

$$\Delta=\begin{vmatrix}1&1&1\\2&5&7\\2&1&-1\end{vmatrix}=1(-5-7)-1(-2-14)+1(2-10)=-12+16-8=-4$$

$$\Delta_1=\begin{vmatrix}9&1&1\\52&5&7\\0&1&-1\end{vmatrix}=9(-5-7)-1(-52-0)+1(52-0)=-108+52+52=-4$$

$$\Delta_2=\begin{vmatrix}1&9&1\\2&52&7\\2&0&-1\end{vmatrix}=1(-52-0)-9(-2-14)+1(0-104)=-52+144-104=-12$$

$$\Delta_3=\begin{vmatrix}1&1&9\\2&5&52\\2&1&0\end{vmatrix}=1(0-52)-1(0-104)+9(2-10)=-52+104-72=-20$$

$\therefore$ By Carmer's rule $x=\frac{\Delta_1}{\Delta}=\frac{-4}{-4}=1,\ y=\frac{\Delta_2}{\Delta}=\frac{-12}{-4}=3,\ z=\frac{\Delta_3}{\Delta}=\frac{-20}{-4}=5$

$\therefore$ The solution is x = 1, y =3, z = 5

21. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

Sol: $\text{L.H.S} = \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$

$$= \begin{vmatrix} 2a+2b+2c & a & b \\ 2a+2b+2c & b+c+2a & b \\ 2a+2b+2c & a & c+a+2b \end{vmatrix} (\because C_1 \to C_1 + C_2 + C_3)$$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix} (\because R_2 \to R_2 - R_1, \ R_3 \to R_3 - R_1)$$

$$= 2(a+b+c)1[(a+b+c)^2 - 0] = 2(a+b+c)^3 = \text{R.H.S}$$

22. **If $\bar{a} = 2\bar{i} + 3\bar{j} + 4\bar{k}, \bar{b} = \bar{i} + \bar{j} - \bar{k}, \bar{c} = \bar{i} - \bar{j} + \bar{k}$, compute $\bar{a}\times(\bar{b}\times\bar{c})$ and verify that it is perpendicular to $\bar{a}$**

Sol: **Given** $\bar{a} = 2\bar{i} + 3\bar{j} + 4\bar{k}, \bar{b} = \bar{i} + \bar{j} - \bar{k}, \bar{c} = \bar{i} - \bar{j} + \bar{k}$

$$\bar{b}\times\bar{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \bar{i}(1-1) - \bar{j}(1+1) + \bar{k}(-1-1) = -2\bar{j} - 2\bar{k}$$

$$\therefore \bar{a}\times(\bar{b}\times\bar{c}) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 3 & 4 \\ 0 & -2 & -2 \end{vmatrix} = \bar{i}(-6+8) - \bar{j}(-4-0) + \bar{k}(-4-0) = 2\bar{i} + 4\bar{j} - 4\bar{k}$$

Now $[(\bar{a}\times\bar{b})\times\bar{c})].\bar{a} = (2\bar{i} + 4\bar{j} - 4\bar{k}).(2\bar{i} + 3\bar{j} + 4\bar{k}) = 4 + 12 - 16 = 0$

$\therefore$ $(\bar{a}\times\bar{b})\times\bar{c})$ is perpendicular to $\bar{a}$.

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23. **If A, B,C are angles in a triangle, then prove that $\cos^2 A + \cos^2 B - \cos^2 C = 1 - 2\sin A\sin B\cos C$.**

Sol: Given A,B,C are angles of a triangle, then A+B+C=180º

L.H.S $= \cos^2 A + (\cos^2 B - \cos^2 C)$

$= \cos^2 A + \sin(C+B)\sin(C-B)$

$= \cos^2 A - \sin(180^\circ - A)\sin(B-C)$ $[\because \sin(-\theta) = -\sin\theta]$

$= \cos^2 A - \sin A\sin(B-C)$

$= 1 - \sin^2 A - \sin A\sin(B-C)$

$= 1 - \sin A[\sin A + \sin(B-C)]$

$= 1 - \sin A[\sin(180^\circ - (B+C)) + \sin(B-C)]$

$= 1 - \sin A[\sin(B+C)) + \sin(B-C)]$

$= 1 - \sin A(2\sin B\cos C)$ $[\because (\sin(A+B)) + \sin(A-B)) = 2\sin A\cos B]$

$= 1 - 2\sin A\sin B\cos C = $ R.H.S

24. In a ΔABC if $a = 13, b = 14, c = 15$ then show that $R = \frac{65}{8}, r = 4, r_1 = \frac{21}{2}, r_2 = 12, r_3 = 14$

A: Given $a = 13, b = 14, c = 15$, then

$$2s = a + b + c = 13 + 14 + 15 = 42 \Rightarrow 2s = 42 \Rightarrow s = 21$$

Now $\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21(21-13)(21-14)(21-15)}$

$$= \sqrt{21\times(8)(7)(6)} = \sqrt{(3\times7)(4\times2)(7)(3\times2)} = \sqrt{3^2\times4^2\times7^2} = 3\times4\times7 = 84$$

(i) $R = \frac{abc}{4\Delta} = \frac{13\times14\times15}{4\times84} = \frac{65}{8}$

(ii) $r = \frac{\Delta}{s} = \frac{84}{21} = 4$;

(iii) $r_1 = \frac{\Delta}{s-a} = \frac{84}{21-13} = \frac{84}{8} = \frac{21}{2}$

(iv) $r_2 = \frac{\Delta}{s-b} = \frac{84}{21-14} = \frac{84}{7} = 12$

(v) $r_3 = \frac{\Delta}{s-c} = \frac{84}{21-15} = \frac{84}{6} = 14$