Previous IPE

SOLVED PAPERS

MARCH -2024(AP)

PREVIOUS PAPERS

IPE: MARCH-2024(AP)

Time : 3 Hours

MATHS-2A

Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **$10 \times 2 = 20$**

1. Write the conjugate of $\frac{5i}{7+i}$
2. Simplify $-2i(3+i)(2+4i)(1+i)$ and obtain the modulus of that complex number.
3. If $1, \omega, \omega^2$ are cube roots of unity, then prove that $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11}) = 49$
4. For what values of x the expression $x^2-5x-14$ is positive?
5. Form polynomical equations of the lowest degree, with roots 1,–1,3.
6. There are 4 copies alike each of 3 different books. Find the number of ways of arranging these 12 books in a shelf in a single row.
7. If ${}^nP_r=5040$ and ${}^nC_r = 210$, find n and r.
8. Write down and simplify the 6^{th} term in $\left(\frac{2x}{3}+\frac{3y}{2}\right)^9$
9. Find the mean deviation about median for the data 4, 6, 9, 3, 10, 13, 2
10. The mean and variance of a binomial distribution are 4, 3 respectively. Find the distribution and find $P(X\geq 1)$.

SECTION-B

II. Answer any FIVE of the following SAQs: **$5 \times 4 = 20$**

11. Show that the points in the Argand plane represented by the complex numbers $-2+7i, \frac{-3}{2}+\frac{1}{2}i, 4-3i, \frac{7}{2}(1+i)$ are the vertices of a rhombus.
12. If the expression $\frac{x-p}{x^2-3x+2}$ takes all real values for $x \in R$ then find the bounds for p.
13. Find the number of 5 letter words that can be formed using the letters of the word CONSIDER. How many of them begin with 'C', how many of them end with 'R' and how many of them begin with 'C' and end with 'R'?
14. A question paper is divided into 3 sections A,B,C containing 3,4,5 questions respectively. Find the number of ways of attempting 6 questions choosing atleast one from each selection.
15. Resolve $\frac{2x^2+3x+4}{(x-1)(x^2+2)}$ into Partial Fractions
16. A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.
17. A fair die is rolled. Consider the events A={1,3,5}, B={2,3} and C={2,3,4,5}. Find (i) $P(A\cap B), P(A\cup B)$ (ii) $P(A|B), P(B|A)$ (iii) $P(A|C), P(C|A)$ (iv) $P(B|C), P(C|B)$

SECTION-C

III. Answer any FIVE of the following LAQs: **$5 \times 7 = 35$**

18. If $\cos\alpha+\cos\beta+\cos\gamma = 0 = \sin\alpha+\sin\beta+\sin\gamma$, then show that (i) $\cos3\alpha+\cos3\beta+\cos3\gamma=3\cos(\alpha+\beta+\gamma)$ (ii) $\sin3\alpha+\sin3\beta+\sin3\gamma=3\sin(\alpha+\beta+\gamma)$
19. Solve the equation $6x^6-25x^5+31x^4-31x^2+25x-6 = 0$.
20. If the coefficients of 4 consecutive terms in $(1+x)^n$ are a_1,a_2,a_3,a_4 respectively, show that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$
21. Find the sum of the infinite series $\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}-......$
22. Find the mean deviation about the mean for the following data:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Students	6	5	8	15	7	6	3

23. State and Prove "Addition theorem on Probability.
24. A random variable x has the following probability distribution

$X=x_i$	0	1	2	3	4	5	6	7
$P(X=x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) the mean (iii) $P(0<X<5)$

IPE AP MARCH-2024
SOLUTIONS

SECTION-A

1. Write the conjugate of $\frac{5i}{7+i}$

Sol: $\frac{5i}{7+i}=\frac{5i(7-i)}{(7+i)(7-i)}=\frac{35i-5i^2}{7^2+1^2}=\frac{35i-5(-1)}{7^2+1^2}=\frac{35i+5}{50}=\frac{5(7i+1)}{5\times10}=\frac{7i+1}{10}=\frac{1+7i}{10}$

$\therefore$ Conjugate of $\frac{1+7i}{10}$ is $\frac{1-7i}{10}$

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2. Simplify $-2i(3+i)\,(2+4i)\,(1+i)$ and obtain the modulus of that complex number.

Sol: G.E $=-2i\,(3+i)\,(2+4i)\,(1+i)=[-2i\,(3+i)]\,[(2+4i)\,(1+i)]$

$=(-6i-2i^2)\,(2+2i+4i+4i^2)=(-6i-2(-1))\,(2+6i+4(-1))$

$=(2-6i)\,(-2+6i)=-4+12i+12i-36i^2=-4+12i+12i-36(-1)=32+24i=8\,(4+3i)$

$\therefore$ Modulus $=|8(4+3i)|=8\sqrt{4^2+3^2}=8\sqrt{25}=8(5)=40$

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3. If $1,\omega,\omega^2$ are the cube roots of unity then prove that $(2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11})=49$

Sol: $\omega^{10}=(\omega^9)\omega=(\omega^3)^3\omega=1(\omega)=\omega$; $\omega^{11}=(\omega^{10})\omega=(\omega)\omega=\omega^2$

$\therefore\ (2-\omega)(2-\omega^2)(2-\omega^{10})(2-\omega^{11})$

$=(2-\omega)(2-\omega^2)(2-\omega)(2-\omega^2)=(2-\omega)^2(2-\omega^2)^2=[(2-\omega)(2-\omega^2)]^2$

$=[4-2\omega^2-2\omega+\omega^3]^2=[4-2(\omega^2+\omega)+1]^2=[5-2(\omega^2+\omega)]^2=[5-2(-1)]^2$

$=(5+2)^2=7^2=49$

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4. For what value of x, the expression $x^2-5x-14$ is positive.

Sol: Comparing $x^2-5x-14$ with $ax^2+bx+c=0$ we get $a=1$, $b=-5$, $c=-14$.

Now $\Delta=b^2-4ac=(-5)^2-4(1)(-14)=25+56=81>0$

Here Δ is positive. $\therefore$ The roots are real

Now, $x^2-5x-14=0 \Rightarrow x^2-7x+2x-14=0 \Rightarrow x(x-7)+2(x-7)=0$

$\Rightarrow (x+2)(x-7)=0 \Rightarrow x=-2$ or 7

Also $a=1>0$.

$\therefore$ 1, $x^2-5x-14$ have the same sign for $x<\alpha$ or $x>\beta$ $\Rightarrow x^2-5x-14$ is positive for $x<-2$ or $x>7$

5. Form a polynomial equation of the lowest degree, with roots 1, –1, 3.

Sol: Equation having roots α, β, γ is $(x-\alpha)(x-\beta)(x-\gamma) = 0$

Required equation is $(x-1)(x+1)(x-3) = 0 \Rightarrow (x^2-1)(x-3) = 0 \Rightarrow x^3 - 3x^2 - x + 3 = 0$

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6. There are 4 copies alike each of 3 different books. Find the number of ways of arranging these 12 books in a shelf in a single row.

Sol: The total number of books = $3\times4=12$

Among 12 books; 4 books are alike of one kind, 4 books are alike of second kind and 4 books are alike of third kind.

$\therefore$ The required number of ways $= \dfrac{n!}{p!q!r!} = \dfrac{12!}{4!\,4!4!}$

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7. If ${}^nP_r = 5040$ and ${}^nC_r = 210$ find n and r.

Sol: We know that $r! = \dfrac{{}^nP_r}{{}^nC_r} = \dfrac{5040}{210} = 24 = 4!$ $\therefore r! = 4! \Rightarrow r = 4$

${}^nP_4 = 5040 = 10\times504 = 10\times9\times56 = 10\times9\times8\times7 = {}^{10}P_4 \Rightarrow n = 10$

$\therefore$ n = 10, r = 4.

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8. Write down and simplify the 6^{th} term in $\left(\dfrac{2x}{3}+\dfrac{3y}{2}\right)^9$

Sol: We know in $(x+y)^n$, $T_{r+1} = {}^nC_r x^{n-r} y^r$.

$$\therefore T_6 = T_{5+1} = {}^9C_5\left(\frac{2x}{3}\right)^{9-5}\left(\frac{3y}{2}\right)^5 = {}^9C_5\left(\frac{2}{3}\right)^4 x^4\left(\frac{3}{2}\right)^5 y^5$$

$$= \frac{9\times8\times7\times6}{1\times2\times3\times4}\times\frac{2^4}{3^4}\times\frac{3^5}{2^5}x^4y^5 = 27\times7x^4y^5 = 189x^4y^5$$

9. Find the mean deviation about median for the data 4, 6, 9, 3, 10, 13, 2

Sol: **Given data:** 4, 6, 9, 3, 10, 13, 2.

Its ascending order : 2,3,4,6,9,10,13.

Number of observations n = 7 is odd .

∴ Median is the middle most term ⇒M=6

Deviations from the median:

2–6= –4; 3–6= –3; 4–6= –2; 6–6=0; 9–6=3;10–6=4; 13–6=7

Absolute values of these deviations:

4, 3, 2, 0, 3, 4, 7

∴ M.D from Median is $M.D=\frac{\Sigma|x_i-M|}{7}=\frac{4+3+2+0+3+4+7}{7}=\frac{23}{7}=3.29$

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10. The mean and variance of a binomial distribution are 4 and 3 respectively. Find the distribution and find P(X≥1)

Sol: Given mean np = 4 ,variance npq = 3

Now, (np)q = 3 $\Rightarrow (4)q=3 \Rightarrow q=\frac{3}{4} \Rightarrow p=1-q=1-\frac{3}{4}=\frac{4-3}{4}=\frac{1}{4}$

Take $np=4 \Rightarrow n\left(\frac{1}{4}\right)=4 \Rightarrow n=4(4)=16$

∴ n=16 , q=3/4 and p=1/4

Binomial distribution is $P(X=r)={}^{n}C_r q^{n-r}.p^r={}^{16}C_r\left(\frac{3}{4}\right)^{16-r}.\left(\frac{1}{4}\right)^r$

∴ $P(X\geq 1)=1-P(X=0)=1-q^n=1-\left(\frac{3}{4}\right)^n=1-\left(\frac{3}{4}\right)^{16}$

SECTION-B

11. Show that the points in the Argand plane represented by the complex numbers $-2+7i, \frac{-3}{2}+\frac{1}{2}i, 4-3i, \frac{7}{2}(1+i)$ are the vertices of a rhombus.

Sol: Given complex numbers are taken as $A(-2,7), B\left(-\frac{3}{2},\frac{1}{2}\right), C(4,-3); D\left(\frac{7}{2},\frac{7}{2}\right)$

$$AB=\sqrt{\left(-2+\frac{3}{2}\right)^2+\left(7-\frac{1}{2}\right)^2}=\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{13}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{169}{4}}=\frac{\sqrt{170}}{2}$$

$$BC=\sqrt{\left(4+\frac{3}{2}\right)^2+\left(-3-\frac{1}{2}\right)^2}=\sqrt{\left(\frac{11}{2}\right)^2+\left(\frac{-7}{2}\right)^2}=\sqrt{\frac{121}{4}+\frac{49}{4}}=\frac{\sqrt{170}}{2}$$

$$CD=\sqrt{\left(4-\frac{7}{2}\right)^2+\left(-3-\frac{7}{2}\right)^2}=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{-13}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{169}{4}}=\frac{\sqrt{170}}{2}$$

$$DA=\sqrt{\left(\frac{7}{2}+2\right)^2+\left(\frac{7}{2}-7\right)^2}=\sqrt{\left(\frac{11}{2}\right)^2+\left(\frac{-7}{2}\right)^2}=\sqrt{\frac{121}{4}+\frac{49}{4}}=\frac{\sqrt{170}}{2}$$

$$AC=\sqrt{(4+2)^2+(-3-7)^2}=\sqrt{6^2+(-10)^2}=\sqrt{36+100}=\sqrt{136}$$

$$BD=\sqrt{\left(\frac{7}{2}+\frac{3}{2}\right)^2+\left(\frac{7}{2}-\frac{1}{2}\right)^2}=\sqrt{\left(\frac{10}{2}\right)^2+\left(\frac{6}{2}\right)^2}=\sqrt{\frac{100}{4}+\frac{36}{4}}=\frac{\sqrt{136}}{2}$$

Hence, the four sides AB, BC, CD, DA are equal.

The two diagonals AC, BD are unequal.

$\therefore$ A, B, C, D form a Rhombus.

12. If the expression $\dfrac{x-p}{x^2-3x+2}$ takes all real values for $x \in R$ then find the bounds for p

Sol: Let $y = \dfrac{x-p}{x^2-3x+2} \Rightarrow y(x^2-3x+2) = x-p$

$\Rightarrow yx^2-3yx+2y=x-p \Rightarrow yx^2+(-3y-1)x+(2y+p)=0$ ...... (1)

(1) is a quadratic in x and its roots are reals.

$\therefore \Delta=b^2-4ac\geq 0$

$\Rightarrow (-3y-1)^2-4y(2y+p)\geq 0 \Rightarrow 9y^2+6y+1-8y^2-4py\geq 0 \Rightarrow y^2+(6-4p)y+1\geq 0$ ..... (2)

But y is real. Also coefficient of y^2 is positive.

$\therefore$ (2) holds true only when the roots of $y^2+(6-4p)y+1=0$ are imaginary or real & equal.

$\Rightarrow \Delta=b^2-4ac \leq 0 \Rightarrow (6-4p)^2-4\leq 0 \Rightarrow 36+16p^2-48p-4\leq 0$

$\Rightarrow 16p^2-48p+32\leq 0 \Rightarrow 16(p^2-3p+2)\leq 0$

$\Rightarrow p^2-3p+2\leq 0 \Rightarrow (p-1)(p-2)\leq 0 \Rightarrow 1\leq p\leq 2$

But if x=p=1 or 2, then $\dfrac{x-p}{x^2-3x+2} = \dfrac{x-p}{(x-1)(x-2)}$ takes the indeterminant form $\dfrac{0}{0}$

$\therefore\ 1 < p < 2$

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13. Find the number of 5 letter words that can be formed using the letters of the word CONSIDER. How many of them begin with 'C', how many of them end with 'R' and how many of them begin with 'C' and end with 'R'?

Sol: **(i)**The given word CONSIDER contains 8 letters.

So, the number of 5 letter words formed from it = $^8P_5 = 8\times 7\times 6\times 5\times 4 = 6720$

(ii) 5 letter words begining with C:

C				

Fill the first place with C. Then the remaining 5 places can be filled with the remaining 7 lettersin $^7P_4 = 7\times 6\times 5\times 4$ =840ways.

(iii) 5 letter words ending with R:

				R

Fill the last place with R. Then the remaining 4 places can be filled with the remaining 7 letters in $^7P_4 = 7\times 6\times 5\times 4$ =840ways.

(iv) 5 letter words beginning with C and ending with R:

C				R

Fill the first place with C and last place with R.

Then the remaining 3 places can be filled with remaining 6 letters in $^6P_3 = 6\times 5\times 4 = 120$ ways

14. A question paper is divided into 3 sections A,B,C containing 3,4,5 questions respectively. Find the number of ways of attempting 6 questions choosing atleast one from each selection.

Sol: The different possible compositions of attempting 6 questions choosing atleast one from each section are shown below:

Section-A 3	Section-B 4	Section-C 5	No. of selections
3	2	1	${}^3C_3 \times {}^4C_2 \times {}^5C_1 = 1\times6\times5=30$
3	1	2	${}^3C_3 \times {}^4C_1 \times {}^5C_2 = 1\times4\times10=40$
2	3	1	${}^3C_2 \times {}^4C_3 \times {}^5C_1 = 3\times4\times5=60$
2	2	2	${}^3C_2 \times {}^4C_2 \times {}^5C_2 = 3\times6\times10=180$
2	1	3	${}^3C_2 \times {}^4C_1 \times {}^5C_3 = 3\times4\times10=120$
1	4	1	${}^3C_1 \times {}^4C_4 \times {}^5C_1 = 3\times1\times5=15$
1	3	2	${}^3C_1 \times {}^4C_3 \times {}^5C_2 = 3\times4\times10=120$
1	2	3	${}^3C_1 \times {}^4C_2 \times {}^5C_3 = 3\times6\times10=180$
1	1	4	${}^3C_1 \times {}^4C_1 \times {}^5C_4 = 3\times4\times5=60$

$\therefore$ The required number = 30+40+60+180+120+15+120+180+60 = 805.

15. Resolve $\dfrac{2x^2+3x+4}{(x-1)(x^2+2)}$ into partial fractions

Sol: Let $\dfrac{2x^2+3x+4}{(x-1)(x^2+2)}=\dfrac{A}{x-1}+\dfrac{Bx+C}{x^2+2}=\dfrac{A(x^2+2)+(Bx+1)(x-1)}{(x-1)(x^2+2)}$

$\therefore A(x^2+2)+(Bx+C)(x-1)=2x^2+3x+4.......(1)$

Putting $x=1$ in(1) we get $A(1^2+2)+(Bx+C)(0)=2(1^2)+3(1)+4$

$\Rightarrow 3A=9\Rightarrow A=3$

Putting $x=0$ in(1) we get $A(0+2)+(0+C)(0-1)=4\Rightarrow 2A-C=4$

$\Rightarrow C=2A-4=2(3)-4=2$

Comparing the coeff.of x^2in(1), we get $A+B=2\Rightarrow 3+B=2\Rightarrow B=-1$

$\therefore \dfrac{2x^2+3x+4}{(x-1)(x^2+2)}=\dfrac{3}{x-1}+\dfrac{(-1)x+2}{x^2+2}=\dfrac{3}{x-1}+\dfrac{2-x}{x^2+2}$

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16. A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.

Sol: Let A,B denote the events of speaking truth by A,B respectively

$P(A)=\dfrac{75}{100}=\dfrac{3}{4};\ P(B)=\dfrac{80}{100}=\dfrac{4}{5}$

$\therefore P(\bar{A})=1-P(A)=1-\dfrac{3}{4}=\dfrac{1}{4};\quad P(\bar{B})=1-P(B)=1-\dfrac{4}{5}=\dfrac{1}{5}$

Let E be the event that A and B contradict to each other

$\Rightarrow P(E)=P\left[(A\cap\bar{B})\cup(\bar{A}\cap B)\right]=P(A\cap\bar{B})+P(\bar{A}\cap B)$

$=P(A)P(\bar{B})+P(\bar{A})P(B)$ [$\because$ A,B are independent] $=\dfrac{3}{4}.\dfrac{1}{5}+\dfrac{1}{4}.\dfrac{4}{5}=\dfrac{7}{20}$

17. A fair die is rolled. Consider the events A={1,3,5}, B={2,3} and C={2,3,4,5}. Find (i) $P(A\cap B)$, $P(A\cup B)$ (ii) $P(A|B)$, $P(B|A)$ (iii) $P(A|C)$, $P(C|A)$ (iv) $P(B|C)$, $P(C|B)$

Sol: When a die is rolled $S = \{1, 2, 3, 4, 5, 6\} \Rightarrow n(S)=6$

Given that $A=\{1,3,5\}$, $B=\{2,3\}$ and $C=\{2,3,4,5\}$

$\therefore A\cap B = \{3\}$, $A\cup B=\{1,2,3,5\}$, $A\cap C = \{3, 5\}$, $B\cap C = \{2, 3\}$

(i) $P(A\cap B)=\frac{1}{6}$, $P(A\cup B)=\frac{4}{6}=\frac{2}{3}$

(ii) $P(A/B)=\frac{n(A\cap B)}{n(B)}=\frac{1}{2}$, $P(B/A)=\frac{n(A\cap B)}{n(A)}=\frac{1}{3}$

(iii) $P(A/C)=\frac{n(A\cap C)}{n(C)}=\frac{2}{4}=\frac{1}{2}$, $P(C/A)=\frac{n(A\cap C)}{n(A)}=\frac{2}{3}$

(iv) $P(B/C)=\frac{n(B\cap C)}{n(C)}=\frac{2}{4}=\frac{1}{2}$, $P(C/B)=\frac{n(B\cap C)}{n(B)}=\frac{2}{2}=1$

SECTION-C

18. If cosα+cosβ+cosγ=0=sinα+sinβ+sinγ, then show that

(i) cos3α+cos3β+cos3γ=3cos(α+β+γ)

(ii) sin3α+sin3β+sin3γ=3sin(α+β+γ)

Sol: **(i)** Given that cosα+cosβ+cosγ=0=sinα+sinβ+sinγ

Let $a = \cos\alpha + i\sin\alpha = \text{cis}\alpha, b = \cos\beta + i\sin\beta = \text{cis}\beta, c = \cos\gamma + i\sin\gamma = \text{cis}\gamma$

Now, $a + b + c = (\cos\alpha + i\sin\alpha) + (\cos\beta + i\sin\beta) + (\cos\gamma + i\sin\gamma)$

$$= (\cos\alpha + \cos\beta + \cos\gamma) + i(\sin\alpha + \sin\beta + \sin\gamma) = 0 + i(0) = 0$$

$\therefore a+b+c=0 \Rightarrow a^3 + b^3 + c^3 = 3abc$

$\Rightarrow (\text{cis}\alpha)^3 + (\text{cis}\beta)^3 + (\text{cis}\gamma)^3 = 3\text{cis}\alpha.\text{cis}\beta.\text{cis}\gamma$

$\Rightarrow \text{cis}3\alpha + \text{cis}3\beta + \text{cis}3\gamma = 3\text{cis}(\alpha + \beta + \gamma)$

$\Rightarrow (\cos 3\alpha + i\sin 3\alpha) + (\cos 3\beta + i\sin 3\beta) + (\cos 3\gamma + i\sin 3\gamma) = 3[\cos(\alpha + \beta + \gamma) + i\sin(\alpha + \beta + \gamma)]$

$\Rightarrow (\cos 3\alpha + \cos 3\beta + \cos 3\gamma) + i(\sin 3\alpha + \sin 3\beta + \sin 3\gamma) = 3\cos(\alpha + \beta + \gamma) + i.3\sin(\alpha + \beta + \gamma)$

(ii) Equating the real parts, we get $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$

Hence (i) is proved.

Equating the imaginary parts, we get $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$

Hence (ii) is proved.

19. Solve $6x^6-25x^5+31x^4-31x^2+25x-6=0$

Sol: The degree of the given equation is n=6, which is even. Also $a_k=-a_{n-k}\ \forall\ k=0,1,2,3,4,5,6$

Hence the given equation is a reciprocal equation of class II of even degree

Hence 1,–1 are the roots of $6x^6-25x^5+31x^4-31x^2+25x-6=0$

On dividing the expression by (x–1), (x+1), we have

1	6	–25	31	0	–31	25	–6
	0	6	–19	12	12	–19	6
-1	6	–19	12	12	–19	6	0
	0	–6	25	–37	25	–6	
	6	–25	37	–25	6	0	

Now, we solve the S.R.E $6x^4-25x^3+37x^2-25x+6=0$

On dividing this equation by x^2, we get

$$6x^2-25x+37-\frac{25}{x}+\frac{6}{x^2}=0\Rightarrow 6\left(x^2+\frac{1}{x^2}\right)-25\left(x+\frac{1}{x}\right)+37=0 \quad \text{....(1)}$$

Put $x+\frac{1}{x}=y$, so that $x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=y^2-2$

$\therefore$ (1) $\Rightarrow 6(y^2-2)-25(y)+37=0\Rightarrow 6y^2-12-25y+37=0$

$\Rightarrow 6y^2-25y+25=0\Rightarrow 6y^2-15y-10y+25=0\Rightarrow 3y(2y-5)-5(2y-5)=0$

$\Rightarrow (2y-5)(3y-5)=0 \Rightarrow y=5/2$ (or) $5/3$

If $y=\frac{5}{2}$ then $x+\frac{1}{x}=\frac{5}{2}=2\frac{1}{2}=2+\frac{1}{2}\Rightarrow x=2$ (or) $\frac{1}{2}$

If $y=\frac{5}{3}$ then $x+\frac{1}{x}=\frac{5}{3}\Rightarrow\frac{x^2+1}{x}=\frac{5}{3}$

$\Rightarrow 3x^2+3=5x\Rightarrow 3x^2-5x+3=0$

$$\Rightarrow x=\frac{5\pm\sqrt{(-5)^2-4(3)(3)}}{6}=\frac{5\pm\sqrt{25-36}}{6}=\frac{5\pm\sqrt{-11}}{6}$$

$$\therefore x=\frac{5\pm\sqrt{11}i}{6}$$

Hence all the six roots of the given equation are $1,-1,2,\frac{1}{2},\frac{5\pm\sqrt{11}i}{6}$

20. If the coefficients of 4 consecutive terms in the expansion of $(1+x)^n$ are a_1, a_2, a_3, a_4 respectively, then show that $\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}=\frac{2a_2}{a_2+a_3}$

Sol : We take the coefficients of 4 consecutive terms of $(1+x)^n$ as follows:

$a_1 = {}^nC_r$, $a_2 = {}^nC_{r+1}$, $a_3 = {}^nC_{r+2}$, $a_4 = {}^nC_{r+3}$.

$$\text{L.H.S} = \frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4} = \frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}+\frac{{}^nC_{r+2}}{{}^nC_{r+2}+{}^nC_{r+3}}$$

$$= \frac{{}^nC_r}{{}^{(n+1)}C_{r+1}}+\frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}} \quad \left(\because {}^nC_r+{}^nC_{r+1} = {}^{(n+1)}C_{r+1}\right)$$

$$= \frac{{}^nC_r}{\left(\frac{n+1}{r+1}\right){}^nC_r}+\frac{{}^nC_{r+2}}{\left(\frac{n+1}{r+3}\right){}^nC_{r+2}} \quad \left(\because {}^nC_r = \left(\frac{n}{r}\right){}^{n-1}C_{r-1}\right)$$

$$= \frac{r+1}{n+1}+\frac{r+3}{n+1} = \frac{r+1+r+3}{n+1} = \frac{2r+4}{n+1} = \frac{2(r+2)}{n+1} \quad ....(1)$$

$$\text{R.H.S} = \frac{2a_2}{a_2+a_3} = \frac{2({}^nC_{r+1})}{{}^nC_{r+1}+{}^nC_{r+2}} = \frac{2({}^nC_{r+1})}{{}^{(n+1)}C_{r+2}} = \frac{2\,({}^nC_{r+1})}{\left(\frac{n+1}{r+2}\right)({}^nC_{r+1})} = \frac{2}{\frac{n+1}{r+2}} = \frac{2(r+2)}{n+1} \quad ...(2)$$

From (1) & (2), L.H.S=R.H.S

21. Find the sum of the infinite series $\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}-......$

Sol: Let $S=\frac{3}{4.8}-\frac{3.5}{4.8.12}+\frac{3.5.7}{4.8.12.16}-.....=\frac{1.3}{4.8}-\frac{1.3.5}{4.8.12}+\frac{1.3.5.7}{4.8.12.16}-........$

On adding $1-\frac{1}{4}$ both sides, we get $1-\frac{1}{4}+S=1-\frac{1}{4}+\frac{1.3}{4.8}-\frac{1.3.5}{4.8.12}+......$

$$\Rightarrow \frac{3}{4}+S=1-\frac{1}{1}.\left(\frac{1}{4}\right)+\frac{1.3}{1.2}\left(\frac{1}{4}\right)^2-\frac{1.3.5}{1.2.3}\left(\frac{1}{4}\right)^3+......$$

Comparing the above series with

$$1-\frac{p}{1}.\left(\frac{x}{q}\right)+\frac{p(p+q)}{1.2}\left(\frac{x}{q}\right)^2-\frac{p(p+q)(p+2q)}{1.2.3}\left(\frac{x}{q}\right)^3+......=(1+x)^{\frac{-p}{q}}$$

we get p =1, p+q = 3 ⇒ 1+q= 3 ⇒q = 2. Also $\frac{x}{q}=\frac{1}{4}\Rightarrow x=\frac{q}{4}=\frac{2}{4}=\frac{1}{2}$

Hence, $\frac{3}{4}+S=(1+x)^{\frac{-p}{q}}=\left(1+\frac{1}{2}\right)^{\frac{-1}{2}}=\left(\frac{3}{2}\right)^{\frac{-1}{2}}=\left(\frac{2}{3}\right)^{\frac{1}{2}}=\sqrt{\frac{2}{3}}\Rightarrow S=\sqrt{\frac{2}{3}}-\frac{3}{4}$

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22. Find the mean deviation about the mean for the following continuous distribution:

Marks	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**
No. of students	**6**	**5**	**8**	**15**	**7**	**6**	**3**

Sol: We take the assumed mean A=35. Here, C=10. Hence we form the following table:

Class interval	**Midpoint(x_i)**	**Number of students(f_i)**	$d_i=\frac{x_i-25}{10}$	f_id_i	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
0-10	5	6	–3	–18	28.4	170.4
10-20	15	5	–2	–10	18.4	92
20-30	25	8	–1	–8	8.4	67.2
30-40	35	15	0	0	1.6	24.0
40-50	45	7	1	7	11.6	81.2
50-60	55	6	2	12	21.6	129.6
60-70	65	3	3	9	31.6	94.8
		$\Sigma f_i=50=N$		$\Sigma f_id_i=-8$		659.2

Here N=50. So, Mean $\bar{x}=A+C\left(\frac{\Sigma f_id_i}{N}\right)=35+10\left(\frac{-8}{50}\right)=35-\frac{8}{5}=35-1.6=33.4$

Mean deviation about the mean = $\frac{1}{N}\Sigma f_i\lvert x_i-\bar{x}\rvert=\frac{1}{50}(659.2)=13.18$

23. State and Prove "Addition theorem on Probability.

Sol: ***Statement*: If E_1, E_2 are the 2 events of a sample space S then $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$**

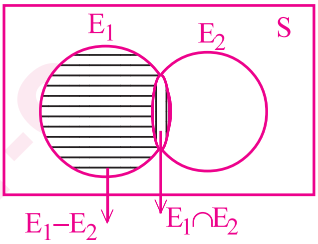
Proof: Case (i): When $E_1 \cap E_2 = \phi$

$$E_1 \cap E_2 = \phi \Rightarrow P(E_1 \cap E_2) = 0$$

$$\therefore P(E_1 \cup E_2) = P(E_1) + P(E_2) \quad \text{[From the Union axiom]}$$

$$= P(E_1) + P(E_2) - 0 = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Case (ii) : When $E_1 \cap E_2 \neq \phi$



$E_1 \cup E_2$ is the union of disjoint sets $(E_1 - E_2), E_2$

$\therefore P(E_1 \cup E_2) = P[(E_1 - E_2) \cup E_2] = P(E_1 - E_2) + P(E_2)$ ....(1)

E_1 is the union of disjoint sets $(E_1 - E_2), (E_1 \cap E_2)$.

$\therefore P(E_1) = P[(E_1 - E_2) \cup (E_1 \cap E_2)] = P(E_1 - E_2) + P(E_1 \cap E_2)$

$\Rightarrow P(E_1 - E_2) = P(E_1) - P(E_1 \cap E_2)$

$\therefore$ from (1), $P(E_1 \cup E_2) = [P(E_1) - P(E_1 \cap E_2)] + P(E_2)$

$= P(E_1) + P(E_2) - P(E_1 \cap E_2)$. Hence proved.

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24. A random variable x has the following probability distribution

$X = x_i$	0	1	2	3	4	5	6	7
$P(X = x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

Find (i) k (ii) the mean (iii) P(0<X<5)

Sol: We know $\Sigma P(X = x_i) = 1$

$\Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1 \Rightarrow 10k^2 + 9k = 1 \Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow 10k^2 + 10k - k - 1 = 0 \Rightarrow 10k(k+1) - 1(k+1) = 0 \Rightarrow (10k-1)(k+1) = 0 \Rightarrow k = 1/10$, (since $k > 0$)

(i) $k = 1/10$

(ii) Mean $\mu = \sum_{i=1}^{n} x_i . P(X = x_i) = 0(0) + 1(k) + 2(2k) + 3(2k) + 4(3k) + 5(k^2) + 6(2k^2) + 7(7k^2 + k)$

$= 0 + k + 4k + 6k + 12k + 5k^2 + 12k^2 + 49k^2 + 7k = 66k^2 + 30k$

$$= 66\left(\frac{1}{100}\right) + 30\left(\frac{1}{10}\right) = 0.66 + 3 = 3.66$$

(iii) $P(0 < X < 5) = P(X=1) + P(X=2) + P(X=3) + P(X=4) = k + 2k + 2k + 3k = 8k = 8\left(\frac{1}{10}\right) = \frac{8}{10} = \frac{4}{5}$