

8. విలాసీమ త్రికోణమితీయ ప్రమేయాలు

IPE : 1 SAQ = 4 Marks

☞ ముఖ్యమైన సూత్రాలు, నిర్వచనాలు ☜

$$1. \sin^{-1}(\sin \theta) = \theta \text{ for } \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\cos^{-1}(\cos \theta) = \theta \text{ for } \theta \in [0, \pi]$$

$$\tan^{-1}(\tan \theta) = \theta \text{ for } \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\sin(\sin^{-1}x) = x \text{ for } x \in [-1, 1]$$

$$\cos(\cos^{-1}x) = x \text{ for } x \in [-1, 1]$$

$$\tan(\tan^{-1}x) = x \text{ for } x \in \mathbb{R}$$

$$2. \sin^{-1}(-x) = -\sin^{-1}x; \cos^{-1}(-x) = \pi - \cos^{-1}x; \tan^{-1}(-x) = -\tan^{-1}x; \cot^{-1}(-x) = \pi - \cot^{-1}x$$

$$3. \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}; \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}; \sec^{-1}x + \csc^{-1}x = \frac{\pi}{2}$$

$$4. (i) \sin^{-1}(x) = \operatorname{Cosec}^{-1} \frac{1}{x} \text{ for } x \in [-1, 0) \cup (0, 1] \quad (ii) \cos^{-1}(x) = \operatorname{Sec}^{-1} \frac{1}{x} \text{ for } x \in [-1, 0) \cup (0, 1]$$

$$(iii) \cot^{-1}x = \tan^{-1} \frac{1}{x} \text{ if } x > 0$$

$$(iv) \cot^{-1}x = \pi + \tan^{-1} \frac{1}{x} \text{ if } x < 0$$

$$5. \sin^{-1}x + \sin^{-1}y = \sin^{-1} \left[x\sqrt{1-y^2} + y\sqrt{1-x^2} \right] \text{ if } x \geq 0, y \geq 0 \text{ and } x^2 + y^2 \leq 1$$

$$6. \cos^{-1}x + \cos^{-1}y = \cos^{-1} \left[xy - \sqrt{(1-x^2)(1-y^2)} \right] \text{ if } x, y \in [0, 1]$$

$$7. \tan^{-1}x + \tan^{-1}y = \begin{cases} \tan^{-1} \left(\frac{x+y}{1-xy} \right), & \text{if } xy < 1 \\ \tan^{-1} \left(\frac{x+y}{1-xy} \right) + \pi, & \text{if } xy > 1, x > 0, y > 0 \\ \tan^{-1} \left(\frac{x+y}{1-xy} \right) - \pi, & \text{if } xy > 1, x < 0, y < 0 \\ \frac{\pi}{2}, & \text{if } xy = 1 \end{cases}$$

$$8. \tan^{-1}x - \tan^{-1}y = \tan^{-1} \left(\frac{x-y}{1+xy} \right)$$

$$9. 2\sin^{-1}x = \sin^{-1} \left(2x\sqrt{1-x^2} \right)$$

$$2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$$

$$2\tan^{-1}x = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$$

$$3\sin^{-1}x = \sin^{-1}(3x - 4x^3)$$

$$3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$$

$$3\tan^{-1}x = \tan^{-1} \left(\frac{3x - x^3}{1-3x^2} \right)$$