

7.నిశ్చిత సమకాలనులు

IPE : 2VSAQ , 1 SAQ & 1 LAQ = 2 + 2 + 4 + 7 = 15 Marks

ముఖ్యమైన సూత్రాలు, నిర్వచనాలు

$$1) [a,b] \text{ లో నిర్వచించబడిన } f(x) \text{ యొక్క సమాకలని } F(x) \text{ అయిన } \int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

$$2) \int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(\theta)d\theta$$

$$3) \int_a^b f(x)dx = - \int_b^a f(x)dx$$

$$4) \int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \text{ for } a < c < b$$

$$5) \int_{-a}^a f(x)dx = \begin{cases} 2 \int_0^a f(x)dx, \\ 0, \end{cases}$$

$$6.1) \int_a^b f(x)dx = \int_a^b f(a+b-x)dx$$

$$6.2) \int_0^a f(x)dx = \int_0^a f(a-x)dx$$

$$7.1) \int_0^a f(x)dx = \begin{cases} 2 \int_0^{a/2} f(x)dx, & \text{if } f(a-x) = f(x) \\ 0, & \text{if } f(a-x) = -f(x) \end{cases} \quad 7.2) \int_0^{2a} f(x)dx = \begin{cases} 2 \int_0^a f(x)dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

$$8) f(x) \text{ అనే ప్రమేయపు పీరియడ్ } a \text{ అయిన } \int_0^{na} f(x)dx = n \int_0^a f(x)dx$$

$$9.1) 'n' \text{ సరి సంఖ్య అయిన } \int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx = \left(\frac{n-1}{n}\right) \left(\frac{n-3}{n-2}\right) \left(\frac{n-5}{n-4}\right) \dots \frac{1}{2} \cdot \frac{\pi}{2}$$

$$9.2) 'n' \text{ బేసి సంఖ్య అయిన } \int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx = \left(\frac{n-1}{n}\right) \left(\frac{n-3}{n-2}\right) \left(\frac{n-5}{n-4}\right) \dots \frac{2}{3} \cdot 1$$

$$\int_0^{\pi/2} \sin^m x \cos^n x dx = \begin{cases} \frac{[(m-1)(m-3)\dots(3)(1)][(n-1)(n-3)\dots(3)(1)]}{(m+n)(m+n-2)\dots(4)(2)} \cdot \frac{\pi}{2}, & \text{if } m \text{ is even \& } n \text{ is even} \\ & \text{(i.e., both } m, n \text{ are even)} \\ \frac{[(m-1)(m-3)\dots(3)(1)][(n-1)(n-3)\dots(4)(2)]}{(m+n)(m+n-2)\dots(3)(1)}, & \text{if } m \text{ is even \& } n \text{ is odd} \\ \frac{[(m-1)(m-3)\dots(4)(2)][(n-1)(n-3)\dots(2 \text{ or } 1)]}{(m+n)(m+n-2)\dots(2 \text{ or } 1)}, & \text{if } m \text{ is odd \& } n \text{ is even or odd} \end{cases}$$