6. త్రికోణమితీయ నిష్పత్తులు, పరివర్తనలు

IPE : 2 VSAQ & 1 SAQ & 1 LAQ = 2 + 2 + 4 + 7 = 15 Marks

ముఖ్యమైన సూత్రాలు, నిర్వచనాలు

1) $\sin^2\theta+\cos^2\theta \equiv 1$; $\sin^2\theta \equiv 1-\cos^2\theta$; $\cos^2\theta \equiv 1-\sin^2\theta$

2.1) $\sec^2\theta - \tan^2\theta=1$; $\sec^2\theta=1+\tan^2\theta$; $\tan^2\theta=\sec^2\theta-1$

2.2) If $\sec\theta+\tan\theta=p$ then $\sec\theta-\tan\theta=\frac{1}{p}$

3) $\csc^2\theta-\cot^2\theta=1$, $\csc^2\theta=1+\cot^2\theta$; $\cot^2\theta=\csc^2\theta-1$

4) (i) $\sin(A+B)=\sin A\cos B+\cos A\sin B$ (ii) $\cos(A+B)=\cos A\cos B-\sin A\sin B$

(iii) $\tan(A+B)=\frac{\tan A+\tan B}{1-\tan A\tan B}$ (iv) $\cot(A+B)=\frac{\cot A\cot B-1}{\cot A+\cot B}$

(v) $\tan\left(\frac{\pi}{4}+\theta\right)=\frac{1+\tan\theta}{1-\tan\theta}=\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}$

5) (i) $\sin(A-B)=\sin A\cos B-\cos A\sin B$ (ii) $\cos(A-B)=\cos A\cos B+\sin A\sin B$

(iii) $\tan(A-B)=\frac{\tan A-\tan B}{1+\tan A\tan B}$ (iv) $\cot(A-B)=\frac{\cot A\cot B+1}{\cot B-\cot A}$

(v) $\tan\left(\frac{\pi}{4}-\theta\right)=\frac{1-\tan\theta}{1+\tan\theta}=\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}$

6) $\sin(A+B)\sin(A-B)=\sin^2A-\sin^2B$

$\cos(A+B)\cos(A-B)=\cos^2A-\sin^2B$

$\sin^2A-\sin^2B = \sin(A+B)\sin(A-B)$
$\cos^2A-\cos^2B = -\sin(A+B)\sin(A-B)$
$\cos^2A-\sin^2B = \cos(A+B)\cos(A-B)$
$\sin^2A-\cos^2B = -\cos(A+B)\cos(A-B)$

7) $\sin 15^\circ=\cos 75^\circ=\frac{\sqrt{3}-1}{2\sqrt{2}}$; $\sin 75^\circ=\cos 15^\circ=\frac{\sqrt{3}+1}{2\sqrt{2}}$; $\tan 15^\circ=2-\sqrt{3}$; $\tan 75^\circ=2+\sqrt{3}$

8) $\tan 22\frac{1}{2}^\circ=\sqrt{2}-1=\cot 67\frac{1}{2}^\circ$

9) $\sin 18^\circ=\cos 72^\circ=\frac{\sqrt{5}-1}{4}$; $\sin 54^\circ=\cos 36^\circ=\frac{\sqrt{5}+1}{4}$

10) $\sin 36^\circ=\cos 54^\circ=\frac{\sqrt{10-2\sqrt{5}}}{4}$; $\sin 72^\circ=\cos 18^\circ=\frac{\sqrt{10+2\sqrt{5}}}{4}$

11.1) $\sin 2A=2\sin A\cos A=\frac{2\tan A}{1+\tan^2 A}$ **11.2)** $\sin A=2\sin\frac{A}{2}\cos\frac{A}{2}=\frac{2\tan\frac{A}{2}}{1+\tan^2\frac{A}{2}}$

12) $\cos 2A=\cos^2A-\sin^2A=2\cos^2A-1=1-2\sin^2A=\frac{1-\tan^2 A}{1+\tan^2 A}$

13.1) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ **13.2)** $\tan A = \dfrac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

14.1) $1-\cos 2A = 2\sin^2 A,\ 1+\cos 2A = 2\cos^2 A$

14.2) $\sin^2 A = \dfrac{1-\cos 2A}{2};\quad \cos^2 A = \dfrac{1+\cos 2A}{2};\quad \tan^2 A = \dfrac{1-\cos 2A}{1+\cos 2A}$

14.3) $\sin\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{2}},\ \cos\dfrac{A}{2} = \pm\sqrt{\dfrac{1+\cos A}{2}},\ \tan\dfrac{A}{2} = \pm\sqrt{\dfrac{1-\cos A}{1+\cos A}}$

15.1) (i) $\sin 3A = 3\sin A - 4\sin^3 A$ (ii) $\cos 3A = 4\cos^3 A - 3\cos A$ (iii) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

15.2) $\sin^3 A = \dfrac{1}{4}(3\sin A - \sin 3A);\ \cos^3 A = \dfrac{1}{4}(3\cos A + \cos 3A)$

16) సంకలనము నుండి లబ్ధమునకు గల పరివర్తనా సూత్రములు:

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \quad \sin(A+B)+\sin(A-B)=2\sin A\cos B$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) \quad \sin(A+B)-\sin(A-B)=2\cos A\sin B$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right) \quad \cos(A+B)+\cos(A-B)=2\cos A\cos B$$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right) = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right) \quad \cos(A+B)-\cos(A-B)=-2\sin A\sin B$$

17) లబ్ధము నుండి సంకలనము గల సూత్రములు:

$2\sin A\cos B=\sin(A+B)+\sin(A-B)$; $2\cos A\sin B=\sin(A+B)-\sin(A-B)$

$2\cos A\cos B=\cos(A+B)+\cos(A-B)$; $2\sin A\sin B=\cos(A-B)-\cos(A+B)$

☞ $A+B+C=180^o$ అయిన $\sin(A+B)=\sin C$, $\cos(A+B)=-\cos C$, $\sin\left(\dfrac{A+B}{2}\right)=\cos\dfrac{C}{2}$, $\cos\left(\dfrac{A+B}{2}\right)=\sin\dfrac{C}{2}$

18.1) $a\cos x+b\sin x$ యొక్క గరిష్ట విలువ $\sqrt{a^2+b^2}$ మరియు కనిష్ట విలువ $-\sqrt{a^2+b^2}$

18.2) $a\cos x+b\sin x+c$ యొక్క గరిష్ట విలువ $c+\sqrt{a^2+b^2}$ మరియు కనిష్ట విలువ $c-\sqrt{a^2+b^2}$

19.1) $a\sin(bx+c)+d$ యొక్క ఆవర్తనము $\dfrac{2\pi}{|b|}$; $a\cos(bx+c)+d$ యొక్క ఆవర్తనము $\dfrac{2\pi}{|b|}$

19.2) $a\tan(bx+c)+d$ యొక్క ఆవర్తనము $\dfrac{\pi}{|b|}$