

10. త్రిభుజ భర్తాలు

IPE : 1SAQ & 1 LAQ = 4+ 7= 11 Marks

ముఖ్యమైన సూత్రాలు, నిర్వచనాలు

- 1) ΔABC యొక్క అర్ధచుట్టుకొలత $s = \frac{a+b+c}{2} \Rightarrow 2s = a+b+c$
- 2.1) $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$ 2.2) $\Delta = \frac{abc}{4R}$ కావున $R = \frac{abc}{4\Delta}$
- 2.3) $\Delta = \frac{1}{2}bc \sin A = \frac{1}{2}ca \sin B = \frac{1}{2}ab \sin C$ 2.4) $\Delta = 2R^2 \sin A \sin B \sin C$
- 3) సైన్ నియమము: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \Rightarrow a = 2R \sin A, b = 2R \sin B, c = 2R \sin C$
- 4) కొసైన్ నియమము: $\cos A = \frac{b^2 + c^2 - a^2}{2bc} \Rightarrow a^2 = b^2 + c^2 - 2bc \cos A$ A
 $\cos B = \frac{c^2 + a^2 - b^2}{2ac} \Rightarrow b^2 = c^2 + a^2 - 2ac \cos B$ c b
 $\cos C = \frac{a^2 + b^2 - c^2}{2ab} \Rightarrow c^2 = a^2 + b^2 - 2ab \cos C$ B a C
- 5) ప్రతిక్షేపణ నియమము: $a = b \cos C + c \cos B$; $b = c \cos A + a \cos C$; $c = a \cos B + b \cos A$
- 6) టాన్‌జెంట్ నియమము: $\tan\left(\frac{A-B}{2}\right) = \left(\frac{a-b}{a+b}\right) \cot \frac{C}{2}$; $\tan\left(\frac{B-C}{2}\right) = \left(\frac{b-c}{b+c}\right) \cot \frac{A}{2}$; $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right) \cot \frac{B}{2}$
- 7) $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$; $\sin \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{ca}}$; $\sin \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{ab}}$
 $\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}}$; $\cos \frac{B}{2} = \sqrt{\frac{s(s-b)}{ca}}$; $\cos \frac{C}{2} = \sqrt{\frac{s(s-c)}{ab}}$
 $\tan \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$; $\tan \frac{B}{2} = \sqrt{\frac{(s-c)(s-a)}{s(s-b)}}$; $\tan \frac{C}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$
- ☞ $\tan \frac{A}{2} = \frac{\Delta}{s(s-a)} = \frac{(s-b)(s-c)}{\Delta}$; $\cot \frac{A}{2} = \frac{s(s-a)}{\Delta} = \frac{\Delta}{(s-b)(s-c)}$
- 8) $r = \frac{\Delta}{s} = 4R \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} = (s-a) \tan \frac{A}{2} = (s-b) \tan \frac{B}{2} = (s-c) \tan \frac{C}{2}$
 $r_1 = \frac{\Delta}{s-a} = 4R \sin \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} = s \tan \frac{A}{2} = (s-b) \cot \frac{C}{2} = (s-c) \cot \frac{B}{2}$
 $r_2 = \frac{\Delta}{s-b} = 4R \cos \frac{A}{2} \sin \frac{B}{2} \cos \frac{C}{2} = s \tan \frac{B}{2} = (s-c) \cot \frac{A}{2} = (s-a) \cot \frac{C}{2}$
 $r_3 = \frac{\Delta}{s-c} = 4R \cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2} = s \tan \frac{C}{2} = (s-a) \cot \frac{B}{2} = (s-b) \cot \frac{A}{2}$
- 9) $\Delta = rs = \sqrt{r_1 r_2 r_3}$ 10) $\frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$