Previous IPE

SOLVED PAPERS

MARCH -2023 (TS)

PREVIOUS PAPERS

IPE: MARCH-2023(TS)

Time : 3 Hours **MATHS-2B** Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **10 × 2 = 20**

1. Find the equation of the circle centre (–1,2) and radius 5.
2. Find the equation of the normal at P=(3, 5) of the circle $S \equiv x^2 + y^2 - 10x - 2y + 6 = 0$
3. Find the equation of the radical axis of $x^2+y^2-3x-4y+5=0, 3(x^2+y^2)-7x+8y-11=0$
4. Find the coordinates of the point on the parabola $y^2=8x$, whose focal distance is 10.
5. Find the value of k if $3x-4y+k=0$ is a tangent to the hyperbola $x^2-4y^2=5$.
6. Evaluate $\int \sqrt{1-\cos 2x}\, dx$
7. Evaluate $\int \frac{x^8}{1+x^{18}} dx$
8. Evaluate $\int_2^3 \frac{2x\,dx}{1+x^2}$
9. Find $\int_0^{\pi/2} \cos^8 x\,dx$.
10. Find the order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 - 3\left(\frac{dy}{dx}\right)^2 - e^x = 4$

SECTION-B

II. Answer any FIVE of the following SAQs: **5 × 4 = 20**

11. Find the length of the chord formed by $x^2 + y^2 = a^2$ on the line $x\cos\alpha + y\sin\alpha = p$.
12. Find the equation of the circle which cut orthogonally the circle $x^2 + y^2 - 4x + 2y - 7 = 0$ and having the centre at (2, 3).
13. Find the eccentricity, coordinates of foci, Length of latus rectum and equations of directrices of the ellipse $9x^2 + 16y^2 - 36x + 32y - 92 = 0$.
14. Find the condition for the line $lx + my + n = 0$ to be a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
15. Find the equations of the tangents to the hyperbola $3x^2 - 4y^2 = 12$, which are (i) parallel and (ii) perpendicular to the line $y = x-7$
16. Evaluate $\int_0^{\pi/2} \frac{\sin^5 x}{\sin^5 x + \cos^5 x} dx$
17. Solve $\frac{dy}{dx} = \frac{xy+y}{xy+x}$

SECTION-C

III. Answer any FIVE of the following LAQs: **5 × 7 = 35**

18. Find the equation of the circle passing through the points (3,4), (3,2) and (1,4).
19. Find the equation of the circle with centre (–2, 3) cutting a chord length 2 units on $3x+4y+4=0$
20. Show that the equation of the parabola in the standard form is $y^2 = 4ax$.
21. Evaluate $\int \frac{dx}{5+4\cos 2x}$
22. Evaluate $\int \frac{dx}{x(x+1)(x+2)}$
23. Evaluate $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$
24. Solve the differential equation $(x^2+y^2)dy = 2xy\,dx$

IPE TS MARCH-2023
SOLUTIONS

SECTION-A

1. Find the equation of the circle centre (–1,2) and radius 5.

Sol: Given centre C=(a,b) = (–1, 2) and radius r = 5

Equation of the circle is $(x-a)^2+(y-b)^2=r^2 \Rightarrow (x+1)^2+(y-2)^2=5^2$

$\Rightarrow (x^2+2x+1)+(y^2-4y+4)=25 \Rightarrow x^2+y^2+2x-4y-20=0$

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2. Find the equation of normal at P(3,5) on the circle $x^2+y^2-10x-2y+6=0$

Sol: Given point $P(x_1,y_1)=(3,5)$ and circle is $x^2+y^2-10x-2y+6=0$. Its centre $C=(-g,-f)=(5,1)$

Slope of the normal through P(3,5) and C(5,1) is

$$m=\frac{y_1-y_2}{x_1-x_2}=\frac{5-1}{3-5}=\frac{4}{-2}=-2$$

$\therefore$ Equation of the normal passing through (3,5) with slope –2 is $(y-y_1)=m(x-x_1)$

$\Rightarrow y-5=-2(x-3) \Rightarrow y-5=-2x+6 \quad \Rightarrow 2x+y-11=0$

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3. Find the equation of the radical axis of $x^2+y^2-3x-4y+5=0$, $3(x^2+y^2)-7x+8y+11=0$

Sol: The general forms of the given circles $S\equiv x^2+y^2-3x-4y+5=0$ & $S'\equiv x^2+y^2-\frac{7}{3}x+\frac{8}{3}y+\frac{11}{3}=0$

Now the equation of radical axis of the given circles is $S-S'=0$

$$\Rightarrow\left(-3+\frac{7}{3}\right)x+\left(-4-\frac{8}{3}\right)y+\left(5-\frac{11}{3}\right)=0 \Rightarrow\left(-\frac{2}{3}\right)x-\frac{20}{3}y+\frac{4}{3}=0 \Rightarrow -2x-20y+4=0$$

$\Rightarrow x+10y-2=0$

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4. Find the coordinates of the point on parabola $y^2=8x$, whose focal distance is 10.

Sol: Given parabola is $y^2=8x \Rightarrow 4a=8 \Rightarrow a=2$

Given focal distance SP=10

Formula: Focal distance $SP=x_1+a \Rightarrow x_1+2=10 \Rightarrow x_1=8$.

But, $y_1^2=8x_1 \Rightarrow y_1^2=8(8) \Rightarrow y_1=\pm 8$

$\therefore P(x_1,y_1)=(8,\pm 8)$

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5. Find the value of k if $3x-4y+k=0$ is a tangent to the hyperbola $x^2-4y^2=5$.

Sol: Given hyperbola $x^2-4y^2=5 \Rightarrow \frac{x^2}{5}-\frac{4y^2}{5}=1 \Rightarrow \frac{x^2}{5}-\frac{y^2}{5/4}=1 \Rightarrow a^2=5$ and $b^2=5/4$

Comparing $3x-4y+k=0$ with $lx+my+n=0$, we get $l=3$, $m=-4$, $n=k$

Tangential condition: $n^2=a^2l^2-b^2m^2$,

$\Rightarrow (k)^2=5(3^2)-\frac{5}{4}(-4)^2=45-20=25$

$\therefore k^2=25 \Rightarrow k=\pm 5$

Try this: Reduce $3x-4y+k=0$ into the form $y=mx+c$ and apply the condition $c^2=a^2m^2-b^2$

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6. Evaluate $\int\sqrt{1-\cos 2x}\,dx$

Sol: $I=\int\sqrt{1-\cos 2x}\,dx=\int\sqrt{2\sin^2 x}\,dx=\sqrt{2}\int\sin x\,dx=-\sqrt{2}\cos x+c$

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7. Evaluate $\int\frac{x^8}{1+x^{18}}dx$

Sol: Put $x^9=t \Rightarrow 9x^8dx=dt \Rightarrow x^8dx=\frac{1}{9}dt$

$$\therefore \int\frac{x^8dx}{1+x^{18}}=\int\frac{x^8dx}{1+(x^9)^2}=\frac{1}{9}\int\frac{dt}{1+t^2}=\frac{1}{9}\mathrm{Tan}^{-1}t+c=\frac{1}{9}\mathrm{Tan}^{-1}(x^9)+c$$

8. **Evaluate** $\int_2^3 \frac{2x\,dx}{1+x^2}$

Sol: $\int_2^3 \frac{2x\,dx}{1+x^2} = [\log|1+x^2|]_2^3 = \log|1+3^2| - \log|1+2^2|$

$= \log 10 - \log 5 = \log\frac{10}{5} = \log 2$

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9. **Find** $\int_0^{\pi/2} \cos^8 x\,dx$

Sol : $\int_0^{\pi/2} \cos^8 x\,dx = \int_0^{\pi/2} \cos^n x\,dx = I_n = I_8 \Rightarrow (n=8)$ Even

$\therefore \int_0^{\pi/2} \cos^8 x\,dx = \left(\frac{n-1}{n}\right) I_{n-2} = \frac{7}{8} x \frac{5}{6} x \frac{3}{4} x \frac{1}{2} x \frac{\pi}{2} = \frac{35\pi}{256}$

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10. **Find the order and degree of the differential equation** $\left(\frac{d^3y}{dx^3}\right)^2 - 3\left(\frac{dy}{dx}\right)^2 - e^x = 4$

Sol: In the given D.E, highest derivative is $\left(\frac{d^3y}{dx^3}\right)^2$

$\therefore$ For the given D.E, order =3 and degree =exponent =2

SECTION-B

11. Find the length of the chord formed by $x^2+y^2=a^2$ on the line $x\cos\alpha+y\sin\alpha=p$.

Sol: Given circle is $x^2+y^2=a^2$

Its centre C= (0,0) and radius r = a

We see that the given line $x\cos\alpha+y\sin\alpha=p$ is in the normal form.

Hence the perpendicular distance from (0,0) to the line is p.

$\therefore$ Length of the chord = $2\sqrt{r^2-p^2}=2\sqrt{a^2-p^2}$

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12. Find the equation of the circle which cut orthogonally the circle $x^2+y^2-4x+2y-7=0$ and having the centre at (2,3).

Sol : The equation of the required circle is taken as $S=x^2+y^2+2gx+2fy+c=0$.....(1)

Given that centre of S =0 is $C(-g,-f)=(2,3)\Rightarrow g=-2, f=-3$

S=0 is orthogonal to $x^2+y^2-4x+2y-7=0$

$\therefore 2gg'+2ff'=c+c' \Rightarrow 2g(-2)+2f(1)=c-7\Rightarrow -4g+2f=c-7$

Now Substituting $g=-2, f=-3$ in the above equation we get

$\Rightarrow -4(-2)+2(-3)=c-7\Rightarrow c=8-6+7=9$

Substituting $g=-2, f=-3, c=9$ in (1) we get the equation of the required circle as

$x^2+y^2+2(-2)x+2(-3)y+9=0\Rightarrow x^2+y^2-4x-6y+9=0$

13. Find the eccentricity, coordinates of foci, length of latus rectum and equations of directrices of the ellipse $9x^2+16y^2-36x+32y-92=0$.

Sol: Given ellipse is $9x^2+16y^2-36x+32y-92=0 \Rightarrow (9x^2-36x)+(16y^2+32y)=92$

$\Rightarrow 9(x^2-4x+4)+16(y^2+2y+1)=92+36+16 \Rightarrow 9(x-2)^2+16(y+1)^2=144$

$$\Rightarrow \frac{9(x-2)^2}{144}+\frac{16(y+1)^2}{144}=1 \Rightarrow \frac{(x-2)^2}{16}+\frac{(y-(-1))^2}{9}=1$$

Comparing with $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, we get $a^2=16$, $b^2=9 \Rightarrow a=4, b=3 \Rightarrow a>b$.

Hence the ellipse is horizontal. Also $(h,k)=(2,-1)$

(i) $e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{16-9}{16}}=\frac{\sqrt{7}}{4}$

(ii) Foci $=(h\pm ae,k)=\left(2\pm\frac{4\sqrt{7}}{4},-1\right)=(2\pm\sqrt{7},-1)$

(iii) Length of latus rectum $=\frac{2b^2}{a}=\frac{2(9)}{4}=\frac{9}{2}$

(iv) Equations of directrices $x=h\pm\frac{a}{e}=2\pm\frac{4x4}{\sqrt{7}}=\frac{2\sqrt{7}\pm16}{\sqrt{7}} \Rightarrow \sqrt{7}x=(2\sqrt{7}\pm16)$

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14. Find the condition for the line $lx+my+n=0$ to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Sol: Let $lx+my+n=0$ be the tangent at $P(\theta)=(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\therefore$ Equation of the tangent at $P(\theta)$ is $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$

Comparing the above equation with $lx+my=-n$, we get

$$\frac{\cos\theta}{al}=\frac{\sin\theta}{bm}=\frac{-1}{n} \Rightarrow \cos\theta=-\frac{al}{n}, \sin\theta=\frac{-bm}{n}$$

Now $\cos^2\theta+\sin^2\theta=1$

$$\Rightarrow \frac{a^2l^2}{n^2}+\frac{b^2m^2}{n^2}=1 \Rightarrow a^2l^2+b^2m^2=n^2$$

15. Find the equations of the tangents to the hyperbola $3x^2-4y^2=12$ which are (a) Parallel (b) Perpendicular to the line $y=x-7$

Sol: Given hyperbola is $3x^2-4y^2=12 \Rightarrow \frac{x^2}{4}-\frac{y^2}{3}=1 \Rightarrow a^2=4, b^2=3$

Slope of the given line $y=x-7$ is $m=1$

$\Rightarrow$ Slope of its perpendicular is -1

Formula:

Tangent with slope m is $y=mx\pm\sqrt{a^2m^2-b^2}$

(i) Parallel tangent with slope m=1 is $y=1.x\pm\sqrt{4(1)^2-3}=x\pm1$

$\Rightarrow x-y\pm1=0$

(ii)Perpendicular tangent with slope m=–1 is $y=(-1)x\pm\sqrt{4(1)^2-3}=-x\pm1$

$\Rightarrow x+y\pm1=0$

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16. Evaluate $\int_0^{\pi/2}\frac{\sin^5 x}{\sin^5 x+\cos^5 x}dx$

Sol: We know $\int_0^a f(x)dx=\int_0^a f(a-x)dx$

$$I=\int_0^{\pi/2}\frac{\sin^5 x}{\sin^5 x+\cos^5 x}dx \quad .....(1) \qquad =\int_0^{\pi/2}\frac{\cos^5\left(\frac{\pi}{2}-x\right)}{\sin^5\left(\frac{\pi}{2}-x\right)+\cos^5\left(\frac{\pi}{2}-x\right)}dx$$

$$=\int_0^{\pi/2}\frac{\sin^5 xdx}{\sin^5 x+\cos^5 x} \quad ......(2)$$

Adding (1) and (2), we get

$$2I=\int_0^{\pi/2}\frac{\sin^5 x+\cos^5 x}{\sin^5 x+\cos^5 x}dx=\int_0^{\pi/2}1\,dx=[x]_0^{\pi/2}=\frac{\pi}{2}-0=\frac{\pi}{2} \qquad \therefore 2I=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$$

17. Solve $\frac{dy}{dx} = \frac{xy+y}{xy+x}$

Sol: Given D.E is $\frac{dy}{dx} = \frac{xy+y}{xy+x} \Rightarrow \frac{dy}{dx} = \frac{y(x+1)}{x(y+1)} \Rightarrow \frac{y+1}{y}dy = \frac{x+1}{x}dx$

$$\Rightarrow \int\left(\frac{y}{y}+\frac{1}{y}\right)dy = \int\left(\frac{x}{x}+\frac{1}{x}\right)dx \Rightarrow \int\left(1+\frac{1}{y}\right)dy = \int\left(1+\frac{1}{x}\right)dx$$

$\Rightarrow y + \log y = x + \log x + \log c \quad \Rightarrow y - x = \log c + \log x - \log y$

$\Rightarrow y - x = \log_e\left(\frac{cx}{y}\right)$

$\therefore$ The solution is $e^{y-x} = \frac{cx}{y}$

SECTION-C

18. Find the equation of the circle passing through the points (3,4), (3,2) and (1,4).

Sol: Let A=(3,4), B=(3,2), C=(1,4)

We take $S(x_1,y_1)$ as the centre of the circle $\Rightarrow$ SA=SB=SC

Now, SA=SB $\Rightarrow SA^2=SB^2$ $\Rightarrow (x_1-3)^2+(y_1-4)^2=(x_1-3)^2+(y_1-2)^2$

$\Rightarrow (y_1-4)^2=(y_1-2)^2$

$\Rightarrow (y_1-4)=\pm(y_1-2)$

$\Rightarrow (y_1-4)=(y_1-2)$ (or) $(y_1-4)=-(y_1-2)$

Now $y_1-4=-(y_1-2)=-y_1+2$

$\Rightarrow 2y_1=6 \Rightarrow y_1=3$ ....(1)

Also, SA = SC $\Rightarrow SA^2=SC^2$

$\Rightarrow (x_1-3)^2+(y_1-4)^2=(x_1-1)^2+(y_1-4)^2$

$\Rightarrow (x_1-3)^2=(x_1-1)^2$

$\Rightarrow (x_1-3)=\pm(x_1-1)$

$\Rightarrow (x_1-3)=(x_1-1)$ (or) $(x_1-3)=-(x_1-1)$

Now $x_1-3=-(x_1-1)=-x_1+1$

$\Rightarrow 2x_1=4 \Rightarrow x_1=2$ ....(2)

From (1) & (2) we get

Centre of the circle $S(x_1,y_1)=(2,3)$.

Also, we have A=(3,4)

So, radius $r=SA \Rightarrow r^2=SA^2$

$\therefore r^2=(2-3)^2+(3-4)^2=1+1=2$

$\therefore$ Circle with centre (2, 3) and $r^2=2$ is $(x-2)^2+(y-3)^2=2$

$\Rightarrow (x^2-4x+4)+(y^2-6y+9)=2$

$\Rightarrow x^2+y^2-4x-6y+11=0$

19. Find the equation of the circle with centre (–2,3) cutting a chord length 2 units on $3x+4y+4=0$

Sol: Perpendicular distance from the centre (–2,3) to the line $3x+4y+4=0$ is $p=\frac{|3(-2)+4(3)+4|}{\sqrt{9+16}}=\frac{10}{5}=2$

Given that Length of the chord is 2 $\Rightarrow 2\sqrt{r^2-p^2}=2$

$\Rightarrow \sqrt{r^2-p^2}=1 \Rightarrow r^2-p^2=1 \Rightarrow r^2-2^2=1 \Rightarrow r^2=1+4=5$

$\therefore$ equation of the circle with centre (–2, 3) and $r^2=5$ is $(x+2)^2+(y-3)^2=5$

$\Rightarrow (x^2+4x+4)+(y^2-6y+9)=5 \Rightarrow x^2+y^2+4x-6y+8=0$

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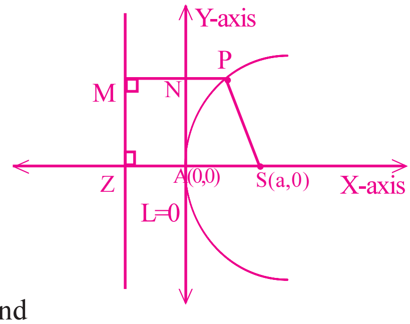
20. Show that the equation of the parabola in the standard form is $y^2 = 4ax$.

A: Let S be the focus and L=0 be the directrix of the parabola.

Let Z be the projection of S on to the directrix



Let A be the mid point of SZ

$\Rightarrow SA=AZ \Rightarrow \frac{SA}{AZ}=1$

$\Rightarrow$ A is a point on the parabola.

Take AS, the principle axis of the parabola as X-axis and

the line perpendicular to AS through A as the Y-axis $\Rightarrow A=(0,0)$

Let AS=a $\Rightarrow S=(a,0),\ Z=(-a,0)$

$\Rightarrow$ the equation of the directrix is $x=-a \Rightarrow x+a=0$

Let $P(x_1,y_1)$ be any point on the parabola.

N be the projection of P on to the Y-axis.

M be the projection of P on to the directrix.

Here $PM=PN+NM=x_1+a$ $\quad (\because PN = x-\text{coordinate of } P \text{ and } NM = AZ = AS = a)$

Now, by the focus directrix property of the parabola,

we have $\frac{SP}{PM}=1 \Rightarrow SP = PM \Rightarrow SP^2 = PM^2$

$\Rightarrow (x_1-a)^2+(y_1-0)^2=(x_1+a)^2$

$\Rightarrow y_1^2=(x_1+a)^2-(x_1-a)^2$

$\Rightarrow y_1^2 = 4ax_1$ $\quad [\because (a+b)^2-(a-b)^2 = 4ab]$

$\therefore$ The equation of locus of $P(x_1,y_1)$ is $y^2=4ax$

21. Evaluate $\int \frac{dx}{5+4\cos 2x}$

Sol: Put tanx=t $\Rightarrow \cos 2x = \frac{1-t^2}{1+t^2}$ and $dx = \frac{dt}{1+t^2}$

$$\therefore\ I = \int \frac{\left(\frac{dt}{1+t^2}\right)}{5+4\left(\frac{1-t^2}{1+t^2}\right)} = \int \frac{dt}{5(1+t^2)+4(1-t^2)} = \int \frac{dt}{t^2+9} = \frac{1}{3}\mathrm{Tan}^{-1}\left(\frac{t}{3}\right)+c = \frac{1}{3}\mathrm{Tan}^{-1}\left(\frac{\tan x}{3}\right)+c$$

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22. Evaluate $\int \frac{dx}{x(x+1)(x+2)}$

Sol: Let $\frac{1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2} = \frac{A(x+1)(x+2)+b(x)(x+2)+C(x)(x+1)}{x(x+1)(x+2)}$

$\Rightarrow A(x+1)(x+2)+b(x)(x+2)+C(x)(x+1)=1$ ......(1)

Putting x =0 in (1), we get A(1)(2) + B(0) + C(0)=1 $\Rightarrow$ A =1/2

Putting x = –1 in (1), we get A(0) + B(–1)(–1+2) +C(0)=1$\Rightarrow$ –B =1 $\Rightarrow$B= –1

Putting x = –2 in (1), we get A(0) + B(0) + C(–2)(–2+1)=1 $\Rightarrow$ C = 1/2

$$\therefore I = \int \frac{1}{x(x+1)(x+2)}dx = \frac{1}{2}\int \frac{1}{x}dx - \int \frac{1}{x+1}dx + \frac{1}{2}\int \frac{1}{x+2}dx$$

$$= \frac{1}{2}\log|x| - \log|(x+1)| + \frac{1}{2}\log|x+2| + c$$

23. Evaluate $\displaystyle\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

Sol: We know $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$$\therefore I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}+\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}dx$$

$$\Rightarrow I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx$$

Now, $$I + I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx + \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$$

$$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx = \int_{\pi/6}^{\pi/3} 1dx = [x]_{\pi/6}^{\pi/3} = \frac{\pi}{3}-\frac{\pi}{6} = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$$

24. Solve $(x^2+y^2)dx=2xydy$

Sol: Given D.E is $(x^2+y^2)dx = 2xydy \Rightarrow \frac{dy}{dx} = \frac{x^2+y^2}{2xy}$ .....(1). This is a homogeneous D.E

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From (1), $v + x\frac{dv}{dx} = \frac{x^2+(vx)^2}{2x(vx)} = \frac{x^2+v^2x^2}{2x^2v} = \frac{\cancel{x^2}(1+v^2)}{2\cancel{x^2}v} = \frac{1+v^2}{2v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{2v} - v = \frac{1+v^2-2v^2}{2v} = \frac{1-v^2}{2v}$

$\therefore x\frac{dv}{dx} = \frac{1-v^2}{2v} \Rightarrow \frac{2vdv}{1-v^2} = \frac{dx}{x} \Rightarrow \int\frac{2vdv}{1-v^2} = \int\frac{dx}{x} \Rightarrow -\int\frac{-2vdv}{1-v^2} = \int\frac{dx}{x}$

$\Rightarrow -\log(1-v^2) = \log x + \log c \Rightarrow \log x + \log(1-v^2) = \log c$

$\Rightarrow \log(x(1-v^2)) = \log c \Rightarrow x(1-v^2) = c \Rightarrow x\left(1-\frac{y^2}{x^2}\right) = c; \quad \left(\because v = \frac{y}{x}\right)$

$\Rightarrow \cancel{x}\left(\frac{x^2-y^2}{\cancel{x^2}}\right) = c \Rightarrow \frac{x^2-y^2}{x} = c \Rightarrow x^2-y^2 = cx$

$\therefore$ The solution is $x^2-y^2 = cx$

☞ 'c' is modified accordingly.