Previous IPE

SOLVED PAPERS

MARCH -2023 (TS)

PREVIOUS PAPERS

IPE: MARCH-2023(TS)

Time : 3 Hours **MATHS-1A** Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **10 × 2 = 20**

1. If A={–2,–1,0,1,2} and f:A→B is a surjection defined by $f(x)=x^2+x+1$ then find B.
2. Find the domain of $\frac{2x^2-5x+7}{(x-1)(x-2)(x-3)}$
3. If $A=\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}, B=\begin{bmatrix}3&2&1\\1&2&3\end{bmatrix}$, find 3B–2A.
4. If $\begin{bmatrix}0&2&1\\-2&0&-2\\-1&x&0\end{bmatrix}$ is a skew symmetric matrix then find the value of x.
5. Let $\bar{a}=2\bar{i}+4\bar{j}-5\bar{k}, \bar{b}=\bar{i}+\bar{j}+\bar{k}, \bar{c}=\bar{j}+2\bar{k}$. Find the unit vector in the opposite direction of $\bar{a}+\bar{b}+\bar{c}$.
6. Find the vector equation of the line passing through the point $2\bar{i}+\bar{j}+3\bar{k}$ and parallel to the vector $4\bar{i}-2\bar{j}+3\bar{k}$.
7. For what values of l the vectors $\bar{i}-\lambda\bar{j}+2\bar{k}, 8\bar{i}+6\bar{j}-\bar{k}$ are at right angles.
8. Find the period of $f(x)=\cos\left(\frac{4x+9}{5}\right)$.
9. Find the maximum and minimum values of f(x)=3cosx+4sinx.
10. If sinh x = 3 then show that $x=\log_e(3+\sqrt{10})$.

SECTION-B

II. Answer any FIVE of the following SAQs: **5 × 4 = 20**

11. If $A=\begin{bmatrix}1&1&3\\5&2&6\\-2&-1&-3\end{bmatrix}$ then find A^3.
12. Let A,B,C and D be four points with position vectors $\bar{a}+2\bar{b}, 2\bar{a}-\bar{b}, \bar{a}, 3\bar{a}+\bar{b}$ respectively. Express the vectors $\overline{AC}, \overline{DA}, \overline{BA}$ and $\overline{BC}$ interms of $\bar{a}$ and $\bar{b}$.
13. Find the volume of the tetrahedron, whose vertices are (1,2,1), (3,2,5), (2,–1,0) and (–1,0,1).
14. Prove that $\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}=2$
15. Solve $7\sin^2\theta+3\cos^2\theta=4$.
16. Prove that $\sin^{-1}\frac{3}{5}+\sin^{-1}\frac{8}{17}=\cos^{-1}\frac{36}{85}$
17. Prove that $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\frac{s^2}{\Delta}$

SECTION-C

III. Answer any FIVE of the following LAQs: **5 × 7 = 35**

18. If f:A→B, g:B→C are two bijective functions then prove that $(gof)^{-1}=f^{-1}og^{-1}$
19. Using mathematical induction, prove that $1.2.3+2.3.4+3.4.5+.....n\text{ terms}=\frac{n(n+1)(n+2)(n+3)}{4}\ \forall\ n\in N$
20. Show that $\begin{vmatrix}a+b+2c&a&b\\c&b+c+2a&b\\c&a&c+a+2b\end{vmatrix}=2(a+b+c)^3$
21. By using Cramer's rule x – y + 3z = 5, 4x+ 2y – z = 0, –x + 3y + z = 5
22. If $\bar{a}=7\bar{i}-2\bar{j}+3\bar{k}, \bar{b}=2\bar{i}+8\bar{k}$ and $\bar{c}=\bar{i}+\bar{j}+\bar{k}$, then compute $\bar{a}\times\bar{b}, \bar{a}\times\bar{c}$ and $\bar{a}\times(\bar{b}+\bar{c})$. Verify whether cross product is distributive over vector addition.
23. If A, B, C are angles in a triangle then prove that $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$.
24. In a ΔABC if a=13, b=14, c=15 then show that $R=\frac{65}{8}, r=4, r_1=\frac{21}{2}, r_2=12, r_3=14$

IPE TS MARCH-2023

SOLUTIONS

SECTION-A

1. If A={–2,–1,0,1,2} and f:A→B is a surjection defined by $f(x)=x^2+x+1$ then find B.

A: Given A={–2,–1,0,1,2} and

$f(x)=x^2+x+1$

$\therefore f(-2)=(-2)^2-2+1=4-2+1=3;$

$f(-1)=(-1)^2-1+1=1;$

$f(0)=0^2+0+1=1;$

$f(1)=1^2+1+1=3;$

$f(2)=2^2+2+1=7$

$\therefore B=f(A)=\{3,1,1,3,7\}=\{3,1,7\}$ [$\because$ For a surjection, Range f(A)= Codomain B]

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2. Find the domain of the real function $\dfrac{2x^2-5x+7}{(x-1)(x-2)(x-3)}$

A: Let $f(x)=\dfrac{2x^2-5x+7}{(x-1)(x-2)(x-3)}$

Here f(x) is not defined when $(x-1)(x-2)(x-3)=0 \Rightarrow x=1,2,3$

$\therefore$ domain of f is $R-\{1,2,3\}$

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3. If $A=\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{bmatrix}$ and $B=\begin{bmatrix}3 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}$, find 3B–2A

A: Given $B=\begin{bmatrix}3 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}; A=\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{bmatrix}$

$$\therefore 3B-2A=3\begin{bmatrix}3 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}-2\begin{bmatrix}1 & 2 & 3\\ 3 & 2 & 1\end{bmatrix}=\begin{bmatrix}9 & 6 & 3\\ 3 & 6 & 9\end{bmatrix}-\begin{bmatrix}2 & 4 & 6\\ 6 & 4 & 2\end{bmatrix}$$

$$=\begin{bmatrix}9-2 & 6-4 & 3-6\\ 3-6 & 6-4 & 9-2\end{bmatrix}=\begin{bmatrix}7 & 2 & -3\\ -3 & 2 & 7\end{bmatrix}$$

4. Define a Skew Symmetric Matrix. If $\begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ is a skew symmetric matrix then find the value of x.

A: Skew Symmetric Matrix: A square Matrix A is said to be a Skew Symmetric Matrix if $A = -A^T$

Given $A = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -2 \\ -1 & x & 0 \end{bmatrix}$ $\Rightarrow -A^T = -\begin{bmatrix} 0 & -2 & -1 \\ 2 & 0 & x \\ 1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & -x \\ -1 & 2 & 0 \end{bmatrix}$

Now $A = -A^T \Rightarrow x = 2$ [On equating the 2×3 elements]

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5. Let $\bar{a} = 2\bar{i} + 4\bar{j} - 5\bar{k}, \bar{b} = \bar{i} + \bar{j} + \bar{k}, \bar{c} = \bar{j} + 2\bar{k}$. Find the unit vector in the opposite direction of $\bar{a} + \bar{b} + \bar{c}$

A: Given $\bar{a} = 2\bar{i} + 4\bar{j} - 5\bar{k}$, $\bar{b} = \bar{i} + \bar{j} + \bar{k}$, $\bar{c} = 0\,\bar{i} + \bar{j} + 2\,\bar{k}$, then

$\bar{a} + \bar{b} + \bar{c} = (2\bar{i} + 4\bar{j} - 5\bar{k}) + (\bar{i} + \bar{j} + \bar{k}) + (0\bar{i} + \bar{j} + 2\bar{k})$

$= 3\bar{i} + 6\bar{j} - 2\bar{k}$

$\Rightarrow |\bar{a} + \bar{b} + \bar{c}| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$

$\therefore$ Opposite Unit vector $= \dfrac{-(\bar{a} + \bar{b} + \bar{c})}{|\bar{a} + \bar{b} + \bar{c}|} = \dfrac{-(3\bar{i} + 6\bar{j} - 2\bar{k})}{7}$

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6. Find the vector equation of the line passing through the point $2\bar{i} + \bar{j} + 3\bar{k}$ and parallel to the vector $4\bar{i} - 2\bar{j} + 3\bar{k}$.

A: Given point $A(\bar{a}) = 2\bar{i} + \bar{j} + 3\bar{k}$ and parallel vector $\bar{b} = 4\bar{i} - 2\bar{j} + 3\bar{k}$

Vector equation of the line is $\bar{r} = \bar{a} + t\bar{b}, t \in R$

$\therefore \bar{r} = (2\bar{i} + \bar{j} + 3\bar{k}) + t(4\bar{i} - 2\bar{j} + 3\bar{k}), t \in R$.

7. For what values of λ the vectors $\bar{i}-\lambda\bar{j}+2\bar{k}, 8\bar{i}+6\bar{j}-\bar{k}$ are at right angles.

A: Let $\bar{a}=\bar{i}-\lambda\bar{j}+2\bar{k}, \bar{b}=8\bar{i}+6\bar{j}-\bar{k}$

If $\bar{a},\bar{b}$ are at right angle then $\bar{a}.\bar{b}=0$

$\Rightarrow(\bar{i}-\lambda\bar{j}+2\bar{k}).(8\bar{i}+6\bar{j}-\bar{k})=0 \Rightarrow 8-6\lambda-2=0 \Rightarrow 6\lambda=6 \Rightarrow \lambda=1$

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8. Find the period of $f(x)=\cos\left(\frac{4x+9}{5}\right)$

A: Given $f(x)=\cos\left(\frac{4x+9}{5}\right)=\cos\left(\frac{4}{5}x+\frac{9}{5}\right)$

Period of $\cos(kx+l)=\frac{2\pi}{|k|}$

$\therefore$ Period $=\frac{2\pi}{\left|\frac{4}{5}\right|}=\frac{10\pi}{4}=\frac{5\pi}{2}$

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9. Find the maximum and minimum values of f(x)=3cosx+4sinx

A: Given function is 3cosx+4sinx

On comparing with acosx+bsinx+c, we get a=3, b=4, c=0

Now, $\sqrt{a^2+b^2}=\sqrt{3^2+4^2}=\sqrt{9+16}=\sqrt{25}=5$

$\therefore$ Maximum value $=c+\sqrt{a^2+b^2}=0+5=5$

Minimum value $=c-\sqrt{a^2+b^2}=0-5=-5$

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10. If sinhx=3 then show that $x=\log_e(3+\sqrt{10})$

A: We know, $\text{Sinh}^{-1}x=\log_e\left(x+\sqrt{x^2+1}\right)$

Given sinhx=3, then $x=\text{Sinh}^{-1}(3)=\log_e(3+\sqrt{3^2+1})$

$=\log_e(3+\sqrt{9+1})=\log_e(3+\sqrt{10})$

$\therefore x=\log_e(3+\sqrt{10})$. Hence proved

SECTION-B

11. If $A=\begin{bmatrix}1 & 1 & 3\\ 5 & 2 & 6\\ -2 & -1 & -3\end{bmatrix}$ **then find** A^3.

A: $A^2=A\times A=\begin{bmatrix}1 & 1 & 3\\ 5 & 2 & 6\\ -2 & -1 & -3\end{bmatrix}\begin{bmatrix}1 & 1 & 3\\ 5 & 2 & 6\\ -2 & -1 & -3\end{bmatrix}$

$$=\begin{bmatrix}1+5-6 & 1+2-3 & 3+6-9\\ 5+10-12 & 5+4-6 & 15+12-18\\ -2-5+6 & -2-2+3 & -6-6+9\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 3 & 3 & 9\\ -1 & -1 & -3\end{bmatrix}$$

$$A^3=A^2\times A=\begin{bmatrix}0 & 0 & 0\\ 3 & 3 & 9\\ -1 & -1 & -3\end{bmatrix}\begin{bmatrix}1 & 1 & 3\\ 5 & 2 & 6\\ -2 & -1 & -3\end{bmatrix}$$

$$=\begin{bmatrix}0+0+0 & 0+0+0 & 0+0+0\\ 3+15-18 & 3+6-9 & 9+18-27\\ -1-5+6 & -1-2+3 & -3-6+9\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Here $A^3=O$. So, A is a Nilpotent matrix of index 3.

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12. Let A,B,C and D be four points with position vectors $\bar{a}+2\bar{b}, 2\bar{a}-\bar{b}, \bar{a}$ **and** $3\bar{a}+\bar{b}$ **respectively. Express the vectors** $\overline{AC}, \overline{DA}, \overline{BA}$ **and** $\overline{BC}$ **interms of** $\bar{a}$ **and** $\bar{b}$**.**

A: P.V's of A,B,C,D w.r.t O are $\overline{OA}=\bar{a}+2\bar{b}$, $\overline{OB}=2\bar{a}-\bar{b}$, $\overline{OC}=\bar{a}$ **and** $\overline{OD}=3\bar{a}+\bar{b}$. **Then**

$\overline{AC}=\overline{OC}-\overline{OA}=\bar{a}-(\bar{a}+2\bar{b})=-2\bar{b}$

$\overline{DA}=\overline{OA}-\overline{OD}=(\bar{a}+2\bar{b})-(3\bar{a}+\bar{b})=-2\bar{a}+\bar{b}$

$\overline{BA}=\overline{OA}-\overline{OB}=(\bar{a}+2\bar{b})-(2\bar{a}-\bar{b})=3\bar{b}-\bar{a}$

$\overline{BC}=\overline{OC}-\overline{OB}=\bar{a}-(2\bar{a}-\bar{b})=\bar{b}-\bar{a}$

13. Find the volume of the tetrahedron, whose vertices are (1,2,1), (3,2,5), (2,–1,0) and (–1,0,1).

A: Let $\overline{OA} = \bar{i} + 2\bar{j} + \bar{k}$, $\overline{OB} = 3\bar{i} + 2\bar{j} + 5\bar{k}$, $\overline{OC} = 2\bar{i} - \bar{j}$, $\overline{OD} = -\bar{i} + \bar{k}$. Then

$$\overline{AB} = \overline{OB} - \overline{OA} = (3\bar{i} + 2\bar{j} + 5\bar{k}) - (\bar{i} + 2\bar{j} + \bar{k}) = 2\bar{i} + 4\bar{k}$$

$$\overline{AC} = \overline{OC} - \overline{OA} = (2\bar{i} - \bar{j}) - (\bar{i} + 2\bar{j} + \bar{k}) = \bar{i} - 3\bar{j} - \bar{k}$$

$$\overline{AD} = \overline{OD} - \overline{OA} = (-\bar{i} + \bar{k}) - (\bar{i} + 2\bar{j} + \bar{k}) = -2\bar{i} - 2\bar{j}$$

Now, $[\overline{AB}\ \overline{AC}\ \overline{AD}] = \begin{vmatrix} 2 & 0 & 4 \\ 1 & -3 & -1 \\ -2 & -2 & 0 \end{vmatrix}$ =[2(0–2)+4(–2–6)]=– 4 –32= –36

$\therefore$ Volume of the tetrahedron $= \frac{1}{6}|-36| = 6$ cubic unit

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14. Prove that $\cos^2\frac{\pi}{8} + \cos^2\frac{3\pi}{8} + \cos^2\frac{5\pi}{8} + \cos^2\frac{7\pi}{8} = 2$

A : $G.E = \cos^2\frac{\pi}{8} + \cos^2\left(\frac{4\pi - \pi}{8}\right) + \cos^2\left(\frac{4\pi + \pi}{8}\right) + \cos^2\left(\frac{8\pi - \pi}{8}\right)$

$$= \cos^2\frac{\pi}{8} + \cos^2\left(\frac{\pi}{2} - \frac{\pi}{8}\right) + \cos^2\left(\frac{\pi}{2} + \frac{\pi}{8}\right) + \cos^2\left(\pi - \frac{\pi}{8}\right)$$

$$= \cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8} + \sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}$$

$$\therefore L.H.S = \left(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}\right) + \left(\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right) = 1 + 1 = 2 = R.H.S$$

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15. Solve $7\sin^2\theta + 3\cos^2\theta = 4$

A: Given equation is $7\sin^2\theta + 3\cos^2\theta = 4 \Rightarrow 7\sin^2\theta + 3(1 - \sin^2\theta) = 4 \Rightarrow 7\sin^2\theta + 3 - 3\sin^2\theta = 4$

$\Rightarrow 4\sin^2\theta = 1 \Rightarrow \sin^2\theta = (1/2)^2 = \sin^2(\pi/6)$

Here Principal value is $\alpha = \pi/6$

$\therefore$ General solution is $\theta = n\pi \pm \frac{\pi}{6}, n \in Z$

16. Prove that $\mathrm{Sin}^{-1}\frac{3}{5}+\mathrm{Sin}^{-1}\frac{8}{17}=\mathrm{Cos}^{-1}\frac{36}{85}$

A: Take $\mathrm{Sin}^{-1}\frac{3}{5}=\alpha$ and $\mathrm{Sin}^{-1}\frac{8}{17}=\beta$

Required To Prove (RTP): $\alpha+\beta=\mathrm{Cos}^{-1}\frac{36}{85}\Rightarrow\cos(\alpha+\beta)=\frac{36}{85}$

$$\mathrm{Sin}^{-1}\frac{3}{5}=\alpha\Rightarrow\sin\alpha=\frac{3}{5}\Rightarrow\cos\alpha=\frac{4}{5}$$

$$\mathrm{Sin}^{-1}\frac{8}{17}=\beta\Rightarrow\sin\beta=\frac{8}{17}\Rightarrow\cos\beta=\frac{15}{17}$$

$$\therefore\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\frac{4}{5}\times\frac{15}{17}-\frac{3}{5}\times\frac{8}{17}=\frac{60-24}{85}=\frac{36}{85}$$

Hence proved.

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17. Prove that $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\frac{s^2}{\Delta}$

A: L.H.S= $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}$

$$=\frac{s(s-a)}{\Delta}+\frac{s(s-b)}{\Delta}+\frac{s(s-c)}{\Delta}=\frac{s(s-a)+s(s-b)+s(s-c)}{\Delta}$$

$$=\frac{s[(s-a)+(s-b)+(s-c)]}{\Delta}$$

$$=\frac{s[3s-(a+b+c)]}{\Delta}=\frac{s[3s-2s]}{\Delta}=\frac{s[s]}{\Delta}=\frac{s^2}{\Delta}=\text{R.H.S}$$

SECTION-C

18. If $f:A\to B$, $g:B\to C$ are two bijective functions then prove that $(gof)^{-1}=f^{-1}og^{-1}$

A: Part -1: Given that $f:A\to B$, $g:B\to C$ are two bijective functions, then

(i) $gof:A\to C$ is bijection $\Rightarrow (gof)^{-1}:C\to A$ is also a bijection

(ii) $f^{-1}:B\to A$, $g^{-1}:C\to B$ are both bijections $\Rightarrow (f^{-1}og^{-1}): C\to A$ is also a bijection.

So, $(gof)^{-1}$ and $f^{-1}og^{-1}$, both have same domain 'C'

Part–2: Given $f:A\to B$ is bijection, then $f(a)=b\Rightarrow a=f^{-1}(b)$........(1), [Here $a\in A$, $b\in B$]

$g:B\to C$ is bijection, then $g(b)=c\Rightarrow b=g^{-1}(c)$........(2), [Here $b\in B$, $c\in C$]

$gof:A\to C$ is bijection, then $gof(a)=c \Rightarrow a=(gof)^{-1}(c)$.........(3)

Now, $(f^{-1}og^{-1})(c)=f^{-1}[g^{-1}(c)]=f^{-1}(b)=a$ ....(4), [From (1) & (2)]

$\therefore (gof)^{-1}(c)=(f^{-1}og^{-1})(c), \forall c\in C$, [from (3) & (4)]

Hence, we proved that $(gof)^{-1}=f^{-1}og^{-1}$

19. Using the principle of finite Mathematical Induction prove that

$$1.2.3 + 2.3.4 + 3.4.5. + ...\text{upto n terms} = \frac{n(n+1)(n+2)(n+3)}{4}, \forall\, n \in N$$

A: To find n^{th} term:

n^{th} term of the given series is T_n=n(n+1)(n+2).

Let S(n) : $1.2.3 + 2.3.4 + ... + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$

Step 1: L.H.S of S(1) = 1.2.3=6

R.H.S of S(1) $= \frac{1(1+1)(1+2)(1+3)}{4} = \frac{2.3.4}{4} = 6$

$\therefore$ L.H.S = R.H.S.

So, S(1) is true.

Step 2: Assume that S(k) is true, for $k \in N$

$$S(k) : 1.2.3. + 2.3.4 + ... + k(k+1)(k+2) = \frac{k(k+1)(k+2)(k+3)}{4} \quad ......(1)$$

Step 3: We show that S(k+1) is true

On adding (k+1)(k+2)(k+3) to both sides of (1), we get

$$\text{L.H.S} = [1.2.3. + 2.3.4 + ... + k(k+1)(k+2)] + (k+1)(k+2)(k+3)$$

$$= \frac{k(k+1)(k+2)(k+3)}{4} + (k+1)(k+2)(k+3)$$

$$= \frac{k(k+1)(k+2)(k+3) + 4(k+1)(k+2)(k+3)}{4}$$

$$= \frac{(k+1)(k+2)(k+3)(k+4)}{4} = \text{R.H.S}$$

$\therefore$ L.H.S=R.H.S .

So, S(k+1) is true whenever S(k) is true

Hence, by P.M.I the given statement is true, for all $n \in N$

20. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

A: $$\text{L.H.S} = \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

$$= \begin{vmatrix} 2a+2b+2c & a & b \\ 2a+2b+2c & b+c+2a & b \\ 2a+2b+2c & a & c+a+2b \end{vmatrix} (\because C_1 \to C_1 + C_2 + C_3)$$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix} \begin{matrix} (\because R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1) \end{matrix}$$

$$= 2(a+b+c)1[(a+b+c)^2 - 0] = 2(a+b+c)^3 = \text{R.H.S}$$

21. By using Cramers rule solve $x-y+3z=5,\ 4x+2y-z=0,\ x+3y+z=5$.

A: Given equations in the matrix equation form: AX = D, where

$$A=\begin{bmatrix}1 & -1 & 3\\4 & 2 & -1\\1 & 3 & 1\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix},\ D=\begin{bmatrix}5\\0\\5\end{bmatrix}$$

$$\Delta=\det A=\begin{vmatrix}1 & -1 & 3\\4 & 2 & -1\\1 & 3 & 1\end{vmatrix}$$

$$=1(2+3)+1(4+1)+3(12-2)$$
$$=5+5+30=40\neq 0$$
$$\Rightarrow A \text{ is non singular}$$

$$\Delta_1=\begin{vmatrix}5 & -1 & 3\\0 & 2 & -1\\5 & 3 & 1\end{vmatrix}$$

$$=5(2\times1-(-1)\times3)+1(0\times1-(-1)\times5)+3(0\times3-5\times2)$$
$$=25+5-30=0;$$

$$\Delta_2=\begin{vmatrix}1 & 5 & 3\\4 & 0 & -1\\1 & 5 & 1\end{vmatrix}$$

$$=1(0\times1-(-1)\times5)-5(4\times1-(-1)\times1)+3(4\times5-0\times1)$$
$$=5-25+60=40;$$

$$\Delta_3=\begin{vmatrix}1 & -1 & 5\\4 & 2 & 0\\1 & 3 & 5\end{vmatrix}$$

$$=1(2\times5-0\times3)+1(4\times5-0\times1)+5(4\times3-2\times1)$$
$$=10+20+50=80$$

$\therefore$ By Cramer's rule, $x=\dfrac{\Delta_1}{\Delta}=\dfrac{0}{40}=0;\ y=\dfrac{\Delta_2}{\Delta}=\dfrac{40}{40}=1$ and $z=\dfrac{\Delta_3}{\Delta}=\dfrac{80}{40}=2$

$\therefore$ The solution is x=0,y=1,z=2

22. **If $\bar{a} = 7\bar{i} - 2\bar{j} + 3\bar{k}, \bar{b} = 2\bar{i} + 8\bar{k}$ and $\bar{c} = \bar{i} + \bar{j} + \bar{k}$, then compute $\bar{a}\times\bar{b}, \bar{a}\times\bar{c}$ and $\bar{a}\times(\bar{b}+\bar{c})$.**

Verify whether cross product is distributive over vector addition.

A: Given that $\bar{a} = 7\bar{i} - \overline{2j} + \bar{k}, \bar{b} = 2\bar{i} + 8\bar{k}$ and $\bar{c} = \bar{i} + \bar{j} + \bar{k}$

Now $\bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 7 & -2 & 3 \\ 2 & 0 & 8 \end{vmatrix} = \bar{i}(-16-0) - \bar{j}(56-6) + \bar{k}(0+4) \quad = -16\bar{i} - 50\bar{j} + 4\bar{k}$

Also, $\bar{a}\times\bar{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 7 & -2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = \bar{i}(-2-3) - \bar{j}(7-3) + \bar{k}(7+2) = -5\bar{i} - 4\bar{j} + 9\bar{k}$

Now $\bar{b}+\bar{c} = (2\bar{i} + 8\bar{k}) + (\bar{i} + \bar{j} + \bar{k}) = 3\bar{i} + \bar{j} + 9\bar{k}$

$\bar{a}\times(\bar{b}+\bar{c}) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 7 & -2 & 3 \\ 3 & 1 & 9 \end{vmatrix} = = \bar{i}(-18-3) - \bar{j}(63-9) + \bar{k}(7+6) = -21\bar{i} - 54\bar{j} + 13\bar{k}$..........(1)

Now $(\bar{a}\times\bar{b}) + (\bar{a}\times\bar{c}) = (-16\bar{i} - 50\bar{j} + 4\bar{k}) + (-5\bar{i} - 4\bar{j} + 9\bar{k}) = -21\bar{i} - 54\bar{j} + 13\bar{k}$.........(2)

From (1) & (2) we have $\bar{a}\times(\bar{b}+\bar{c}) = (\bar{a}\times\bar{b}) + (\bar{a}\times\bar{c})$

$\therefore$ Vector product is distributive over vector addition.

23. If A+B+C=180^{0} then prove that cosA+cosB+cosC=$1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$

A: Given A,B,C are angles of a triangle, then A+B+C=180º $\Rightarrow \frac{A+B+C}{2}=90^{o}$

$$\text{L.H.S} =(\cos A+\cos B)+\cos C=2\cos\frac{A+B}{2}.\cos\frac{A-B}{2}+\cos C$$

$$=2\cos\left(90^{o}-\frac{C}{2}\right).\cos\frac{A-B}{2}+\cos C$$

$$=2\sin\frac{C}{2}.\cos\frac{A-B}{2}+\left(1-2\sin^{2}\frac{C}{2}\right) \qquad \left[\because \cos\theta=1-2\sin^{2}\frac{\theta}{2}\right]$$

$$=1+2\sin\frac{C}{2}.\cos\frac{A-B}{2}-2\sin^{2}\frac{C}{2} \;=1+2\sin\frac{C}{2}\left(\cos\frac{A-B}{2}-\sin\frac{C}{2}\right)$$

$$=1+2\sin\frac{C}{2}\left[\cos\frac{A-B}{2}-\sin\left(90^{o}-\frac{A+B}{2}\right)\right]$$

$$=1+2\sin\frac{C}{2}\left(\cos\frac{A-B}{2}-\cos\frac{A+B}{2}\right) \qquad [\because \sin(90^{o}-\theta)=\cos\theta]$$

$$=1+2\sin\frac{C}{2}\left(2\sin\frac{A}{2}.\sin\frac{B}{2}\right) \qquad [\because \cos(A-B)-\cos(A+B)=2\sin A\sin B]$$

$$=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}=\text{R.H.S}$$

24. In a ΔABC if a=13, b=14, c=15 then show that $R=\frac{65}{8}, r=4, r_1=\frac{21}{2}, r_2=12, r_3=14$

A: Given a=13, b=14, c=15 , then

$$2s=a+b+c=13+14+15=42 \Rightarrow \not{2}s=\not{42} \Rightarrow s=21$$

Now, $\Delta=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{21(21-13)(21-14)(21-15)}$

$$=\sqrt{21\times(8)(7)(6)}=\sqrt{(3\times7)(4\times2)(7)(3\times2)}=\sqrt{3^2\times4^2\times7^2}=3\times4\times7=84$$

(i) $R=\frac{abc}{4\Delta}=\frac{13\times14\times15}{4\times84}=\frac{65}{8}$

(ii) $r=\frac{\Delta}{s}=\frac{\not{84}}{\not{21}}=4$;

(iii) $r_1=\frac{\Delta}{s-a}=\frac{84}{21-13}=\frac{\not{84}}{\not{8}}=\frac{21}{2}$

(iv) $r_2=\frac{\Delta}{s-b}=\frac{84}{21-14}=\frac{\not{84}}{\not{7}}=12$

(v) $r_3=\frac{\Delta}{s-c}=\frac{84}{21-15}=\frac{\not{84}}{\not{6}}=14$