Previous IPE

SOLVED PAPERS

MARCH -2020 (TS)

PREVIOUS PAPERS

IPE: MARCH-2020(TS)

MATHS-2B

Time : 3 Hours Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: $10 \times 2 = 20$

1. Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 41 = 0$.
2. Find the value of k if the length of the tangent from (5,4) to $x^2 + y^2 + 2ky = 0$ is 1.
3. Find the equation of the common chord of the circles:
   $x^2 + y^2 - 4x - 4y + 3 = 0$, $x^2 + y^2 - 5x - 6y + 4 = 0$
4. Find the equation of the tangent and normal at the positive end of L.R on the parabola $y^2 = 6x$.
5. If the angle between the asymptotes is 30^0 then find its eccentricity.
6. Evaluate $\int \frac{\sin^2 x}{1+\cos 2x} dx$
7. Evaluate $\int \frac{2x+1}{x^2+x+1} dx$
8. Evaluate $\int_2^3 \frac{2x dx}{1+x^2}$
9. Find the area of the region bounded by $y = x^3 + 3$, x-axis and $x = -1$, $x = 2$
10. Form the differential equation corresponding to $y = A\cos 3x + B\sin 3x$, where A&B are parameters.

SECTION-B

II. Answer any FIVE of the following SAQs: $5 \times 4 = 20$

11. Find the length of the chord intercepted by the circle $x^2+y^2-x+3y-22=0$ on the line $y = x - 3$
12. Show that the angle between the circles $x^2 + y^2 = a^2$, $x^2 + y^2 = ax + ay$ is $3\pi/4$.
13. Find the eccentricity, length of latus rectum, centre, foci, equation of the directrices of the ellipse $9x^2 + 16y^2 = 144$
14. Find the equation of tangent and normal to the ellipse $x^2 + 8y^2 - 33 = 0$ at (–1,2)ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.
15. Find the equations of the tangents to the hyperbola $3x^2 - 4y^2 = 12$, which are
    (i) parallel and (ii) perpendicular to the line $y = x - 7$
16. Evaluate $\int_{\pi/6}^{\pi/3} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$
17. Solve $(xy^2 + x)dx + (yx^2 + y)\, dy = 0$

SECTION-C

III. Answer any FIVE of the following LAQs: $5 \times 7 = 35$

18. Find the equation of the circle, which passes through (2, –3), (–4, 5) and whose centre lies on the line $4x + 3y + 1 = 0$.
19. Show that the circles $x^2 + y^2 - 6x - 9y + 13 = 0$ and $x^2 + y^2 - 2x - 16y = 0$ touch each other. Find the point of contact and the equation of the common tangent at that point.
20. Show that the equation of the parabola in the standard form is $y^2 = 4ax$.
21. Evaluate $\int \frac{1}{(1+x)\sqrt{3+2x-x^2}} dx$
22. Evaluate the reduction formula for $I_n = \int \sin^n x dx$ and hence find $\int \sin^4 x dx$.
23. Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$
24. Solve $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$

BABY BULLET-Q

IPE TS MARCH-2020
SOLUTIONS

SECTION-A

1. Find the centre and radius of the circle $x^2 + y^2 - 4x - 8y - 41 = 0$.

A: Given circle is $x^2+y^2-4x-8y-41=0$

Comparing with $x^2+y^2+2gx+2fy+c=0$, we get $2g=-4$; $2f=-8$; $c=-41 \Rightarrow g=-2, f=-4, c=-41$

Centre $C=(-g,-f)=(-(-2),-(-4))=(2,4)$

Radius $r=\sqrt{g^2+f^2-c}=\sqrt{2^2+(4)^2-(-41)}=\sqrt{4+16+41}=\sqrt{61}$

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2. Find the value of k if the length of the tangent from (5,4) to $x^2+y^2+2ky=0$ is 1

Sol: Length of the tangent from (5,4) to

$S=x^2+y^2+2ky=0$ is $\sqrt{S_{11}}=1$;

On squaring both sides, we get $S_{11}=1$

$\Rightarrow 5^2+4^2+2k(4)=1 \Rightarrow 25+16+8k=1 \Rightarrow 41+8k=1$

$\Rightarrow 8k=-40 \Rightarrow k=-40/8=-5$

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3. Find the equation of the common chord of the circles:

$x^2+y^2-4x-4y+3=0$, $x^2+y^2-5x-6y+4=0$

A: Given Cirlces $S=x^2+y^2-4x-4y+3=0$, $S'=x^2+y^2-5x-6y+4=0$

Equation of common of chord (radical axis) is $S-S'=0 \Rightarrow -4x+5x-4y+6y+3-4=0$

$\Rightarrow x+2y-1=0$

4. Find the equation of the tangent and normal at the positive end of L.R on the parabola $y^2=6x$.

Sol: $y^2=6x \Rightarrow 4a=6 \Rightarrow 2a=3 \Rightarrow a=3/2$

$\therefore$ positive end of L.R is (a,2a)= (3/2, 3)

The equation of the tangent at (x_1,y_1) on $S=y^2-4ax=0$ is $S_1=yy_1-2a(x+x_1)=0$

$\therefore$ the equation of the tangent at (3/2, 3) on $y^2=6x$ is $y(3)-3(x+3/2)=0 \Rightarrow 2x-2y+3=0$

The slope of the tangent $2x-2y+3=0$ is 1

$\Rightarrow$ the slope of its normal is -1

$\therefore$ the equation of the normal at (3/2, 3) with slope -1 is $y-3=-1(x-3/2) \Rightarrow 2x+2y-9=0$

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5. If the angle between the asymptotes is 30° then find its eccentricity.

Sol: The angle between the asymptotes of the hyperbola S=0 is $2\text{Sec}^{-1}e$

$\therefore\ 2\text{Sec}^{-1}e = 30^\circ \Rightarrow \text{Sec}^{-1}e=15^\circ \Rightarrow e=\sec 15^\circ=\sqrt{6}-\sqrt{2}$

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6. Evaluate $\int \frac{\sin^2 x}{1+\cos 2x}dx$

Sol : $I=\int \frac{\sin^2 x}{1+\cos 2x}dx=\int \frac{\sin^2 x}{2\cos^2 x}dx=\frac{1}{2}\int \tan^2 x\,dx=\frac{1}{2}\int(\sec^2 x-1)\,dx=\frac{1}{2}(\tan x-x)+c$

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7. Evaluate $\int \frac{2x+1}{x^2+x+1}dx$

Sol: Put $x^2+x+1=t \Rightarrow (2x+1)dx=dt$

$\therefore I=\int \frac{2x+1}{x^2+x+1}dx=\int \frac{dt}{t}=\log|t|+c=\log|x^2+x+1|+c$

BABY BULLET-Q

8. Evaluate $\int_2^3 \frac{2x\,dx}{1+x^2}$

Sol: $\int_2^3 \frac{2x\,dx}{1+x^2} = [\log|1+x^2|]_2^3 = \log|1+3^2| - \log|1+2^2|$

$$= \log 10 - \log 5 = \log\frac{10}{5} = \log 2$$

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9. Find the area of the region bounded by $y=x^3+3$, x-axis and $x=-1$, $x=2$

Sol : The area bounded by the curve $y=x^3+3$, the x-axis and the lines $x=-1$, $x=2$ is

$$A = \int_{-1}^{2} y\,dx = \int_{-1}^{2}(x^3+3)dx = \left[\frac{x^4}{4}+3x\right]_{-1}^{2} = \left\{\left(\frac{16}{4}+6\right)-\left(\frac{1}{4}-3\right)\right\} = \frac{40}{4}+\frac{11}{4} = \frac{51}{4} \text{ sq.units}$$

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10. Form the differential equation corresponding to $y = A\cos 3x + B\sin 3x$, where A and B are parameters.

Sol: Given $y=A\cos3x+B\sin3x \Rightarrow \frac{dy}{dx} = -3A(\sin 3x) + 3B(\cos 3x)$

$$\Rightarrow \frac{d^2y}{dx^2} = -9A\cos 3x - 9B\sin 3x = -9(A\cos 3x + B\sin 3x) = -9y$$

$$\therefore \frac{d^2y}{dx^2} + 9y = 0$$

BABY BULLET-Q

SECTION-B

11. Find the length of the chord intercepted by the circle $x^2+y^2-x+3y-22=0$ onthe line $y=x-3$

Sol: Given circle $x^2+y^2-x+3y-22=0$

It's Centre C=(1/2,–3/2)

$$\text{radius } r=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{3}{2}\right)^2+22}=\sqrt{\frac{1}{4}+\frac{9}{4}+22}=\sqrt{\frac{1+9+88}{4}}=\sqrt{\frac{98}{4}}=\sqrt{\frac{49}{2}}=\frac{7}{\sqrt{2}}$$

Perpendicular distance from the centre(1/2,–3/2) to the line y=x–3=0 ⇒ x–y–3=0

$$\text{is } p=\frac{\left|\frac{1}{2}+\frac{3}{2}-3\right|}{\sqrt{1^2+1^2}}=\frac{\left|\frac{1+3-6}{2}\right|}{\sqrt{2}}=\frac{\left|\frac{-2}{2}\right|}{\sqrt{2}}=\frac{1}{\sqrt{2}}$$

$$\therefore \text{Length of the chord } =2\sqrt{r^2-p^2}=2\sqrt{\left(\frac{49}{2}\right)-\frac{1}{2}}=2\sqrt{\frac{48}{2}}=2\sqrt{24}$$

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12. Show that the angle between the circles $x^2+y^2 = a^2$, $x^2+y^2= ax +ay$ is $3\pi/4$.

Sol: Given circle is $x^2+y^2-a^2=0$. Its centre $C_1=(0,0)$

and radius $r_1=a$

Other circle is $x^2+y^2-ax-ay=0$. Its centre $C_2=\left(\frac{a}{2},\frac{a}{2}\right)$

$$\text{Radius } r_2=\sqrt{g^2+f^2-c}=\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{a}{2}\right)^2-0}=\sqrt{\frac{a^2}{4}+\frac{a^2}{4}}=\sqrt{\frac{a^2}{2}}=\frac{a}{\sqrt{2}}$$

$$\therefore d=C_1C_2=\sqrt{\frac{a^2}{4}+\frac{a^2}{4}}=\sqrt{\frac{2a^2}{4}}=\frac{a}{\sqrt{2}}$$

$$\therefore \cos\theta=\frac{d^2-r_1^2-r_2^2}{2r_1r_2}=\frac{\frac{a^2}{2}-a^2-\frac{a^2}{2}}{2.a.\frac{a}{\sqrt{2}}}=\frac{-a^2}{\sqrt{2}.a^2}=\frac{-1}{\sqrt{2}}=\cos 135^{o}$$

∴ Angle between the circles is θ= 135°=3π/4

☺ HINT BOX ☺

- Angle between two circles
$$\cos\theta=\frac{d^2-r_1^2-r_2^2}{2r_1r_2}$$
- Orthogonal Condition: **$r_1^2+r_2^2=d^2$.**

2gg' + 2ff' = c + c'

BABY BULLET-Q

13. Find the eccentricity, coordinates of foci, length of latus rectum and equations of directrices of the ellipse $9x^2+16y^2=144$

Sol: Equation of ellipse is $9x^2+16y^2=144 \Rightarrow \frac{9x^2}{144}+\frac{16y^2}{144}=1 \Rightarrow \frac{x^2}{16}+\frac{y^2}{9}=1$.

Here, $a^2=16$, $b^2=9 \Rightarrow a>b$. Hence the ellipse is horizontal

(i) Eccentricity $e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{16-9}{16}}=\sqrt{\frac{7}{16}}=\frac{\sqrt{7}}{4}$

(ii) Foci $=(\pm ae,0)=\left(\pm 4\left(\frac{\sqrt{7}}{4}\right),0\right)=(\pm\sqrt{7},0)$

(iii) Length of latus rectum $=\frac{2b^2}{a}=\frac{2(9)}{4}=\frac{9}{2}$

(iv) Equation of directrices is $x=\pm\frac{a}{e}=\pm 4\left(\frac{4}{\sqrt{7}}\right)=\frac{\pm 16}{\sqrt{7}} \Rightarrow \sqrt{7}x=\pm 16 \Rightarrow \sqrt{7}x\pm 16=0$

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14. Find the equation of tangent and normal to the ellipse $x^2+8y^2-33=0$ at $(-1,2)$

Sol: The equation of the ellipse is $S=x^2+8y^2-33=0$

$\Rightarrow$The equation of the tangent at $(-1,2)$ is $S_1=0 \Rightarrow x_1x+8y_1y-33=0$

$\Rightarrow(-1)x+8(2)(y)-33=0 \Rightarrow -x+16y-33=0 \Rightarrow x-16y+33=0$

$\therefore$ Slope of the tangent is $\frac{1}{16} \Rightarrow$slope of its normal is -16

$\therefore$ Equation of the normal at $(-1,2)$ with slope -16 is $y-2=-16(x+1)=-16x-16 \Rightarrow 16x+y+14=0$

BABY BULLET-Q

15. Find the equations of the tangents to the hyperbola $3x^2-4y^2=12$ which are (a) Parallel (b) Perpendicular to the line $y=x-7$

Sol: Given hyperbola is $3x^2-4y^2=12 \Rightarrow \frac{x^2}{4}-\frac{y^2}{3}=1 \Rightarrow a^2=4, b^2=3$

Slope of the given line $y=x-7$ is $m=1$

$\Rightarrow$ Slope of its perpendicular is -1

Formula:

Tangent with slope m is $y=mx\pm\sqrt{a^2m^2-b^2}$

(i) Parallel tangent with slope m=1 is $y=1.x\pm\sqrt{4(1)^2-3}=x\pm 1$

$\Rightarrow x-y\pm 1=0$

(ii)Perpendicular tangent with slope m=–1 is $y=(-1)x\pm\sqrt{4(1)^2-3}=-x\pm 1$

$\Rightarrow x+y\pm 1=0$

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16. Evaluate $\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

Sol: We know $\int_a^b f(x)dx=\int_a^b f(a+b-x)dx$. $\therefore I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$ ....(1)

$$=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}+\sqrt{\cos\left(\frac{\pi}{6}+\frac{\pi}{3}-x\right)}}dx$$

$$=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}}{\sqrt{\sin\left(\frac{\pi}{2}-x\right)}+\sqrt{\cos\left(\frac{\pi}{2}-x\right)}}dx=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}dx \quad ....(2)$$

From (1) and (2), $I+I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx+\int_{\pi/6}^{\pi/3}\frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx$

$$\Rightarrow 2I=\int_{\pi/6}^{\pi/3}\frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}dx=\int_{\pi/6}^{\pi/3}1dx=[x]_{\pi/6}^{\pi/3}=\frac{\pi}{3}-\frac{\pi}{6}=\frac{\pi}{6}\Rightarrow 2I=\frac{\pi}{6}\Rightarrow I=\frac{\pi}{12}$$

BABY BULLET-Q

17. Solve $(xy^2 + x)\,dx + (yx^2 + y)\,dy = 0$

Sol: The given D.E is in the variables and separable form

$$\therefore (xy^2+x)dx+(yx^2+y)dy=0 \Rightarrow (xy^2+x)dx=-(yx^2+y)dy$$

$$\Rightarrow x(y^2+1)dx=-y(x^2+1)dy \Rightarrow \frac{x}{x^2+1}dx = -\frac{y}{y^2+1}dy$$

$$\Rightarrow \int\frac{2x}{x^2+1}dx = -\int\frac{2y}{y^2+1}dy \Rightarrow \log(x^2+1) = -\log(y^2+1)+\log c$$

$$\Rightarrow \log(x^2+1)+\log(y^2+1) = \log c \Rightarrow \log(x^2+1)(y^2+1) = \log c \Rightarrow (x^2+1)(y^2+1) = c$$

$\therefore$ The solution is $(x^2+1)(y^2+1) = c$

BABY BULLET-Q

SECTION-C

18. Find the equation of the circle, which passes through (2,–3), (–4,5) and whose centre lies on the line 4x+3y+1=0.

Sol : Let A=(2,–3), B=(–4,5)

We take $S(x_1,y_1)$ as the centre of the circle

$\Rightarrow SA=SB \Rightarrow SA^2=SB^2$.

$\Rightarrow (x_1-2)^2+(y_1+3)^2=(x_1+4)^2+(y_1-5)^2$

$\Rightarrow (x_1^2-4x_1+4)+(y_1^2+6y_1+9)$

$=(x_1^2+8x_1+16)+(y_1^2-10y_1+25)$

$\Rightarrow 12x_1-16y_1+28=0$

$\Rightarrow 4(3x_1-4y_1+7)=0$

$\Rightarrow 3x_1-4y_1+7=0$ ....(1)

But centre (x_1,y_1) lies on 4x+3y+1=0

$\Rightarrow 4x_1+3y_1+1=0$ ....(2)

Solving (1) & (2) we get the centre $S(x_1,y_1)$

$$\frac{x_1}{3\times7+4\times1}=\frac{-y_1}{4\times7-3\times1}=\frac{1}{4(-4)-3(3)}$$

$$\Rightarrow \frac{x_1}{25}=\frac{-y_1}{25}=-\frac{1}{25}\Rightarrow x_1=-1,\ y_1=1$$

$\therefore S(x_1,y_1)=(-1,1)$

Also, we have A=(2,–3)

So, radius $r=SA\Rightarrow r^2=SA^2$.

$\Rightarrow r^2=SA^2=(-1-2)^2+(1+3)^2=9+16=25$

$\therefore$ Circle with centre (–1,1) and $r^2=25$ is $(x+1)^2+(y-1)^2=25$

$\Rightarrow(x^2+2x+1)+(y^2-2y+1)=25 \Rightarrow x^2+y^2+2x-2y-23=0$

BABY BULLET-Q

19. Show that the circles $x^2+y^2-6x-9y+13=0$, $x^2+y^2-2x-16y=0$ touch each other. Find the point of contact and the equation of the common tangent at that point.

Sol: First circle is $S\equiv x^2+y^2-6x-9y+13=0$

Centre $C_1=(3,9/2)$,

$$\text{radius } r_1=\sqrt{3^2+\left(\frac{9}{2}\right)^2-13}=\sqrt{9+\frac{81}{4}-13}=\sqrt{\frac{36+81-52}{4}}=\sqrt{\frac{65}{4}}=\frac{\sqrt{65}}{2}$$

Second circle is $S'\equiv x^2+y^2-2x-16y=0$

Centre $C_2=(1,8)$, radius $r_2=\sqrt{1^2+8^2-0}=\sqrt{1+64}=\sqrt{65}$

$$C_1C_2=\sqrt{(3-1)^2+\left(\frac{9}{2}-8\right)^2}=\sqrt{2^2+\left(\frac{9-16}{2}\right)^2}=\sqrt{4+\frac{49}{4}}=\sqrt{\frac{16+49}{4}}=\sqrt{\frac{65}{4}}=\frac{\sqrt{65}}{2}$$

Also, $r_2-r_1=\sqrt{65}-\frac{\sqrt{65}}{2}=\frac{\sqrt{65}}{2}=C_1C_2$

$\therefore$ The circles touch each other **internally**.

Now, $r_1:r_2=\frac{\sqrt{65}}{2}:\sqrt{65}=\frac{1}{2}:1=1:2$

So, the point of contact P divides the join of $C_1(3,9/2)$, $C_2(1,8)$ externally in the ratio 1:2

$$\therefore P=\left(\frac{1(1)-2(3)}{1-2},\frac{1(8)-2(\frac{9}{2})}{1-2}\right)=(5,1)$$

The equation of the common tangent to the circles S=0 and S'=0 at the point of contact is given by S–S'=0

$\Rightarrow(x^2+y^2-6x-9y+13)-(x^2+y^2-2x-16y)=0$

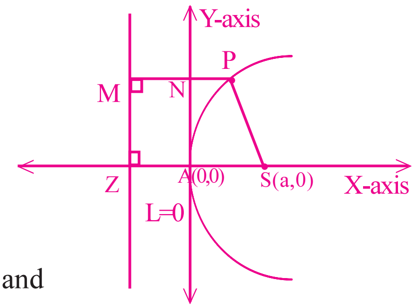
$\Rightarrow -4x+7y+13=0\Rightarrow 4x-7y-13=0$

BABY BULLET-Q

20. Show that the equation of the parabola in the standard form is $y^2 = 4ax$.

Sol: Let S be the focus and L=0 be the directrix of the parabola.

Let Z be the projection of S on to the directrix



Let A be the mid point of SZ

$\Rightarrow SA=AZ \Rightarrow \frac{SA}{AZ}=1$

$\Rightarrow$ A is a point on the parabola.

Take AS, the principle axis of the parabola as X-axis and the line perpendicular to AS through A as the Y-axis $\Rightarrow A=(0,0)$

Let $AS=a \Rightarrow S=(a,0),\ Z=(-a,0)$

$\Rightarrow$ the equation of the directrix is $x=-a \Rightarrow x+a=0$

Let $P(x_1,y_1)$ be any point on the parabola.

N be the projection of P on to the Y-axis.

M be the projection of P on to the directrix.

Here $PM=PN+NM=x_1+a$ $(\because PN = x - \text{coordinate of } P \text{ and } NM = AZ = AS = a)$

Now, by the focus directrix property of the parabola,

we have $\frac{SP}{PM}=1 \Rightarrow SP = PM \Rightarrow SP^2 = PM^2$

$\Rightarrow (x_1-a)^2+(y_1-0)^2=(x_1+a)^2$

$\Rightarrow y_1^2=(x_1+a)^2-(x_1-a)^2$

$\Rightarrow y_1^2=4ax_1$ $[\because (a+b)^2-(a-b)^2=4ab]$

$\therefore$ The equation of locus of $P(x_1,y_1)$ is $y^2=4ax$

BABY BULLET-Q

21. **Evaluate** $\int \frac{dx}{(1+x)\sqrt{3+2x-x^2}}$

Sol: Put $1+x=\frac{1}{t} \Rightarrow dx=-\frac{1}{t^2}dt$. Also $x=\frac{1}{t}-1=\frac{1-t}{t}$

$$\therefore 3+2x-x^2=3+2\left(\frac{1-t}{t}\right)-\left(\frac{1-t}{t}\right)^2=3+2\left(\frac{1}{t}-1\right)-\left(\frac{1-2t+t^2}{t^2}\right)$$

$$=3+\frac{2}{t}-2-\left(\frac{1}{t^2}-\frac{2}{t}+1\right)=3+\frac{2}{t}-2-\frac{1}{t^2}+\frac{2}{t}-1=\frac{4}{t}-\frac{1}{t^2}=\frac{4t-1}{t^2}$$

$$\therefore I=\int\frac{1}{(1+x)\sqrt{3+2x-x^2}}dx=\int\frac{1}{\frac{1}{t}\sqrt{\frac{4t-1}{t^2}}}\left(\frac{-1}{t^2}\right)dt$$

$$=-\int\frac{dt}{\sqrt{4t-1}}=\frac{-1}{4}\left(2\sqrt{4t-1}\right)+c=-\frac{1}{2}\sqrt{\frac{4}{1+x}-1}+c=-\frac{1}{2}\sqrt{\left[\frac{3-x}{1+x}\right]}+c$$

BABY BULLET-Q

22. Evaluate the reduction formula for $I_n=\int \sin^n x dx$ and hence find $\int \sin^4 x dx$

Sol: Given $I_n=\int \sin^n x dx=\int \sin^{n-1}x(\sin x)dx$.

We take First function $u=\sin^{n-1}x$ and

Second function $v=\sin x \Rightarrow \int v=-\cos x$

From By parts rule, we have

$I_n=\sin^{n-1}x\,(-\cos x)-\int(n-1)\sin^{n-2}x\cos x\,(-\cos x)dx$

$=-\sin^{n-1}x\cos x+(n-1)\int \sin^{n-2}x\cos^2 x dx$

$=-\sin^{n-1}x\cos x+(n-1)\int \sin^{n-2}x(1-\sin^2 x)dx$

$=-\sin^{n-1}x\cos x\ +(n-1)[\int \sin^{n-2}xdx-\int \sin^n xdx]$

$=-\sin^{n-1}x\cos x+(n-1)[I_{n-2}-I_n]$

$I_n=-\sin^{n-1}x\cos x+(n-1)I_{n-2}-(n-1)I_n$ $\quad=-\sin^{n-1}x\cos x+(n-1)I_{n-2}-nI_n+I_n$

$\Rightarrow nI_n=-\sin^{n-1}x\cos x+(n-1)I_{n-2}+\cancel{I_n}-\cancel{I_n}$

$\Rightarrow I_n=\dfrac{-\sin^{n-1}x\cos x}{n}+\left(\dfrac{n-1}{n}\right)I_{n-2}$ ....(1)

Put n=4, 2,0 successively in (1), we get $I_4=-\dfrac{\sin^3 x\cos x}{4}+\dfrac{3}{4}I_2$

$=-\dfrac{\sin^3 x\cos x}{4}+\dfrac{3}{4}\left[-\dfrac{\sin x\cos x}{2}+\dfrac{1}{2}I_0\right]$

$=\dfrac{-\sin^3 x\cos x}{4}-\dfrac{3}{8}\sin x\cos x+\dfrac{3}{8}I_0$

$=-\dfrac{\sin^3 x\cos x}{4}-\dfrac{3}{8}\sin x\cos x+\dfrac{3}{8}x+c$ $\qquad [\because I_0=x]$

BABY BULLET-Q

23. **Evaluate** $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$

Sol: Put $x = \tan\theta \Rightarrow dx = \sec^2\theta d\theta$ Also, $x = 0 \Rightarrow \theta = 0; x = 1 \Rightarrow \theta = \frac{\pi}{4}$

And $1+x^2 = 1+\tan^2\theta = \sec^2\theta$

$$\therefore \int_0^1 \frac{\log(1+x)}{1+x^2}dx = \int_0^{\pi/4} \frac{\log(1+\tan\theta)}{\cancel{\sec^2\theta}}\cancel{\sec^2\theta}\,d\theta = \int_0^{\pi/4} \log(1+\tan\theta)d\theta$$

$$\therefore I = \int_0^{\pi/4} \log[1+\tan\theta]d\theta$$

$$= \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-\theta\right)\right]d\theta$$

$$= \int_0^{\pi/4} \log\left[1+\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right]d\theta$$

$$= \int_0^{\pi/4} \log\left[1+\frac{1-\tan\theta}{1+\tan\theta}\right]d\theta$$

$$= \int_0^{\pi/4} \log\left[\frac{(1+\tan\theta)+(1-\tan\theta)}{1+\tan\theta}\right]d\theta$$

$$= \int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right)d\theta$$

$$= \int_0^{\pi/4} [\log 2 - \log(1+\tan\theta)]\,d\theta$$

$$= \log 2\int_0^{\pi/4} 1\,d\theta - \int_0^{\pi/4} \log(1+\tan\theta)d\theta = \log 2\int_0^{\pi/4} 1\,d\theta - I$$

$$= \log 2[\theta]_0^{\pi/4} - I$$

$$\Rightarrow I + I = (\log 2)\left(\frac{\pi}{4}\right) \Rightarrow 2I = \left(\frac{\pi}{4}\right)(\log 2)$$

$$\Rightarrow I = \left(\frac{\pi}{8}\right)(\log 2)$$

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24. **Solve $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$**

Sol: Given D.E is $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y \Rightarrow \frac{dy}{dx} = \sin(x + y)$ ....(1)

Put $x + y = t \Rightarrow 1 + \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$ From (1), $\frac{dt}{dx} - 1 = \sin t \Rightarrow \frac{dt}{dx} = 1 + \sin t$

$\Rightarrow \frac{dt}{1+\sin t} = dx \Rightarrow \int \frac{(1-\sin t)dt}{(1+\sin t)(1-\sin t)} = \int dx \Rightarrow \int\left(\frac{1-\sin t}{\cos^2 t}\right)dt = \int dx \Rightarrow \int(\sec^2 t - \tan t \sec t)dt = x + c$

$\Rightarrow \tan t - \sec t = x + c \Rightarrow \tan(x+y) - \sec(x+y) = x + c$ $[\because t = x+y]$

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