Previous IPE

SOLVED PAPERS

MARCH-2020 (TS)

PREVIOUS PAPERS

IPE: MARCH-2020(TS)

Time : 3 Hours **MATHS-2A** Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **10 × 2 = 20**

1. Write the complex number (2–3i) (3 + 4i) in the form A+iB
2. Express $-1-i\sqrt{3}$ in the mod-amp form. **3.** Find the value of $(1-i)^8$.
4. Find the maximum of the expression $2x+5-3x^2$ as x varies over R.
5. If $1,1,\alpha$ are the roots of $x^3-6x^2+9x-4=0$ then find α. **6.** If ${}^nC_5={}^nC_6$, then find ${}^{13}C_n$
7. Find the number of permutations that can be made by using all the letters of the word MATHEMATICS.
8. Find the coeff. of x^{11} in $\left(2x^2+\frac{3}{x^3}\right)^{13}$ **9.** Find the mean deviation about median for the data 4, 6, 9, 3, 10, 13, 2
10. For a binomial distribution with mean 6 and variance 2. Find the first two terms of the distribution.

SECTION-B

II. Answer any FIVE of the following SAQs: **5 × 4 = 20**

11. If $z=2-i\sqrt{7}$, then show that $3z^3-4z^2+z+88=0$ **12.** Find the range of $\frac{x^2+x+1}{x^2-x+1}$
13. A round table conference is attended by 3 Indians, 3 Chinese, 3 Canadians and 2 Americans. Find the number of ways of arranging them at the round table so that the delegates belonging to same country sit together.
14. Find the number of ways of selecting a cricket team of 11 players from 7 batsmen and 6 bowlers such that there will be atleast 5 bowlers in the team. **15.** Resolve $\frac{3x-1}{(1-x+x^2)(2+x)}$ into partial fractions.
16. A problem in calculus is given to two students A and B whose chances of solving it are 1/3 and 1/4. What is the probability that the problem will be solved if both of them try independently?
17. A bag contains 12 two rupee coions. 7 one rupee coins and 4 half a rupee coins. If three coins are selected at random, then find the probability that: (i) The sum of three coins is maximum (ii) The sum of three coins is minimum (iii) Each con is of different value.

SECTION-C

III. Answer any FIVE of the following LAQs: **5 × 7 = 35**

18. Show that one value of $\left(\frac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^{8/3}=-1$
19. Solve $18x^3+81x^2+121x+60=0$, given that a root is equal half the sum of the remaining roots.
20. If C_r denotes ${}^nC_r$ then prove that $C_0+C_1.\frac{x}{2}+C_2.\frac{x^2}{3}+.....+C_n.\frac{x^n}{n+1}=\frac{(1+x)^{n+1}-1}{(n+1)x}$ Also deduce that (i) $C_0+\frac{C_1}{2}+\frac{C_2}{3}+..........+\frac{C_n}{n+1}=\frac{2^{n+1}-1}{n+1}$ **21.** If $t=\frac{4}{5}+\frac{4.6}{5.10}+\frac{4.6.8}{5.10.15}+....$ then prove that 9t = 16.
22. A,B,C are three news papers published from a city. 20% of the population readA, 16% read B, 14% read C, 8% read both A and B, 5% read both A andC, 4% readboth B and C and 2% all the three. Find the percentage of the population who read atleast one news paper.
23. Find the mean deviation about mean for the following data:

x_i	2	5	7	8	10	35
f_i	6	8	10	6	8	2

24. A random variable X has the following probability distribution

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) the mean and (iii) P(0<x<5)

IPE TS MARCH-2020
SOLUTIONS

SECTION-A

1. Write the complex number (2–3i) (3 + 4i) in the form A+iB

Sol: $(2-3i)(3+4i)$

$= 2(3)+2(4i)-3i(3)-3i(4i) = 6+8i-9i+12 \quad [\because i^2 = -1]$

$= 18-i$

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2. Express $-1-i\sqrt{3}$ in the mod-amp form.

Sol: Let $-1-i\sqrt{3} = x+iy \Rightarrow x=-1,\ y=-\sqrt{3}$

$\therefore r = \sqrt{x^2+y^2} = \sqrt{(-1)^2+(-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\text{Now, } \theta = \text{Tan}^{-1}\left(\frac{y}{x}\right) = \text{Tan}^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \text{Tan}^{-1}\sqrt{3} = \frac{-2\pi}{3} \qquad [\because (-1,-\sqrt{3}) \in Q_3,\ \theta \in (-\pi,\pi]]$

$\therefore$ Mod-amp form is $r(\cos\theta + i\sin\theta) = \sqrt{2}\left(\cos\left(\frac{-2\pi}{3}\right) + i\sin\left(\frac{-2\pi}{3}\right)\right)$

$= \sqrt{2}\left(\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right) [\because \cos(-\theta) = \cos\theta, \sin(-\theta) = -\sin\theta]$

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3. Find the value of $(1-i)^8$.

Sol: $1-i = \sqrt{2}\left[\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right] = \sqrt{2}\left[\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right]$

$\therefore (1-i)^8 = \left[\sqrt{2}\left(\cos\frac{\pi}{4} - i\sin\frac{\pi}{4}\right)\right]^8 = (\sqrt{2})^8\left[\cos 8\frac{\pi}{4} - i\sin 8\frac{\pi}{4}\right]$

$= 2^4(\cos 2\pi - i\sin 2\pi) = 16[1-i(0)] = 16$

4. Find the maximum or minimum of the expression $2x+5-3x^2$ as x varies over R.

Sol: Comparing $2x+5-3x^2$ with $ax^2+bx+c=0$ we get $a=-3, b=2, c=5$.

Here $a=-3<0$. So the given expression has maximum value

$$\therefore \text{The maximum value is } \frac{4ac-b^2}{4a}=\frac{4(-3)(5)-2^2}{4(-3)}=\frac{-60-4}{-12}=\frac{-64}{-12}=\frac{16}{3}$$

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5. If 1,1,α are the roots of $x^3-6x^2+9x-4=0$ then find α.

Sol: From the given equation we get, $a_0=1, a_1=-6, a_2=9, a_3=-4$

Product of roots $1.1.\alpha = S_3=\frac{-a_3}{a_0}=\frac{4}{1} \quad \therefore \alpha=4.$

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6. If ${}^nC_5={}^nC_6$, then find ${}^{13}C_n$

Sol : **Formula:** ${}^nC_r={}^nC_s \Rightarrow r+s=n$ (or) $r=s$

$$\therefore {}^nC_5={}^nC_6 \Rightarrow n=5+6=11$$

$$\therefore {}^{13}C_n={}^{13}C_{11}={}^{13}C_{13-11}={}^{13}C_2=\frac{13\times \cancel{12}}{1\times \cancel{2}}=13\times 6=78$$

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7. Find the number of ways of arranging letters of the word MATHEMATICS.

Sol: Given word MATHEMATICS contains 11 letters.

Here, 2 'M's, 2'A's, 2'T's are alike

$$\therefore \text{Number of arrangements} = \frac{n!}{p!q!r!}=\frac{11!}{2!2!2!}$$

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8. Find the coeff. of x^{11} in $\left(2x^2+\frac{3}{x^3}\right)^{13}$

Sol: The general term of $\left(2x^2+\frac{3}{x^3}\right)^{13}$ is $T_{r+1}={}^{13}C_r\left(2x^2\right)^{13-r}\left(\frac{3}{x^3}\right)^r$

$$={}^{13}C_r(2)^{13-r}.3^r.x^{26-2r}.x^{-3r}={}^{13}C_r(2)^{13-r}(3)^r.x^{26-5r}\text{ ....(1)}$$

To get the coefficient of x^{11}, we take $26-5r=11 \Rightarrow 5r=15 \Rightarrow r=3$

From (1), the coefficient of $x^{11}={}^{13}C_3(2)^{10}(3)^3=(286)(2^{10})(3^3)$

9. Find the mean deviation about median for the data 4, 6, 9, 3, 10, 13, 2

Sol: **Given data:** 4, 6, 9, 3, 10, 13, 2.

Its ascending order : 2,3,4,$\underline{6}$,9,10,13.

Number of observations n = 7 is odd .

$\therefore$ Median is the middle most term $\Rightarrow$M=6

Deviations from the median:

2–6= –4; 3–6= –3; 4–6= –2; 6–6=0; 9–6=3;10–6=4; 13–6=7

Absolute values of these deviations:

4, 3, 2, 0, 3, 4, 7

$\therefore$ M.D from Median is $M.D=\frac{\Sigma|x_i-M|}{7}=\frac{4+3+2+0+3+4+7}{7}=\frac{23}{7}=3.29$

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10. For a binomial distribution with mean 6 and variance 2. Find the first two terms of the distribution.

Sol: Given mean np=6, variance npq=2

$\therefore (np)q=2\Rightarrow 6(q)=2\Rightarrow q=\frac{2}{6}=\frac{1}{3}$

$\Rightarrow p=1-q=1-\frac{1}{3}=\frac{2}{3}$

Take $np=6\Rightarrow n.\frac{2}{3}=6\Rightarrow n=\frac{18}{2}=9$ $\therefore n=9, q=\frac{1}{3}$ and $p=\frac{2}{3}$

(i) $P(X=0)={}^9C_0\left(\frac{1}{3}\right)^9=\frac{1}{3^9}$;

(ii) $P(X=1)={}^9C_1\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right)=\frac{2}{3^7}$

SECTION-B

11. If $z = 2 - i\sqrt{7}$ then show that $3z^3 - 4z^2 + z + 88 = 0$.

Sol: Given that $z = 2 - i\sqrt{7}$

Now, $z - 2 = -i\sqrt{7} \Rightarrow (z-2)^2 = \left(-i\sqrt{7}\right)^2$

$\Rightarrow z^2 - 4z + 4 = -7 \Rightarrow z^2 - 4z + 11 = 0$

On dividing $3z^3 - 4z^2 + z + 88$ with $z^2 - 4z + 11$, we get the quotient $3z+8$

$\therefore 3z^3 - 4z^2 + z + 88 = (3z^2 - 4z + 11)(3z+8) = 0(3z+8) = 0$

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12. Find the range of $\dfrac{x^2 + x + 1}{x^2 - x + 1}$ for $x \in R$.

Sol: Let $y = \dfrac{x^2 + x + 1}{x^2 - x + 1}$

$\Rightarrow y(x^2 - x + 1) = x^2 + x + 1$

$\Rightarrow yx^2 - yx + y = x^2 + x + 1$

$\Rightarrow yx^2 - x^2 - yx - x + y - 1 = 0$

$\Rightarrow x^2(y-1) - x(y+1) + (y-1) = 0$

$\Rightarrow (y-1)x^2 - (y+1)x + (y-1) = 0 \,..........(1)$

(1) is a quadratic in x and its roots are reals.

$\therefore \Delta = b^2 - 4ac \geq 0$

$\Rightarrow (y+1)^2 - 4(y-1)^2 \geq 0$

$\Rightarrow (y+1)^2 - (2y-2)^2 \geq 0$

$\Rightarrow \left(y+1+2y-2\right)\left(y+1\right) - \left(2y-2\right) \geq 0$ $\left[\because a^2 - b^2 = (a+b)(a-b)\right]$

$\Rightarrow (3y-1)\left(3-y\right) \geq 0 \Rightarrow (3y-1)(y-3) \leq 0$

$\Rightarrow y \in \left[\dfrac{1}{3}, 3\right]$ $\therefore$ Range$=\left[\dfrac{1}{3}, 3\right]$

13. A round table conference is attended by 3 Indians, 3 Chinese, 3 Canadians and 2 Americans. Find the number of ways of arranging them at the round table so that the delegates belonging to same country sit together.

Sol: 3 Indians can be arranged among themselves in 3! ways.

3 Chinese can be arranged among themselves in 3! ways.

3 Canadians can be arranged among themselves in 3! ways.

2 Americans can be arranged among themselves in 2! ways.

Since the delegates belonging to the same country sit together, the delegates of 4 countries in a round table are arranged in (4–1)!= 3! ways

$\therefore$ The required number of arrangements $=3!\times3!\times3!\times2!\times3!=6\times6\times6\times2\times6= 2592$

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14. Find the number of ways of selecting a cricket team of 11 players from batsmen and 6 bowlers such that there will be atleast 5 bowlers in the team.

Sol: A Team of 11 players with atleast 5 bowlers can be selected in the following compositions:

Bowlers(6)	Batsmen(7)	No. of selections
5	6	${}^6C_5\times{}^7C_6= 6\times7=42$
6	5	${}^6C_6\times{}^7C_5= 1\times21=21$

$\because {}^7C_5={}^7C_2$
$=\dfrac{7\times6}{2\times1}=21$

$\therefore$ the total number of selections=42+21= 63

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15. Resolve $\dfrac{3x-1}{(1-x+x^2)(x+2)}$ into partial fractions.

Sol: Let $\dfrac{3x-1}{(1-x+x^2)(x+2)}=\dfrac{A}{x+2}+\dfrac{Bx+C}{1-x+x^2}=\dfrac{A(1-x+x^2)+(Bx+C)(x+2)}{(x+2)(1-x+x^2)}$

$\Rightarrow A(1-x+x^2)+(Bx+C)(x+2)=3x-1$ ....**(1)**

Putting $x=-2$ in (1) , we get $A(1+2+4)=-7 \Rightarrow 7A=-7 \Rightarrow A=-1$

Equating the coefficients of x^2 in (1), we get $A+B=0 \Rightarrow B=-A \Rightarrow B=1$

Equating the constant terms in (1), we get $A+2C=-1 \Rightarrow 2C=-1-A=-1+1=0 \Rightarrow C=0$

$\therefore \dfrac{3x-1}{(1-x+x^2)(x+2)}=\dfrac{A}{x+2}+\dfrac{Bx+C}{1-x+x^2}=-\dfrac{1}{x+2}+\dfrac{x}{1-x+x^2}$

16. A problem in calculus is given to two students A and B whose chances of solving it are1/3, 1/4 respectively. Find the probability of the problem being solved if both of them try independently.

Sol: Let A,B denote the events of solving the problem by A, B respectively $\Rightarrow P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$

$$\therefore P(\bar{A})=1-P(A)=1-\frac{1}{3}=\frac{2}{3};\quad P(\bar{B})=1-P(B)=1-\frac{1}{4}=\frac{3}{4}$$

$$\therefore P(A\cup B)=1-P(\bar{A}\cap\bar{B})=1-P(\bar{A}).P(\bar{B})=1-\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)=1-\frac{1}{2}=\frac{1}{2}$$

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17. A bag contains 12 two rupee coins. 7 one rupee coins and 4 half a rupee coins. If three coins are selected at random, then find the probability that:
(i) The sum of three coins is maximum
(ii) The sum of three coins is minimum
(iii) Each coin is of different value.

Sol: In the bag, there are 12 two rupee, 7 one rupee and 4 half rupee coins.

Total number of coins = 12 +7 + 4= 23

Number of ways of drawing 3 coins $=n(S)={}^{23}C_3$

i) We get maximum amount, if all the 3 coins are 2 rupee coins.

Number of drawing 3 two rupee coins $= n(E_1) = {}^{12}C_3$

$$P(E_1)=\frac{n(E_1)}{n(S)}=\frac{{}^{12}C_3}{{}^{23}C_3}$$

ii) We get minimum amount, if 3 coins are taken from 4 half rupee coins.

Number of ways of drawing 3 half rupee coins $= n(E_2)={}^{4}C_3$

$$P(E_2)=\frac{n(E_2)}{n(S)}=\frac{{}^{4}C_3}{{}^{23}C_3}$$

iii) Each coin is of different value if we draw one coin from each type.

Number of ways of drawing different valued coins $= n(E_3)={}^{12}C_1\times {}^{7}C_1\times {}^{4}C_1$

$$P(E_3)=\frac{n(E_3)}{n(S)}=\frac{{}^{12}C_1\times {}^{7}C_1\times {}^{4}C_1}{{}^{23}C_3}$$

SECTION-C

18. Show that one value of $\left(\dfrac{1+\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}{1+\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}}\right)^{8/3}=-1$

Sol: Let $z=\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}$.

$$\text{Then }\frac{1}{z}=\frac{1}{\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}}=\frac{1\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)}{\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)}=\frac{\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)}{\sin^2\frac{\pi}{8}+\cos^2\frac{\pi}{8}}=\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)$$

$$\therefore \text{G.E}=\left(\frac{1+\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)}{1+\left(\sin\frac{\pi}{8}-i\cos\frac{\pi}{8}\right)}\right)^{8/3}=\left(\frac{1+z}{1+\frac{1}{z}}\right)^{8/3}=\left(\frac{\cancel{1+z}}{\frac{\cancel{z+1}}{z}}\right)^{8/3}=(z)^{8/3}$$

$$=\left(\sin\frac{\pi}{8}+i\cos\frac{\pi}{8}\right)^{8/3}=\left[\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)+i\sin\left(\frac{\pi}{2}-\frac{\pi}{8}\right)\right]^{8/3}$$

$$=\left(\cos\frac{4\pi-\pi}{8}+i\sin\frac{4\pi-\pi}{8}\right)^{8/3}=\left(\cos\frac{3\pi}{8}+i\sin\frac{3\pi}{8}\right)^{8/3}=\left(\cos\frac{\cancel{8}}{\cancel{3}}\left(\frac{\cancel{3}\pi}{\cancel{8}}\right)+i\sin\frac{\cancel{8}}{\cancel{3}}\left(\frac{\cancel{3}\pi}{\cancel{8}}\right)\right)$$

$$=\cos\pi+i\sin\pi=\cos 180^\circ+i\sin 180^\circ=-1+i(0)=-1$$

19. Solve $18x^3+81x^2+121x+60=0$, given that a root is equal half the sum of the remaining roots

Sol: Easy Method: Let α,β,γ be the roots of $18x^3+81x^2+121x+60=0$

Given that one root is half the sum of remaining two roots. Let $\beta=\frac{\alpha+\gamma}{2}$

$\Rightarrow \alpha, \beta, \gamma$ are in A.P

$\therefore$ we take $\alpha=a-d$, $\beta=a$, $\gamma=a+d$

The given equation is $18x^3+81x^2+121x+60=0$

$$S_1=\alpha+\beta+\gamma=-\frac{b}{a}\Rightarrow (a-d)+a+(a+d)=-\frac{81}{18}=-\frac{9}{2}\Rightarrow 3a=-\frac{9}{2}\Rightarrow a=-\frac{3}{2}$$

$$S_3=\alpha\beta\gamma=-\frac{d}{a}\Rightarrow (a-d)(a)(a+d)=-\frac{60}{18}=-\frac{10}{3}\Rightarrow a(a^2-d^2)=-\frac{10}{3}$$

$$\Rightarrow -\frac{3}{2}\left(\left(-\frac{3}{2}\right)^2-d^2\right)=-\frac{10}{3}\Rightarrow \frac{9}{4}-d^2=\frac{10}{3}\times\frac{2}{3}=\frac{20}{9}\Rightarrow d^2=\frac{9}{4}-\frac{20}{9}=\frac{81-80}{36}=\frac{1}{36}\Rightarrow d=\frac{1}{6}$$

Now $a-d=-\frac{3}{2}-\frac{1}{6}=\frac{-9-1}{6}=-\frac{10}{6}=-\frac{5}{3}$, Also, $a+d=-\frac{3}{2}+\frac{1}{6}=\frac{-9+1}{6}=-\frac{8}{6}=-\frac{4}{3}$

$\therefore$ The roots a–d, a, a+d are $-\frac{5}{3},-\frac{3}{2},-\frac{4}{3}$

20. If n is a positive integer and x is any nonzero real number, then prove that

$$C_0 + C_1\frac{x}{2} + C_2\frac{x^2}{3} + C_3\frac{x^3}{4} + ... + C_n\frac{x^n}{n+1} = \frac{(1+x)^{n+1} - 1}{(n+1)x}$$

Also deduce that (i) $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$

Sol: Let $S = C_0 + C_1.\frac{x}{2} + C_2.\frac{x^2}{3} + ... + C_n.\frac{x^n}{n+1} = {}^nC_0 + {}^nC_1\frac{x}{2} + {}^nC_2\frac{x^2}{3} + ... + {}^nC_n.\frac{x^n}{n+1}$

$$\Rightarrow x.S = {}^nC_0.x + {}^nC_1\frac{x^2}{2} + {}^nC_2.\frac{x^3}{3} + ... + {}^nC_n.\frac{x^{n+1}}{n+1}$$

$$\Rightarrow (n+1)xS = \frac{n+1}{1}\cdot {}^nC_0.x + \frac{n+1}{2}.{}^nC_1.x^2 + \frac{n+1}{3}.{}^nC_2x^3 + ... + \frac{n+1}{n+1}\,{}^nC_n.x^{n+1}$$

$$= {}^{n+1}C_1.x + {}^{n+1}C_2x^2 + {}^{n+1}C_3x^3 + ... + {}^{n+1}C_{n+1}.x^{n+1} \qquad \left(\because \left(\frac{n+1}{r+1}\right).{}^nC_r = {}^{(n+1)}C_{r+1}\right)$$

$$\Rightarrow (n+1)xS = (1+x)^{n+1} - 1 \qquad (\because {}^nC_1x + {}^nC_2x^2 + ... + {}^nC_nx^n = (1+x)^n - 1)$$

$$\therefore S = \frac{(1+x)^{n+1} - 1}{(n+1).x}$$

Corollary: **Prove that** $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$

Proof: $S = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ... \frac{C_n}{n+1}$

$$\Rightarrow S = {}^nC_0 + \frac{1}{2}.{}^nC_1 + \frac{1}{3}.{}^nC_2 + ... + \frac{1}{n+1}.C_n$$

$$\Rightarrow (n+1)S = \frac{n+1}{1}.{}^nC_0 + \frac{n+1}{2}.{}^nC_1 + \frac{n+1}{3}.{}^nC_2 + ... + \frac{n+1}{n+1}\,{}^nC_n$$

$$= {}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+1}C_3 + ... + {}^{n+1}C_{n+1} \qquad \left(\text{Since } \frac{n+1}{r+1}.{}^nC_r = {}^{n+1}C_{r+1}\right)$$

$$= 2^{n+1} - 1 \qquad (\because {}^nC_1 + {}^nC_2 + ... + {}^nC_n = 2^n - 1)$$

$$\therefore S = \frac{2^{n+1} - 1}{n+1}$$

21. **If $t=\frac{4}{5}+\frac{4.6}{5.10}+\frac{4.6.8}{5.10.15}+....$ then prove that $9t=16$.**

Sol: Given that $t=\frac{4}{5}+\frac{4.6}{5.10}+\frac{4.6.8}{5.10.15}+....$

Adding 1 on both sides, we get

$$1+t=1+\frac{4}{1!}\left(\frac{1}{5}\right)+\frac{4.6}{2!}\left(\frac{1}{5}\right)^2+\frac{4.6.8}{3!}\left(\frac{1}{5}\right)^3+......$$

Comparing the above series with $1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+.......=(1-x)^{-p/q}$

we get $p=4, p+q=6 \Rightarrow 4+q=6 \Rightarrow q=2$. Also $\frac{x}{q}=\frac{1}{5} \Rightarrow x=\frac{q}{5}=\frac{2}{5}$

$$\therefore 1+t=(1-x)^{-p/q}=(1-\frac{2}{5})^{-4/2}=\left(\frac{3}{5}\right)^{-2}=\left(\frac{5}{3}\right)^2=\frac{25}{9}$$

$$\Rightarrow 1+t=\frac{25}{9} \Rightarrow 9(1+t)=25 \Rightarrow 9+9t=25 \Rightarrow 9t=16$$

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22. **A,B,C are three news papers published from a city. 20% of the population readA, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C and 2% all the three. Find the percentage of the population who read atleast one news paper.**

Sol: Given that $P(A)=\frac{20}{100}=0.2$, $P(B)=\frac{16}{100}=0.16$, $P(C)=\frac{14}{100}=0.14$

$P(A\cap B)=\frac{8}{100}=0.08$, $P(B\cap C)=\frac{4}{100}=0.04$, $P(A\cap C)=\frac{5}{100}=0.05$,

$P(A\cap B\cap C)=\frac{2}{100}=0.02$

$\therefore P(A\cup B\cup C)= P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(C\cap A)+P(A\cap B\cap C)$

$= 0.2 + 0.16+ 0.14 - 0.08 - 0.04 - 0.05 + 0.02 = 0.52 - 0.17 = 0.35$

Percentage of population who read atleast one newspaper = $0.35\times 100\% = 35\%$.

23. Find the mean deviation about mean for the following data:

x_i	2	5	7	8	10	35
f_i	6	8	10	6	8	2

Sol: We form the following table from the given data.

x_i	f_i	f_ix_i	$\lvert x_i - \bar{x}\rvert$	$f_i\lvert x_i - \bar{x}\rvert$
2	6	12	6	36
5	8	40	3	24
7	10	70	1	10
8	6	48	0	0
10	8	80	2	16
35	2	70	27	54
	40	**320**		**140**

Here, $N = \Sigma f_i = 40$; $\Sigma f_i x_i = 320$ $\Rightarrow$ Mean $\bar{x} = \frac{\Sigma f_i x_i}{N} = \frac{320}{40} = 8$

From the table $\Sigma f_i \lvert x_i - \bar{x}\rvert = 140$

$\therefore$ Mean deviation about mean is $M.D = \frac{\Sigma f_i \lvert x_i - \bar{x}\rvert}{N} = \frac{140}{40} = 3.5$

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24. A random variable x has the following probability distribution

$X=x_i$	0	1	2	3	4	5	6	7
$P(X=x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) the mean (iii) P(0<X<5)

Sol: We know $\Sigma P(X=x_i)=1$

$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1 \Rightarrow 10k^2+9k=1 \Rightarrow 10k^2+9k-1=0$

$\Rightarrow 10k^2+10k-k-1=0 \Rightarrow 10k(k+1)-1(k+1)=0 \Rightarrow (10k-1)(k+1)=0 \Rightarrow k=1/10$, (since $k>0$)

(i) $k=1/10$

(ii) Mean $\mu = \sum_{i=1}^{n} x_i . P(X=x_i) = 0(0)+1(k)+2(2k)+3(2k)+4(3k)+5(k^2)+6(2k^2)+7(7k^2+k)$

$=0+k+4k+6k+12k+5k^2+12k^2+49k^2+7k=66k^2+30k$

$= 66\left(\frac{1}{100}\right) + 30\left(\frac{1}{10}\right) = 0.66 + 3 = 3.66$

(iii) $P(0<X<5)=P(X=1)+P(X=2)+P(X=3)+P(X=4) = k+2k+2k+3k=8k = 8\left(\frac{1}{10}\right) = \frac{8}{10} = \frac{4}{5}$