Previous IPE

SOLVED PAPERS

MARCH -2020 (TS)

PREVIOUS PAPERS

IPE: MARCH-2020(TS)

Time : 3 Hours **MATHS-1A** Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: $10 \times 2 = 20$

1. If f:R→R is defined by $f(x)=\frac{1-x^2}{1+x^2}$ then show that $f(\tan\theta)=\cos 2\theta$.
2. Find the domain of the real function $f(x)=\frac{1}{(x^2-1)(x+3)}$
3. If $\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix}=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$ then find the values of x, y,z and a. **4.** Define Rank of a matrix.
5. If the vectors $-3\bar{i}+4\bar{j}+\lambda\bar{k}, \mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear vectors then find λ & μ.
6. Find the vector equation of the plane passing through the points (0,0,0), (0,5,0) and (2,0,1)
7. Find the angle between the planes $\bar{r}.(2\bar{i}-\bar{j}+2\bar{k})=3,\ \bar{r}.(3\bar{i}+6\bar{j}+\bar{k})=4$
8. Find a cosine function whose period is 7.
9. What is the value of $\tan 20^\circ+\tan 40^\circ+\sqrt{3}\tan 20^\circ \tan 40^\circ$?
10. Prove that $\cosh^4 x-\sinh^4 x=\cosh(2x)$

SECTION-B

II. Answer any FIVE of the following SAQs: $5 \times 4 = 20$

11. If $A=\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then find A^4.
12. If the points whose position vectors are $3\bar{i}-2\bar{j}-\bar{k}, 2\bar{i}+3\bar{j}-4\bar{k}, -\bar{i}+\bar{j}+2\bar{k}, 4\bar{i}+5\bar{j}+\lambda\bar{k}$ are coplanar, then show that $\lambda=-146/17$
13. If $|\bar{a}|=13, |\bar{b}|=13$ and $\bar{a}.\bar{b}=60$, then find $|\bar{a}\times\bar{b}|$
14. Prove that $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{8\pi}{7}=\frac{1}{8}$ **15.** Solve $\sqrt{3}\sin\theta-\cos\theta=\sqrt{2}$
16. Prove that $\sin^{-1}\frac{4}{5}+2\tan^{-1}\frac{1}{3}=\frac{\pi}{2}$ **17.** In ΔABC,show that $b\cos^2\frac{C}{2}+c\cos^2\frac{B}{2}=s$

SECTION-C

III. Answer any FIVE of the following LAQs: $5 \times 7 = 35$

18. If f:A→B, g:B→C are two bijective functions then prove that $(gof)^{-1}=f^{-1}og^{-1}$
19. Using Mathematical Induction prove that $a+ar+ar^2+....+n$ terms $=\frac{a(r^n-1)}{r-1}, r\neq 1$
20. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix}=0$, then show that abc= –1
21. By using Cramer's solve $x+y+z=1, 2x+2y+3z=6, x+4y+9z=3$
22. Find the volume of the tetrahedron, whose vertices are (1,2,1), (3,2,5), (2,–1,0) and (–1,0,1).
23. If A, B, C are angles in a triangle, then prove that $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$
24. If $r : R : r_1 = 2 : 5 : 12$, then prove that the triangle is right angled at A.

IPE TS MARCH-2020
SOLUTIONS

SECTION-A

1. If f:R→R is $f(x)=\frac{1-x^2}{1+x^2}$ then show that f(tanθ) = cos2θ.

A: Given $f(x)=\frac{1-x^2}{1+x^2}$

$\therefore$ f(tan θ) = $\frac{1-\tan^2\theta}{1+\tan^2\theta}$ =cos2θ. [Formula from Trigonometry]

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2. Find the domain of the real function $\frac{1}{(x^2-1)(x+3)}$

A: Given f(x) is defined when $(x^2-1)(x+3)\neq 0$

$\Rightarrow (x-1)(x+1)(x+3)\neq 0 \Rightarrow x\neq 1,-1,-3$

$\therefore$ Domain = R–{1,–1,–3}

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3. If $\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix}=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$ then find the values of x, y,z and a.

A: Given $\begin{bmatrix} x-1 & 2 & 5-y \\ 0 & z-1 & 7 \\ 1 & 0 & a-5 \end{bmatrix}=\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 1 & 0 & 0 \end{bmatrix}$

On equating corresponding elements,we get

x–1=1 $\Rightarrow$ x=1+1=2;

5–y=3 $\Rightarrow$ y=5–3=2;

z–1=4$\Rightarrow$ z=4+1=5;

a–5 = 0 $\Rightarrow$ a=5 $\therefore$ x=2, y=2, z= 5, a=5

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4. Define Rank of a matrix.

A: **Rank of a Matrix:** The rank of a non-zero matrix A is the order of any highest order non-zero minor of the given matrix A.

5. If $-3\bar{i}+4\bar{j}+\lambda\bar{k}$, $\mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear vectors then find λ & μ.

A: Given that the vectors $\bar{a}=-3\bar{i}+4\bar{j}+\lambda\bar{k}$, $\bar{b}=\mu\bar{i}+8\bar{j}+6\bar{k}$ are collinear.

$$\therefore \frac{-3}{\mu}=\frac{4}{8}=\frac{\lambda}{6}$$

$$\Rightarrow \frac{-3}{\mu}=\frac{1}{2}\Rightarrow \mu=2\times -3=-6 \text{ and } \frac{\lambda}{6}=\frac{1}{2}\Rightarrow \lambda=\frac{6}{2}=3$$

$\therefore$ $\lambda=3$, $\mu=-6$

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6. Find the vector equation of the plane passing through the points (0,0,0), (0,5,0) and (2,0,1)

A: Given $A(\bar{a})=\bar{0}$, $B(\bar{b})=5\bar{j}$,

$C(\bar{c})=2\bar{i}+\bar{k}$

Vector equation of the plane is $\bar{r}=(1-s-t)\bar{a}+s\bar{b}+t\bar{c}$, s, t$\in$R

$\bar{r}=(1-s-t)\bar{0}+s(5\bar{j})+t(2\bar{i}+\bar{k})$

$\therefore \bar{r}=(5\bar{j})s+t(2\bar{i}+\bar{k}), s,t\in R$

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7. Find the angle between the planes $\bar{r}.(2\bar{i}-\bar{j}+2\bar{k})=3$, $\bar{r}.(3\bar{i}+6\bar{j}+\bar{k})=4$

A: Comparing the given planes with $\bar{r}.\bar{n}_1=p_1$, $\bar{r}.\bar{n}_2=p_2$, we get

$\bar{n}_1=2\bar{i}-\bar{j}+2\bar{k}$, $\bar{n}_2=3\bar{i}+6\bar{j}+\bar{k}$

$$\therefore \cos\theta=\frac{\bar{n}_1.\bar{n}_2}{|\bar{n}_1|\,|\bar{n}_2|}$$

$$=\frac{(2\bar{i}-\bar{j}+2\bar{k}).(3\bar{i}+6\bar{j}+\bar{k})}{\sqrt{4+1+4}.\sqrt{9+36+1}}=\frac{2(3)-1(6)+2(1)}{\sqrt{9}.\sqrt{46}}=\frac{6-6+2}{\sqrt{9}.\sqrt{46}}=\frac{2}{3\sqrt{46}}$$

$$\therefore \cos\theta=\frac{2}{3\sqrt{46}}\Rightarrow \theta=\cos^{-1}\frac{2}{3\sqrt{46}}$$

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8. Find a cosine function whose period is 7.

A: Let coskx be the required cosine function

Period of coskx$=\frac{2\pi}{k}$

$\therefore$ Period $\frac{2\pi}{k}=7 \Rightarrow k=\frac{2\pi}{7}$

$\therefore \cos\left(\frac{2\pi}{7}\right)x$ is the required cosine function

9. What is the value of $\tan 20^\circ + \tan 40^\circ + \sqrt{3}\tan 20^\circ \tan 40^\circ$?

A: Consider $20^\circ+40^\circ=60^\circ \Rightarrow \tan(20^\circ+40^\circ)=60^\circ$ Apply tan(A+B) formula

$$\Rightarrow \frac{\tan 20^\circ + \tan 40^\circ}{1-\tan 20^\circ \tan 40^\circ} = \sqrt{3} \Rightarrow \tan 20^\circ + \tan 40^\circ = \sqrt{3}(1-\tan 20^\circ \tan 40^\circ)$$

$$\Rightarrow \tan 20^\circ + \tan 40^\circ = \sqrt{3} - \sqrt{3}\tan 20^\circ \tan 40^\circ \Rightarrow \tan 20^\circ + \tan 40^\circ + \sqrt{3}\tan 20^\circ \tan 40^\circ = \sqrt{3}$$

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10. Prove that $\cosh^4 x - \sinh^4 x = \cosh 2x$

A: L.H.S= $\cosh^4 x - \sinh^4 x = (\cosh^2 x - \sinh^2 x)(\cosh^2 x + \sinh^2 x) = (1)(\cosh 2x) = \cosh 2x$ =R.H.S

SECTION-B

11. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ then find A^4.

A : $A^2 = A \times A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 3\times3 & 0 & 0 \\ 0 & 3\times3 & 0 \\ 0 & 0 & 3\times3 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}$

$$\Rightarrow A^4 = A^2 \times A^2 = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 81 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 81 \end{bmatrix} = 81\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 81\,I$$

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12. If the points whose position vectors are $3\bar{i} - 2\bar{j} - \bar{k}, 2\bar{i} + 3\bar{j} - 4\bar{k}, -\bar{i} + \bar{j} + 2\bar{k}, 4\bar{i} + 5\bar{j} + \lambda\bar{k}$ are coplanar, then show that $\lambda = -146/17$

A: We take $\overline{OP} = 3\bar{i} - 2\bar{j} - \bar{k}, \overline{OQ} = 2\bar{i} + 3\bar{j} - 4\bar{k}$,

$\overline{OR} = -\bar{i} + \bar{j} + 2\bar{k}, \overline{OS} = 4\bar{i} + 5\bar{j} + \lambda\bar{k}$,

where 'O' is the origin.

$\therefore \overline{PQ} = \overline{OQ} - \overline{OP} = (2\bar{i} + 3\bar{j} - 4\bar{k}) - (3\bar{i} - 2\bar{j} - \bar{k}) = -\bar{i} + 5\bar{j} - 3\bar{k}$

$\overline{PR} = \overline{OR} - \overline{OP} = (-\bar{i} + \bar{j} + 2\bar{k}) - (3\bar{i} - 2\bar{j} - \bar{k}) = -4\bar{i} + 3\bar{j} + 3\bar{k}$

$\overline{PS} = \overline{OS} - \overline{OP} = (4\bar{i} + 5\bar{j} + \lambda\bar{k}) - (3\bar{i} - 2\bar{j} - \bar{k}) = \bar{i} + 7\bar{j} + (\lambda + 1)\bar{k}$

But $[\overline{PQ}\ \overline{PR}\ \overline{PS}] = 0$ [Since P,Q,R,S are coplanar]

$\Rightarrow \begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & \lambda+1 \end{vmatrix} = 0 \Rightarrow (-1)[3(\lambda+1)-21]-5[-4(\lambda+1)-3]-3[(-28)-3]=0$

$\Rightarrow -1(3\lambda-18) -5(-4\lambda-7)-3(-31)=0$

$\Rightarrow -3\lambda+18+20\lambda+35+93=0$

$\Rightarrow -3\lambda+20\lambda+35+93+18=0$

$\Rightarrow 17\lambda + 146 = 0$

$\Rightarrow 17\lambda = -146$

$\Rightarrow \lambda = -146/17$

13. If $|\bar{a}| = 13$, $|\bar{b}| = 5$ and $\bar{a}.\bar{b} = 60$ then find $|\bar{a} \times \bar{b}|$.

A: We have $|\bar{a} \times \bar{b}|^2 = |\bar{a}|^2 |\bar{b}|^2 - (\bar{a}.\bar{b})^2 = (13)^2 (5)^2 - (60)^2$

$= 169(25) - 3600 = 4225 - 3600 = 625$

$\therefore |\bar{a} \times \bar{b}| = \sqrt{625} = 25$

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14. Prove that $\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}.\cos\frac{8\pi}{7} = \frac{1}{8}$

A: Let $C = \cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}.\cos\frac{8\pi}{7}$ and $S = \sin\frac{2\pi}{7}.\sin\frac{4\pi}{7}.\sin\frac{8\pi}{7}$

$\Rightarrow CS = \left(\sin\frac{2\pi}{7}\cos\frac{2\pi}{7}\right)\left(\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}\right)\left(\sin\frac{8\pi}{7}\cos\frac{8\pi}{7}\right)$

$= \frac{1}{2^3}\left(2\sin\frac{2\pi}{7}\cos\frac{2\pi}{7}\right)\left(2\sin\frac{4\pi}{7}\cos\frac{4\pi}{7}\right)\left(2\sin\frac{8\pi}{7}\cos\frac{8\pi}{7}\right)$

$= \frac{1}{8}\left(\sin\frac{4\pi}{7}\right)\left(\sin\frac{8\pi}{7}\right)\left(\sin\frac{16\pi}{7}\right) = \frac{1}{8}\sin\frac{4\pi}{7}\sin\frac{8\pi}{7}\sin\left(\frac{14\pi + 2\pi}{7}\right)$

$= \frac{1}{8}\sin\frac{4\pi}{7}\sin\frac{8\pi}{7}\sin\left(2\pi + \frac{2\pi}{7}\right)$

$= \frac{1}{8}\sin\frac{4\pi}{7}\sin\frac{8\pi}{7}\sin\frac{2\pi}{7} \qquad = \frac{1}{8}S \qquad [\because \sin(2\pi+\theta) = \sin\theta]$

$\Rightarrow C\cancel{S} = \frac{1}{8}\cancel{S} \Rightarrow C = \frac{1}{8} \Rightarrow \cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}.\cos\frac{8\pi}{7} = \frac{1}{8}$

15. Solve $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$

A: Given equation is $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2}$

On dividing by $\sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = \sqrt{4} = 2$, we get

$$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{\sqrt{2}}{2}$$

$$\Rightarrow \sin\theta\left(\frac{\sqrt{3}}{2}\right) - \cos\theta\left(\frac{1}{2}\right) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sin\theta\cos 30^\circ - \cos\theta\sin 30^\circ = \sin 45^\circ$$

$$\Rightarrow \sin\theta\cos\frac{\pi}{6} - \cos\theta\sin\frac{\pi}{6} = \sin\frac{\pi}{4}$$

$$\Rightarrow \sin\left(\theta - \frac{\pi}{6}\right) = \sin\frac{\pi}{4} \qquad [\because \sin A\cos B - \cos A\sin B = \sin(A-B)]$$

Here, P.V is $\alpha = \frac{\pi}{4}$

$\therefore$ General solution is $\theta = n\pi + (-1)^n\alpha, n \in Z$

$$\Rightarrow \theta - \frac{\pi}{6} = n\pi + (-1)^n\frac{\pi}{4}, n \in Z$$

$$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{4} + \frac{\pi}{6}, n \in Z$$

16. Prove that $\mathrm{Sin}^{-1}\frac{4}{5}+2\mathrm{Tan}^{-1}\frac{1}{3}=\frac{\pi}{2}$

A: We know $2\mathrm{Tan}^{-1}x=\mathrm{Cos}^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

$$\therefore 2\mathrm{Tan}^{-1}\frac{1}{3}=\mathrm{Cos}^{-1}\left(\frac{1-\left(\frac{1}{3}\right)^2}{1+\left(\frac{1}{3}\right)^2}\right)=\mathrm{Cos}^{-1}\left(\frac{1-\frac{1}{9}}{1+\frac{1}{9}}\right)$$

$$=\mathrm{Cos}^{-1}\left(\frac{\frac{8}{9}}{\frac{10}{9}}\right)=\mathrm{Cos}^{-1}\left(\frac{8}{10}\right)=\mathrm{Cos}^{-1}\left(\frac{4}{5}\right)$$

$$\therefore \text{L.H.S}=\mathrm{Sin}^{-1}\frac{4}{5}+\mathrm{Cos}^{-1}\frac{4}{5}=\frac{\pi}{2}=\text{R.H.S}\qquad (\because \mathrm{Sin}^{-1}x+\mathrm{Cos}^{-1}x=\frac{\pi}{2})$$

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17. In ΔABC, find $b\cos^2\frac{C}{2}+c\cos^2\frac{B}{2}$

A: G.E $= b.\cos^2\frac{C}{2}+c.\cos^2\frac{B}{2}=b.\frac{s(s-c)}{ab}+c.\frac{s(s-b)}{ca}=\frac{s(s-c)}{a}+\frac{s(s-b)}{a}=\frac{s}{a}(s-c+s-b)$

$$=\frac{s}{a}(2s-c-b)=\frac{s}{a}(a+b+c-c-b)=\frac{s}{a}(a)=s$$

SECTION-C

18. If $f:A\to B$, $g:B\to C$ are two bijective functions then prove that $(gof)^{-1}=f^{-1}og^{-1}$

A: **Part -1:** Given that $f:A\to B$, $g:B\to C$ are two bijective functions, then

(i) $gof:A\to C$ is bijection $\Rightarrow (gof)^{-1}:C\to A$ is also a bijection

(ii) $f^{-1}:B\to A$, $g^{-1}:C\to B$ are both bijections $\Rightarrow (f^{-1}og^{-1}): C\to A$ is also a bijection.

So, $(gof)^{-1}$ and $f^{-1}og^{-1}$, both have same domain 'C'

Part–2: Given $f:A\to B$ is bijection, then $f(a)=b\Rightarrow a=f^{-1}(b)$........(1), [Here $a\in A$, $b\in B$]

$g:B\to C$ is bijection, then $g(b)=c\Rightarrow b=g^{-1}(c)$........(2), [Here $b\in B$, $c\in C$]

$gof:A\to C$ is bijection, then $gof(a)=c \Rightarrow a=(gof)^{-1}(c)$.........(3)

Now, $(f^{-1}og^{-1})(c)=f^{-1}[g^{-1}(c)]=f^{-1}(b)=a$ ....(4), [From (1) & (2)]

$\therefore (gof)^{-1}(c)=(f^{-1}og^{-1})(c)$, $\forall c\in C$, [from (3) & (4)]

Hence, we proved that $(gof)^{-1}=f^{-1}og^{-1}$

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19. Prove that $a+ar+ar^2+....+n$ terms $=\dfrac{a(r^n-1)}{r-1}, r\neq 1$

A : The n^{th} term of the Geometric series $a, ar, ar^2...$ is ar^{n-1}. $\therefore T_n=ar^{n-1}$

Let $S(n): a+ar+ar^2+....+ar^{n-1}=\dfrac{a(r^n-1)}{r-1}$

Step 1: L.H.S of S(1)=a and R.H.S of S(1) $=\dfrac{a(r^1-1)}{r-1}=a$

L.H.S of S(1)= R.H.S of S(1) $\Rightarrow$ S(1) is true

Step 2: Assume that S(k) is true for $k\in N$

$$S(k): a+ar+ar^2+...+ar^{k-1}=\frac{a(r^k-1)}{r-1} \quad ....(1)$$

Step 3: We show that S(k+1) is true

On adding $ar^{(k+1)-1}=ar^k$ to both sides of (1), we get

$$\text{L.H.S of } S(k+1)=(a+ar+ar^2+....+ar^{k-1})+ar^k=\frac{a(r^k-1)}{r-1}+ar^k, \text{ [From (1)]}$$

$$=\frac{a(r^K-1)+(r-1)ar^K}{r-1}=\frac{ar^K-a+rar^K-ar^K}{r-1}=\frac{r.ar^K-a}{r-1}$$

$$=\frac{a.r^{k+1}-a}{r-1}=\frac{a(r^{k+1}-1)}{r-1}=\text{R.H.S of } S(k+1)$$

$\therefore$ L.H.S of S(k+1)=R.H.S of S(k+1) $\Rightarrow$ S(k+1) is true whenever S(k) is true

Hence, by the principle of Mathematical induction, the given statement is true $\forall\, n\in N$

20. If $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$ then show that abc= –1.

A: Given that $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} = 0$

$$\Rightarrow -\begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+abc)\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$\Rightarrow$ 1+abc=0 $\Rightarrow$ abc = –1

21. By using Cramer's rule solve x+y+z=1, 2x+2y+3z=6, x+4y+9z=3

A: Given equations in the matrix equation form: AX = D, where

$$A=\begin{bmatrix}1&1&1\\2&2&3\\1&4&9\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}, D=\begin{bmatrix}1\\6\\3\end{bmatrix}$$

$$\text{Now, } \Delta = \det A = \begin{vmatrix}1&1&1\\2&2&3\\1&4&9\end{vmatrix}$$

$$=1(18-12)-1(18-3)+1(8-2)$$

$$=1(6)-1(15)+1(6)=6-15+6=-3$$

$$\Delta_1=\begin{vmatrix}1&1&1\\6&2&3\\3&4&9\end{vmatrix}$$

$$=1(18-12)-1(54-9)+1(24-6)$$

$$=1(6)-1(45)+1(18)=6-45+18=-21$$

$$\Delta_2=\begin{vmatrix}1&1&1\\2&6&3\\1&3&9\end{vmatrix}$$

$$=1(54-9)-1(18-3)+1(6-6)$$

$$=1(45)-1(15)+1(0)=45-15+0=30$$

$$\Delta_3=\begin{vmatrix}1&1&1\\2&2&6\\1&4&3\end{vmatrix}$$

$$=1(6-24)-1(6-6)+1(8-2)$$

$$=1(-18)-1(0)+1(6)=-18-0+6=-12$$

$\therefore$ By Cramer's rule, $x=\frac{\Delta_1}{\Delta}=\frac{\cancel{-21}}{\cancel{-3}}=7;\ y=\frac{\Delta_2}{\Delta}=\frac{\cancel{30}}{\cancel{-3}}=10\,;z=\frac{\Delta_3}{\Delta}=\frac{\cancel{-12}}{\cancel{-3}}=4$

$\therefore$ The solution is x=7, y= –10, z=4

22. Find the volume of the tetrahedron, whose vertices are (1,2,1),(3,2,5), (2,–1,0), (–1,0,1).

A: We take $\overline{OA}=\overline{i}+2\overline{j}+\overline{k}$, $\overline{OB}=3\overline{i}+2\overline{j}+5\overline{k}$, $\overline{OC}=2\overline{i}-\overline{j}$, $\overline{OD}=-\overline{i}+\overline{k}$ where 'O' is origin.

Then, $\overline{AB}=\overline{OB}-\overline{OA}=(3\overline{i}+2\overline{j}+5\overline{k})-(\overline{i}+2\overline{j}+\overline{k})=2\overline{i}+4\overline{k}$

$\overline{AC}=\overline{OC}-\overline{OA}=(2\overline{i}-\overline{j})-(\overline{i}+2\overline{j}+\overline{k})=\overline{i}-3\overline{j}-\overline{k}$

$\overline{AD}=\overline{OD}-\overline{OA}=(-\overline{i}+\overline{k})-(\overline{i}+2\overline{j}+\overline{k})=-2\overline{i}-2\overline{j}$

Now, $[\overline{AB}\ \overline{AC}\ \overline{AD}]=\begin{vmatrix}2 & 0 & 4\\ 1 & -3 & -1\\ -2 & -2 & 0\end{vmatrix}$ =[2(0–2)+4(–2–6)]=[– 4 –32]= –36

$\therefore$ Volume of the tetrahedron $V=\frac{1}{6}[\overline{AB}\ \overline{AC}\ \overline{AD}]=\frac{1}{6}|-36|=6$ cubic unit

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23. If A,B,C are angles in a triangle then prove that $\mathbf{\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}$

A: Given A,B,C are angles of a triangle, then A+B+C=180° $\Rightarrow \frac{A+B+C}{2}=90^\circ$

L.H.S = (sinA+sinB)+sinC

$=2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)+2\sin\frac{C}{2}\cos\frac{C}{2}$ $\left[\because \sin\theta=2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right]$

$=2\sin\left(90^\circ-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right)+2\sin\frac{C}{2}\cos\frac{C}{2}$

$=2\cos\frac{C}{2}\cos\left(\frac{A-B}{2}\right)+2\sin\frac{C}{2}\cos\frac{C}{2}$

$=2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)+\sin\frac{C}{2}\right]$

$=2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)+\sin\left(90^\circ-\frac{A+B}{2}\right)\right]$

$=2\cos\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)+\cos\left(\frac{A+B}{2}\right)\right]$ $[\because \sin(90^\circ-\theta)=\cos\theta]$

$=2\cos\frac{C}{2}\left(2\cos\frac{A}{2}\cos\frac{B}{2}\right)$ $[\because \cos(A+B)+\cos(A-B)=2\cos A\cos B]$

$=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=$ R.H.S

24. If $r:R:r_1$ =2:5:12, then prove that the triangle is right angled at A.

A: Given that $r:R:r_1 = 2:5:12$ then $r=2k$, $R=5k$ and $r_1=12k$ for some proportionality constant k.

Now $r_1 - r = 12k - 2k = 10k = 2(5k) = 2R$

$$\therefore r_1 - r = 2R \Rightarrow 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} - 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2} = 2R$$

$$\Rightarrow 4R\sin\frac{A}{2}\left[\cos\frac{B}{2}\cos\frac{C}{2} - \sin\frac{B}{2}\sin\frac{C}{2}\right] = 2R \Rightarrow 2\sin\frac{A}{2}\cos\left(\frac{B+C}{2}\right) = 1$$

$$\Rightarrow \sin^2\frac{A}{2} = \frac{1}{2} \qquad \Rightarrow \sin\frac{A}{2} = \frac{1}{\sqrt{2}} = \sin 45^\circ$$

$$\Rightarrow \frac{A}{2} = 45^\circ \Rightarrow A = 90^\circ \left[\because \cos\left(\frac{B+C}{2}\right) = \sin\frac{A}{2}\right]$$

Hence the triangle is right angled at A.