Previous IPE

SOLVED PAPERS

MARCH -2019 (TS)

PREVIOUS PAPERS

IPE: MARCH-2019(TS)

Time : 3 Hours **JR.PHYSICS** **Max.Marks : 60**

SECTION-A

I. Answer ALL questions : **$10 \times 2 = 20$**

1. What is the contribution of S. Chandrasekhar to physics ?
2. Define unified atomic mass unit and write its value in kg.
3. What is the acceleration of the projectile at the top of its trajectory ?
4. What happens to coefficient of friction if the weight of the body is doubled ?
5. What is Magnus effect ?
6. What is meant by hydrostatic paradox ?
7. If the maximum intensity of radiation for a black body is found at 1.45 μm. What is the temperature of radiating body?(Wein's constant = 3.9×10^{-11} mk)
8. Why do liquids have no linear and areal expansions?
9. When does real gas behave like an ideal gas ?
10. The absolute temperature of gas is increased by 3 times. What will the increase in rms velocity of the gas molecule ?

SECTION-B

II. Answer any SIX of the following Questions. **$6 \times 4 = 24$**

11. Derive the equation $S = ut + \frac{1}{2}at^2$ from v – t graph.
12. O is the point on the ground chosen as origin. A body first suffers a displacement of $10\sqrt{2}$ m North - East, next 10 m North and finally $10\sqrt{2}$ m North - West. How far is it from the origin?
13. Explain the various methods to minimise the friction.
14. What is the moment of inertia of a rod of mass M and length L about an axis perpendicular to it and passing through one end?
15. Define vector product of two vectors and write any two properties of it.
16. Define escape velocity and derive an expression for it.
17. Define strain energy and derive an expression for it.
18. Pendulum clocks generally go fast in winter and slow in summer. Why ?

SECTION-C

III. Answer any TWO of the following Questions. **$2 \times 8 = 16$**

19. State law of conservation of energy and verify in the case of freely falling body. Calculate the power of a pump required to lift 600 kg of water per minute from a well of 25 m deep.
20. Define simple harmonic motion. Show that the motion of (point) projection of a particle performing uniform circular motion on any diameter is simple harmonic. A mass of 2 kg is attached to a spring of force constant 200 Nm^{-1}. Find its time period.
21. Explain reversible and irreversible process. Describe the working of Carnot's engine. Obtain the expression for the efficiency.

IPE TS MARCH-2019
ANSWERS

SECTION-A

1. What is the contribution of S. Chandra Sekhar to physics ?

A: Chandra Sekhar limit, structure and evolution of stars, motion of Galaxy.

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2. Define unified atomic mass unit and write its value in kg.

A: 1 unified atomic unit (u) = 1.67×10^{-27} kg

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3. What is the acceleration of a projectile at the top of its projectory?

A: 1) At the top of its projectory, acceleration is $9.8 ms^{-2}$.

2) The direction of acceleration is vertically downwards.

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4. What happens to the coefficient of friction if weight of the body is doubled.

A: Coefficient of friction is independent of weight of the body. So it remains constant.

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5. What is magnus effect?

A: 1) The 'dynamic lift' arised due to spinning of a moving ball is called Magnus effect.

2) When a ball is 'moving forward with spinning' then the velocity of air above the ball is larger and below the ball is smaller. This difference in the velocities of air results in the 'pressure difference' between the upper and and lower surfaces. This causes a spining in its track.

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6. What is hydrostatic paradox? [TS 19]

A: **Hydrostatic paradox:** It states that the liquid pressure is same at all points at the same horizontal level (same depth) and is independent of the shape of the container (or) base area.

7. If the maximum intensity of radiation for a black body is found at 2.65 μm. Find the temperature of the radiating body. **[TS 19]**

A: Given $\lambda_m = 2.65\ \mu m = 2.65 \times 10^{-6}$ m

From Wein's displacement law $\lambda_m T = b$

Where b is Wein's constant, $b = 2.9 \times 10^{-3}$ mK,

$$\therefore T = \frac{b}{\lambda_m} = \frac{2.9\times10^{-3}}{2.65\times10^{-6}} = 1.094 \times 10^3\ K = 1094\ K$$

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8. Why do liquids have no linear and areal expansions? **[TS 19][Imp.Q]**

A: Liquids have no shape of their own. They always take the shape of the vessel. Hence they do not have coefficients of Linear and Areal expansion.

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9. When does a real gas behave like an ideal gas?

A: At 'low pressures and high temperatures', a real gas behaves like an ideal gas.

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10. The absolute temperature of a gas is increased 3 times. What will be the increase in rms velocity of the gas molecule?

A: 1) Let $T_2 = 3T_1$

2) rms velocity $\Rightarrow C \propto \sqrt{T} \Rightarrow \frac{C_1}{C_2} = \sqrt{\frac{T_1}{T_2}}$ of the gas molecule $= C = \sqrt{\frac{3RT}{M}}$

$$\therefore \frac{C_1}{C_2} = \sqrt{\frac{\cancel{T_1}}{3\cancel{T_1}}} \Rightarrow C_2 = \sqrt{3}C_1$$

3) The r.m.s velocity of the gas molecule becomes $\sqrt{3}$ times of initial rms velocity.

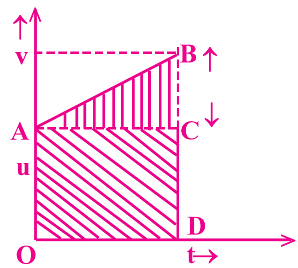
4) Increase in r.m.s velocity of the gas molecule $= C_2 - C_1 = \sqrt{3}C_1 - C_1 = 1.732C_1 - C_1 = 0.732C_1$

5) Percentage increase in rms velocity $= \frac{C_2 - C_1}{C_1} \times 100 = 73.2\%$

SECTION-B

11. Derive the equation $S = ut + \frac{1}{2}at^2$ from v – t graph. **[TS 19]**

A: A graph is drawn between v & t for uniformly accelerated motion. The area under v-t curve gives the displacement .



So Area between the instants 0 to t

= Area of triangle ABC + Area of rectangle OACD

$$S = ut + \frac{1}{2}(v-u)t$$

$$S = ut + \frac{1}{2}(at)t \quad (\because v-u = at)$$

$$\therefore\ S = ut + \frac{1}{2}at^2$$

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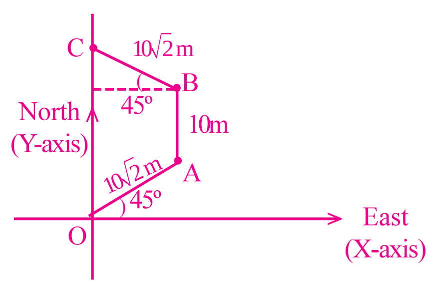
12. O is a point on the ground chosen as origin. A body first suffers a displacement of $10\sqrt{2}$m North-East, next 10m North and finally $10\sqrt{2}$m in North-West. How far it is from the origin? **[TS 19]**



Sol: $\overline{OA} = 10\sqrt{2}\cos 45^o\,\bar{i} + 10\sqrt{2}\sin 45^o\,\bar{j}$

$$= 10\sqrt{2} \times \frac{1}{\sqrt{2}}\bar{i} + 10\sqrt{2} \times \frac{1}{\sqrt{2}}\bar{j} = 10\bar{i} + 10\bar{j}$$

$$\overline{AB} = 10\bar{j}$$

$$\overline{BC} = -10\sqrt{2}\cos 45^o\,\bar{i} + 10\sqrt{2}\sin 45^o\,\bar{j}$$

$$= -10\sqrt{2} \times \frac{1}{\sqrt{2}}\bar{i} + 10\sqrt{2} \times \frac{1}{\sqrt{2}}\bar{j} = -10\bar{i} + 10\bar{j}$$

$\therefore$ Resultant displacement, $\overline{OC} = \overline{OA} + \overline{AB} + \overline{BC}$

$$\Rightarrow \overline{OC} = (10\bar{i} + 10\bar{j}) + (10\bar{j}) + (-10\bar{i} + 10\bar{j}) = 30\bar{j}$$

$\therefore$ The body is at a distance of 30m from the origin in north direction.

13. Explain the various methods to minimise the friction.

A: **1) Polishing:** It reduces the frictional force of the polished surfaces.

2) Lubricants: They reduce the friction by forming thin layers between the surfaces in contact.

3) Ball bearings: They reduce friction in the wheels of motor vehicles while revolving.

4) Stream lining: It reduces friction due to air when Aeroplanes and Cars are streamlined in their front surfaces.

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14. What is the moment of inertia of a rod of mass M, length l about an axis perpendicular to it and passing through one end? [TS 19]

Sol: For the rod of mass M and length l, the M.I of the rod about an axis passing through its centre of mass and perpendicular its plane is $I = Ml^2/12$.

Using the parallel axes theorem, $I' = I + Ma^2$.

With $a = l/2$ we get, $I' = M\frac{l^2}{12} + M\left(\frac{l}{2}\right)^2 = \frac{Ml^2}{3}$

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15. Define vector product. Explain the properties of vector product with 2 examples.

A: **1) Vector Product:** The vector product of two vectors $\bar{a}, \bar{b}$ with angle θ between them, is

$$\bar{a} \times \bar{b} = |\bar{a}||\bar{b}| \sin\theta\, \hat{n}$$

Here, $\hat{n}$ is the unit vector normal to the plane of $\bar{a}, \bar{b}$.

2) Properties of Vector Product :

(i)Commutative law is not satisfied: $\bar{a} \times \bar{b} \neq \bar{b} \times \bar{a}$

(ii)Distributive law is satisifed : $\bar{a} \times (\bar{b} + \bar{c}) = (\bar{a} \times \bar{b}) + (\bar{a} \times \bar{c})$

(iii)Vector product of two parallel vectors is null vector.

Ex: $\bar{i} \times \bar{i} = \bar{j} \times \bar{j} = \bar{k} \times \bar{k} = \bar{o}$

(iv)Vector product of two perpendicular unit vectors is unit normal vector.

Ex: $\bar{i} \times \bar{j} = \bar{k}$, $\bar{j} \times \bar{k} = \bar{i}$, $\bar{k} \times \bar{i} = \bar{j}$

3)Examples : Torque $\bar{\tau} = \bar{r} \times \bar{F}$; Velocity $\bar{v} = \bar{\omega} \times \bar{r}$

16. Define escape velocity and derive an expression for it.

A: **1) Escape Velocity(V_e) :** The **minimum velocity** required for an body to escape from the gravitational influence of a planet is known as "escape velocity".

2) Derivation: Consider an body of mass 'm' at rest on the surface of a planet of mass M and radius R.

3) The gravitational potential on the surface of a planet = $\frac{-GM}{R}$

The gravitational P.E of the system=Gravitational potential×mass of the body=$\frac{-GMm}{R}$ ....(i)

4) When a body of mass m is projected with a velocity V_e then its K.E=$\frac{1}{2}mV_e^2$ ....(ii)

After crossing the Gravitational limits, the total energy becomes zero.

5) Applying the Law of conservation of energy, from (i) & (ii) we have

$$\frac{1}{2}mV_e^2 = -\left(\frac{-GMm}{R}\right) \Rightarrow \frac{1}{2}mV_e^2 = \frac{GMm}{R} \Rightarrow V_e^2 = \frac{2GM}{R}$$

6) $V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2g\cancel{R}^2}{\cancel{R}}}$ $[\because GM = gR^2]$

$\therefore V_e = \sqrt{2gR}$. (Its value for the earth is V_e=11.2 km/s)

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17. Define strain energy and derive the equation for the same.

A: **1) Strain energy:** The energy stored in a body due to its deformation is called strain energy.

2) Derivation: Consider a thin uniform wire of length L and area of cross section A fixed at one end. If an external force 'F' is applied on the wire then the elongation in it is 'e'.

3) If the force on the wire is increased from 0 to F then

'average force' on the wire = $\frac{0+F}{2} = \frac{F}{2}$

4) Work done W = Average force × elongation = $\frac{1}{2}Fe$ $[\because W = Fs]$

5) This work done is stored as 'strain energy' in the wire.

$$\therefore W = \frac{1}{2}Fe = \frac{1}{2} x \left(\frac{F}{A}\right) x \left(\frac{e}{L}\right) x\,(AL) = \frac{1}{2}\ \text{Stress} \times \text{Strain} \times \text{Volume of wire}$$

$\therefore$ Strain energy per unit volume = $\frac{1}{2}$ × Stress × Strain

18. Pendulum clocks generally go fast in winter and slow in summer. Why?

Sol: The time period of pendulum clock is given by $T = 2\pi\sqrt{\frac{l}{g}}$.

At a given place, $T \propto \sqrt{l}$

The pendulum of a clock expands in summer, so its time period increases.

Hence, it makes less number of oscillations than required per day. Hence it will lose time or clock goes slow.

The pendulum of a clock contracts in winter, its length decreases so its time period decreases.

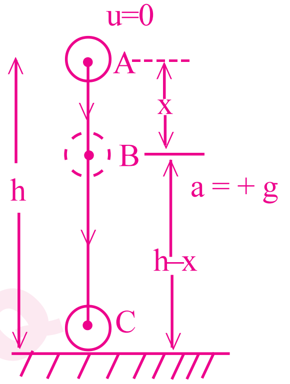
Hence, it makes more number of oscillations than required per day. Hence it will gain time or clock goes fast.

SECTION-C

19. State and prove law of conservation of energy in case of freely falling body.

A: **1) Law of conservation of energy:** Energy can neither be created nor be destroyed.

Total energy remains constant in a given system.



2) Proof: Consider a freely falling body of mass 'm' released from a point 'A' .

The acceleration of the body is a= +g

3) At Point A :

Let the height of the body from the ground is 'h'.

$\therefore$ Potential Energy P.E = mgh...............(i)

At A, velocity $v_A = u = 0$

$\therefore$ Kinetic Energy K.E= $\frac{1}{2}mv_A^2 = \frac{1}{2}m(0)^2 = 0$...............(ii)

From (i)& (ii) Total Energy T.E= P.E + K.E = mgh + 0 = mgh .....(A)

4) At Point B :

Let the body travels a displacement x and reaches the point B.

So height of the body from the ground is (h–x)

$\therefore$ P.E = mg(h–x) = mgh – mgx...............(i)

At B, displacement s=x, u=0, v= v_B, a= +g

We know $v^2-u^2=2as \Rightarrow v_B^2 - 0^2 = 2gx \Rightarrow v_B^2 = 2gx$

$\therefore$ K.E $= \frac{1}{2}mv_B^2 = \frac{1}{\cancel{2}}m(\cancel{2}gx) = mgx$...............(ii)

From (i) & (ii) T.E = P.E + K.E $= (mgh - \cancel{mgx}) + \cancel{mgx} = mgh$ ....(B)

5) At Point C:

Let the body hits the ground at C.

So height of the body h = 0

$\therefore$ P.E = mgh= mg (0) = 0 ....(i)

At C, displacement s=h, u=0, v=v_C, a= +g

We know $v^2-u^2=2as \Rightarrow v_C^2 - 0^2 = 2gh \Rightarrow v_C^2 = 2gh$

$\therefore$ K.E $= \frac{1}{2}mv_C^2 = \frac{1}{\cancel{2}}m(\cancel{2}gh) = mgh$ ....(ii)

From (i)& (ii) T.E = P.E + K.E = 0 + mgh = mgh ....(C)

6) From (A) ,(B), (C) it is clear that the total energy 'T.E' is always constant.

Hence, the law of conservation of energy is proved.

b) A pump is required to lift 600kg of water per minute from a well 25m deep and to eject it with a speed of $50ms^{-1}$. Calculate the power required to perform the above task?

Sol: 1) Given Mass of water lifted (m) = 600kg, depth of well (h) = 25m

2) Work done to lift water (W_1) = mgh = $600 \times 9.8 \times 25 = 147000$ J

3) Speed of water (v) = $50ms^{-1}$; Mass of water (m) = 600kg

4) Work done to give K.E to water, $w_2 = \frac{1}{2}mv^2 = \frac{1}{\not{2}} \times \not{600} \times (50)^2 = 300(2500) = 750000$ J

5) Total work done, $w = w_1 + w_2 = 147000 + 750000 = 897000J$

Time taken (t) = 1 minute = 60 s.

6) $\therefore$ Power required $= \frac{\text{work done}}{\text{time}} = \frac{897000}{60} = 14950\text{ w} = 14.95\text{ kW}$

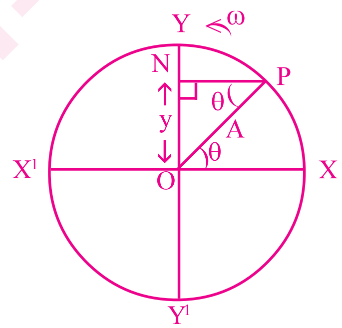
20. Define simple harmonic motion. Show that the motion of (point) projection of a particle performing uniform circular motion, on any diameter, is simple harmonic.

A: **1) Simple Harmonic Motion (SHM):** The 'to and fro motion' of a particle along a straight line, about a fixed point is said to be **Simple Harmonic motion**, when the acceleration is always proportional to its displacement, but in opposite direction.

2) Proof: Suppose a particle P is moving along the circumference of a circle of radius A.

Let N be the projection of P on the diameter Y-axis.

If P completes one revolution then its projection point N makes one oscillation on the diameter.



3) If θ is the angular displacement of P at time t and ω is uniform angular velocity then $\theta = \omega t$

4) From ΔOPN, $\sin\theta = \dfrac{ON}{OP} = \dfrac{y}{A} \Rightarrow y = A\sin\theta$

∴ Displacement $y = A \sin(\omega t)$.......... (i)

5) Velocity is the 'rate of change of displacement'.

$$\therefore \text{Velocity } v = \frac{d}{dt}(y) = \frac{d}{dt}A\sin(\omega t) = A\frac{d}{dt}\sin(\omega t)$$

$$= A\omega\cos(\omega t)\left[\because \frac{d}{dx}\sin(kx) = k\cos(kx)\right]$$

6) Acceleration is the 'rate of change of velocity'.

$$\therefore \text{Acceleration } a = \frac{d}{dt}(v) = \frac{d}{dt}A\omega\cos(\omega t) = A\omega\frac{d}{dt}[\cos(\omega t)]$$

$$= -A\omega(\omega)[\sin(\omega t)] = -\omega^2[A\sin(\omega t)] = -\omega^2 y, \text{ [from (i)]}\left[\because \frac{d}{dx}\cos(kx) = -k\sin(kx)\right]$$

7) $\therefore a \propto -y \quad (\because \omega \text{ is a constant})$

8) Hence the motion of projection N on any diameter is S.H.M.

b) A mass of 2kg attached to a spring of force constant 260 Nm^{-1} makes 100 oscillations. What is the time taken?

Sol: Given mass attached to the spring, m = 2kg

Force constant of spring, K = 260 Nm^{-1}

No. of oscillations completed = 100

Time taken to complete 100 oscillations (t) = ?

We know that, time period of a spring pendulum, $T = 2\pi\sqrt{\frac{m}{k}}$

$$\Rightarrow T = 2\pi\sqrt{\frac{2}{260}} = 2\times3.14\times0.088 = 0.55 sec$$

∴ Time taken to complete 100 oscillations t = 100 × 0.55 = 55 sec

21. Explain reversible and irreversible processes. Describe the working of Carnot engine. Obtain an expression for the efficiency.

A: **1) Reversible process:** A process that can be 'retraced back' in the opposite direction is called a reversible process.

Ex: Fusion of ice and vaporisation of water.

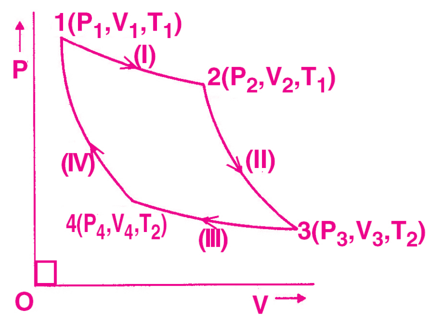
2) Irreversible process: A process that can not be retraced back in the opposite direction is called an irreversible process.

Ex: Work done against friction.

3) Carnot engine: A reversible heat engine operating between two temperatures is called Carnot engine.

Working of Carnot engine: The carnot engine undergoes a cycle of four processes called carnot cycle. It consists of two isothermal processes connected by two adiabatic processes. Ideal gas acts as the working substance in the carnot engine.

4) The 4 steps of Carnot cycle:



a) Step I: Isothermal Expansion(IE) of the gas from $1(P_1,V_1,T_1)$ to $2(P_2,V_2,T_1)$.

Work done by the gas on the environment = Heat(Q_1) absorbed by the gas, from the reservoir, at constant temperature (T_1).

$$W_1 = Q_1 = nRT_1\log_e\left(\frac{V_2}{V_1}\right) \text{ - - - - -(i)}$$

b) Step II : Adiabatic Expansion(AE) of the gas from $2(P_2,V_2,T_1)$to $3(P_3,V_3,T_2)$.

Work done by the gas in this adiabatic process is $W_2 = \frac{nR}{(\gamma-1)}(T_1 - T_2)$ - - - - (ii)

c) Step III : Isothermal Compression(IC) of the gas from $3(P_3,V_3,T_2)$ to $4(P_4,V_4,T_2)$.

Work done on the gas by the environment = Heat(Q_2) released by the gas to the reservoir, at constant temperature(T_2).

$$W_3 = Q_2 = nRT_2\log_e\left(\frac{V_3}{V_4}\right) \text{ - - - - (iii)}$$

d) Step IV: Adiabatic Compression (AC) of the gas from 4(P_4,V_4,T_2) to 1(P_1,V_1,T_1).

Work done on the gas in this adiabatic process is $W_4 = \frac{nR}{(\gamma-1)}(T_1 - T_2)$ - - - - (iv)

5) $\therefore$ Total work done by the gas in one complete cycle is

$$W = W_1 + W_2 - W_3 - W_4$$

$$= nRT_1\log_e\left(\frac{V_2}{V_1}\right) + \frac{nR}{(\gamma-1)}(T_1-T_2) - nRT_2\log_e\left(\frac{V_3}{V_4}\right) - \frac{nR}{(\gamma-1)}(T_1-T_2)$$

6) $\therefore W = nRT_1\log_e\left(\frac{V_2}{V_1}\right) - nRT_2\log_e\left(\frac{V_3}{V_4}\right)$

7) The efficiency of the Carnot engine is

$$\eta = \frac{W}{Q_1} = \frac{nRT_1\log_e\left(\frac{V_2}{V_1}\right) - nRT_2\log_e\left(\frac{V_3}{V_4}\right)}{nRT_1\log_e\left(\frac{V_2}{V_1}\right)}$$

$$= \frac{T_1\log_e\left(\frac{V_2}{V_1}\right) - T_2\log_e\left(\frac{V_3}{V_4}\right)}{T_1\log_e\left(\frac{V_2}{V_1}\right)}$$

$$= \frac{T_1\log_e\left(\frac{V_2}{V_1}\right)}{T_1\log_e\left(\frac{V_2}{V_1}\right)} - \frac{T_2\log_e\left(\frac{V_3}{V_4}\right)}{T_1\log_e\left(\frac{V_2}{V_1}\right)}$$

$$= 1 - \left(\frac{T_2}{T_1}\right)\frac{\log_e\left(\frac{V_3}{V_4}\right)}{\log_e\left(\frac{V_2}{V_1}\right)}$$

$\because$ **Step (b) & (d) are adiabatic processes.**

$\therefore TV^{\gamma-1}$ = constant

$\Rightarrow T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$ and $T_1V_1^{\gamma-1} = T_2V_4^{\gamma-1}$

Dividing the above two equations, we get

$$\frac{T_1V_2^{\gamma-1}}{T_1V_1^{\gamma-1}} = \frac{T_2V_3^{\gamma-1}}{T_2V_4^{\gamma-1}} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

8) $\therefore \eta = 1 - \frac{T_2}{T_1}$