Previous IPE

SOLVED PAPERS

MARCH -2019 (TS)

PREVIOUS PAPERS

IPE: MARCH-2019(TS)

Time : 3 Hours **MATHS-2B** Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: $10 \times 2 = 20$

1. Write the parametric equations of the circle $2x^2 + 2y^2 = 7$
2. Find the value of k if the points (1, 3), (2, k) are conjugate w.r.to the circle $x^2 + y^2 = 35$.
3. Find the equation of the radical axis of the circles $x^2+y^2+4x+6y+7=0$, $4(x^2 + y^2) + 8x + 12y - 9 = 0$
4. Find the equation of the normal to the parabola $y^2 = 4x$ which is parallel to $y - 2x + 5 = 0$
5. If the eccentricity of a hyperbola is 5/4, then find the eccentricity of its conjugate hyperbola.
6. Evaluate $\int \frac{1+\cos^2 x}{1-\cos 2x} dx$
7. Evaluate $\int \frac{1}{x\log x[\log(\log x)]} dx$
8. Evaluate $\int_0^a \left(\sqrt{a}-\sqrt{x}\right)^2 dx$
9. Evaluate $\int_0^{\pi/2} \cos^{11} x\, dx$
10. Find the general solution of : $\frac{dy}{dx} = \frac{2y}{x}$

SECTION-B

II. Answer any FIVE of the following SAQs: $5 \times 4 = 20$

11. Find the equation of the circle which cut orthogonally the circle $x^2 + y^2 - 4x + 2y - 7 = 0$ and having the centre at (2, 3).
12. The line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ intersect at A and B. If $AB = 2\lambda$, then show that $c^2 = (1 + m^2)(a^2 - \lambda^2)$
13. Find the equation of the ellipse, if focus = (1,–1), e = 2/3 and directrix is $x + y + 2 = 0$
14. Find the equations of the tangent and normal to the ellipse $2x^2 + 3y^2 = 11$ at the point whose ordinate is 1.
15. Find the centre, eccentricity, foci, length of latus rectum and equations of the directrices of the Hyperbola $x^2 - 4y^2 = 4$
16. Evaluate $\int_0^{2\pi} \sin^4 x \cos^6 x\, dx$
17. Solve $\cos x.\frac{dy}{dx} + y\sin x = \sec^2 x$

SECTION-C

III. Answer any FIVE of the following Long Answer Questions: $5 \times 7 = 35$

18. Show that the points (9,1), (7,9), (–2,12), (6,10) are concyclic and find the equation of the circle on which they lie.
19. Show that, four common tangents can be drawn for the circles given by $x^2 + y^2 - 14x + 6y + 38 = 0$ and $x^2 + y^2 + 30x - 2y + 1 = 0$
20. From an external point P, tangents are drawn to the parabola $y^2 = 4ax$ and these tangents make angles θ_1, θ_2 with its axis, such that $\cot\theta_1 + \cot\theta_2$ is a constant d. Then show that all such P lie on a horizontal line.
21. Evaluate $\int e^{ax} \sin(bx + c)dx; (a, b, c \in R; b \neq 0)$ on R
22. Obtain reduction formula for $I_n = \int \cot^n x\, dx$, n being a positive integer, $n \geq 2$ and deduce the value of $\int \cot^4 x\, dx$
23. Evaluate $\int_0^{\pi} x\sin^7 x.\cos^6 x\, dx$
24. Solve the differential equation : $\frac{dy}{dx} = \frac{2y + x + 1}{2x + 4y + 3}$

IPE TS MARCH-2019
SOLUTIONS

SECTION-A

1. Obtain the parametric equation of $2x^2 + 2y^2 = 7$

Sol: Reducing the given equation into standard form, we get $x^2 + y^2 = \frac{7}{2} = \left(\sqrt{\frac{7}{2}}\right)^2$

$\therefore$ Centre (0, 0) radius $r = \sqrt{\frac{7}{2}}$

The parametric equations of $x^2 + y^2 = r^2$ are given by $x = r\cos\theta$, $y = r\sin\theta$.

$\Rightarrow x = \sqrt{\frac{7}{2}}\cos\theta, y = \sqrt{\frac{7}{2}}\sin\theta,\ 0 \le \theta \le 2\pi$

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2. Find the value of k if the points (1,3), (2,k) are conjugate w.r.to the circle $x^2+y^2=35$.

Sol: The points (1,3), (2,k) are conjugate to w.r.t the circle $S= x^2+y^2-35=0 \Rightarrow S_{12} =0$

$\Rightarrow x_1x_2+y_1y_2-35=0$

$\Rightarrow (1)(2)+(3)(k) -35=0$

$\Rightarrow 3k = 33 \Rightarrow k = 11$

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3. Find the equation of the radical axis of $x^2+y^2+4x+6y-7=0$, $4(x^2+y^2)+8x+12y-9=0$

Sol: Let $S \equiv x^2 + y^2 + 4x + 6y - 7 = 0$ and $S' \equiv x^2 + y^2 + 2x + 3y - \frac{9}{4} = 0$

Radical axis is $S - S' = 0$

$\Rightarrow (4-2)x + (6-3)y + \left(-7 + \frac{9}{4}\right) = 0$

$\Rightarrow 2x + 3y - \frac{19}{4} = 0 \Rightarrow 8x + 12y - 19 = 0$

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4. Find the equation of the normal to the parabola $y^2 = 4x$ which is parallel to $y - 2x + 5 = 0$

Sol: Given equation of parabola is $y^2 = 4x$ ......(1)

Comparing with $y^2 = 4ax$, we get $4a = 4 \Rightarrow a = 1$

Given line is $y - 2x + 5 = 0 \Rightarrow y = 2x - 5$

$\therefore$ Slope m = 2.

The equation of normal to the parabola which is parallel to the given line with slope m = 2 is

$y = mx - 2am - am^3 \Rightarrow y = 2x - 2(1)(2) - 1(2)^3$

$\Rightarrow y = 2x - 12 \Rightarrow 2x - y - 12 = 0$

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5. If the eccentricity of a hyperbola is 5/4, then find eccentricity of its conjugate hyperbola.

Sol: Let e= 5/4 and the eccentricity of the conjugate hyperbola be e_1

Then, $\frac{1}{e^2} + \frac{1}{e_1^2} = 1 \Rightarrow \frac{1}{(5/4)^2} + \frac{1}{e_1^2} = 1 \Rightarrow \frac{1}{e_1^2} = 1 - \frac{16}{25} = \frac{9}{25} \Rightarrow e_1^2 = \frac{25}{9} \Rightarrow e_1 = \frac{5}{3}$

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6. Evaluate $\int \frac{1 + \cos^2 x}{1 - \cos 2x} dx$

Sol: $\int \frac{1 + \cos^2 x}{1 - \cos 2x} dx = \int \frac{1 + \cos^2 x}{2\sin^2 x} dx$

$= \frac{1}{2} \int \left(\frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} \right) dx$

$= \frac{1}{2} \int (\operatorname{cosec}^2 x + \cot^2 x) dx$

$= \frac{1}{2} \int (\operatorname{cosec}^2 x + (\operatorname{cosec}^2 x - 1)) dx$

$= \frac{1}{2} \int (2\operatorname{cosec}^2 x - 1) dx$

$= \int \operatorname{cosec}^2 x dx - \frac{1}{2} \int dx = -\cot x - \frac{1}{2} x + c$

7. Evaluate $\int \frac{1}{x\log x[\log(\log x)]}dx$

Sol: Put log(logx)=t $\Rightarrow \frac{1}{\log x}.\frac{1}{x}dx = dt$

$$\therefore I = \int\frac{dt}{t} = \log t + c = \log(\log(\log x)) + c$$

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8. Evaluate $\int_0^a \left(\sqrt{a}-\sqrt{x}\right)^2 dx$

Sol: $I = \int_0^a \left(\sqrt{a} - \sqrt{x}\right)^2 dx = \int_0^a (a + x - 2\sqrt{a}\sqrt{x})\, dx$

$$= \left[ax + \frac{x^2}{2} - 2\sqrt{a}\frac{2}{3}x\sqrt{x}\right]_0^a = a^2 + \frac{a^2}{2} - \frac{4}{3}a^2 = \frac{6a^2 + 3a^2 - 8a^2}{6} = \frac{a^2}{6}$$

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9. Evaluate $\int_0^{\pi/2} \cos^{11}x\, dx$

Sol : When n is odd, $\int_0^{\pi/2} \cos^n x dx = \frac{(n-1)(n-3)....}{n(n-2)....}\frac{2}{3}.1$

$$\therefore \int_0^{\pi/2} \cos^{11} x\, dx = \frac{(10)(8)(6)(4)(2)}{(11)(9)(7)(5)(3)}(1) = \frac{256}{693}$$

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10. Find the general solution of $\frac{dy}{dx} = \frac{2y}{x}$

Sol: Given D.E is $\frac{dy}{dx} = \frac{2y}{x} \Rightarrow \frac{dy}{y} = \frac{2dx}{x} \Rightarrow \int\frac{dy}{y} = 2\int\frac{dx}{x}$

$$\Rightarrow \log y = 2 \log x + \log c \Rightarrow \log y = \log x^2 + \log c \Rightarrow \log y = \log cx^2 \Rightarrow y = cx^2$$

$\therefore$ The general solution is $y = cx^2$

SECTION-B

11. Find the equation of the circle which cut orthogonally the circle $x^2+y^2-4x+2y-7=0$ and having the centre at (2,3).

Sol : The equation of the required circle is taken as $S=x^2+y^2+2gx+2fy+c=0$.....(1)

Given that centre of S =0 is $C(-g, -f) = (2,3) \Rightarrow g = -2, f = -3$

S=0 is orthogonal to $x^2+y^2-4x+2y-7=0$

$\therefore 2gg'+2ff'=c+c' \Rightarrow 2g(-2)+2f(1)= c-7 \Rightarrow -4g+2f = c-7$

Now Substituting $g = -2, f = -3$ in the above equation we get

$\Rightarrow -4(-2)+2(-3) = c-7 \Rightarrow c = 8-6+7 = 9$

Substituting $g = -2, f = -3, c = 9$ in (1) we get the equation of the required circle as

$x^2+y^2+2(-2)x+2(-3)y+9=0 \Rightarrow x^2+y^2-4x-6y+9=0$

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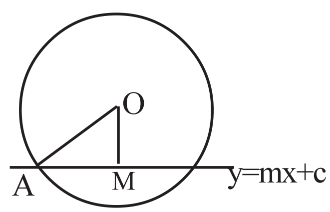
12. The line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ intersect at A and B. If $AB = 2\lambda$, then show that $(a^2 - \lambda^2)(1 + m^2) = c^2$

Sol: Given Equation of the circle $x^2 + y^2 = a^2$, Centre $= O(0,0)$

The line $y = mx + c$ intersects circle at A and B.

OM = perpendicular distance from (0, 0) to $mx - y + c = 0$



$$\Rightarrow OM = \frac{|c|}{\sqrt{1+m^2}}$$

From ΔOAM, $AM^2 + OM^2 = AO^2$

$$\Rightarrow AM^2 = OA^2 - OM^2 = a^2 - \frac{c^2}{1+m^2} = \frac{a^2(1+m^2)-c^2}{1+m^2}$$

$$\text{Length of the chord } AB = 2AM = 2\sqrt{a^2 - \frac{c^2}{1+m^2}}$$

$$\text{Given } AB = 2\lambda, \therefore \cancel{2}\lambda = \cancel{2}\sqrt{a^2 - \frac{c^2}{1+m^2}} \Rightarrow \lambda = \sqrt{\frac{a^2(1+m^2)-c^2}{1+m^2}}$$

$$\Rightarrow \frac{a^2(1+m^2)-c^2}{1+m^2} = \lambda^2 \Rightarrow a^2(1+m^2) - c^2 = \lambda^2(1+m^2) \Rightarrow a^2(1+m^2) - \lambda^2(1+m^2) = c^2$$

$\Rightarrow (a^2 - \lambda^2)(1 + m^2) = c^2$

13. Find the equation of ellipse, if focus=(1,–1), e = 2/3 and directrix is x+y+2=0

Sol: Given that Focus S=(1,–1), e=2/3, directrix is x+y+2=0

Let $P(x_1, y_1)$ be any point on the ellipse $\Rightarrow SP = ePM$

$$\Rightarrow \sqrt{(x_1-1)^2+(y_1+1)^2} = \frac{2}{3}\frac{|x_1+y_1+2|}{\sqrt{2}} \Rightarrow 9[(x_1-1)^2+(y_1+1)^2] = 2(x_1+y_1+2)^2$$

$$\Rightarrow 9[x_1^2-2x_1+1+y_1^2+2y_1+1] = 2[x_1^2+y_1^2+4+2x_1y_1+4y_1+4x_1]$$

$$\Rightarrow 9(x_1^2+y_1^2-2x_1+2y_1+2) = 2(x_1^2+y_1^2+2x_1y_1+4x_1+4y_1+4)$$

$$\Rightarrow 9x_1^2+9y_1^2-18x_1+18y_1+18 = 2x_1^2+2y_1^2+4x_1y_1+8x_1+8y_1+8$$

$$\Rightarrow 7x_1^2-4x_1y_1+7y_1^2-26x_1+10y_1+10=0$$

$\therefore$ Equation of the ellipse is $7x^2-4xy+7y^2-26x+10y+10=0$

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14. Find the equations of tangent and normal to the ellipse $2x^2+3y^2=11$ at the point whose ordinate is 1.

Sol: Let $P(x_1,y_1)$ be a point on the given ellipse, whose ordinate is 1.

$\Rightarrow P(x_1,y_1)$ is a point on the ellipse $S=2x^2+3y^2=11 \Rightarrow 2{x_1}^2+3=11 \Rightarrow x_1=\pm2$

$\therefore$ the coordinates of P are (2,1) are (–2,1)

Hence, equation of tangent at (2,1) is $S_1=0 \Rightarrow 4x+3y=11 \Rightarrow$ slope of the tangent is –4/3

$\Rightarrow$ slope of its normal is 3/4

$\therefore$ Equation of normal at (2,1) is $y-1=3/4\,(x-2) \Rightarrow 4y-4=3x-6 \Rightarrow 3x-4y-2=0$

Also equation of tangent at (–2,1) is $S_1=0 \Rightarrow 4x-3y+11=0 \Rightarrow$ slope of the tangent is 4/3

$\Rightarrow$ slope of its normal is –3/4

Equation of normal at (–2,1) is $y-1=-3/4\,(x+2) \Rightarrow 3x+4y+2=0$

15. Find the centre, eccentricity, foci, length of latus rectum and equations of the directrices of the Hyperbola $x^2-4y^2=4$

Sol: Given hyperbola is $x^2-4y^2=4$

$\Rightarrow \frac{x^2}{4}-\frac{y^2}{1}=1$. Here $a^2=4$, $b^2=1$

(i) Centre C= (0,0)

(ii) Eccentricity $e=\sqrt{\frac{a^2+b^2}{a^2}}=\sqrt{\frac{4+1}{4}}=\sqrt{\frac{5}{4}}=\frac{\sqrt{5}}{2}$

(iii) Foci $=(\pm ae,0)=\left(\pm 2\left(\frac{\sqrt{5}}{2}\right),0\right)=\left(\pm\sqrt{5},0\right)$

(iv) Equation of the directrices is $x=\pm\frac{a}{e}$ $\Rightarrow x=\pm\frac{2}{\frac{\sqrt{5}}{2}}\Rightarrow x=\pm\frac{4}{\sqrt{5}}$

(v) Length of latusrectum $=\frac{2b^2}{a}=\frac{2(1)}{2}=1$

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16. Evaluate $\int_0^{2\pi}\sin^4 x\cos^6 x\,dx$

Sol: Let $f(x)=\sin^4 x\cos^6 x$

Here, $f(2\pi-x) = f(\pi-x)=f(x)$

$$\therefore \int_0^{2\pi}\sin^4 x\cos^6 x\,dx = 2\int_0^{\pi}\sin^4 x\cos^6 x\,dx = 2(2)\int_0^{\pi/2}\sin^4 x\cos^6 x\,dx$$

$$=4\frac{[(3)(1)][(5)(3)(1)]}{(10)(8)(6)(4)(2)}\left(\frac{\pi}{2}\right)=\frac{3\pi}{128}$$

17. Solve $\cos x.\frac{dy}{dx} + y\sin x = \sec^2 x$

Sol: Given D.E is $\cos x.\frac{dy}{dx} + y\sin x = \sec^2 x \Rightarrow \frac{dy}{dx} + y\left(\frac{\sin x}{\cos x}\right) = \frac{\sec^2 x}{\cos x} \Rightarrow \frac{dy}{dx} + y(\tan x) = \sec^3 x$

The given D.E is in the form $\frac{dy}{dx} + P(x)y = Q(x)$. This is a linear D.E in y.

Here $P = \tan x \Rightarrow \int P dx = \int \tan x dx = \log \sec x$ $\therefore I.F = e^{\int P dx} = e^{\log \sec x} = \sec x$

Hence the solution is $y(I.F) = \int (I.F)Q\, dx$

$\Rightarrow y.\sec x = \int (\sec x)(\sec^3 x)dx = \int \sec^4 x dx = \int (\sec^2 x)(\sec^2 x)dx$

$= \int (1+\tan^2 x)\sec^2 x dx = \int \sec^2 x dx + \int \sec^2 x \tan^2 x dx = \tan x + \frac{\tan^3 x}{3} + c \left[\because f(x) = \tan x, f'(x) = \sec^2 x\right]$

SECTION-C

18. Show that the points (9,1), (7,9), (–2,12), (6,10) are concyclic and find the equation of the circle on which they lie.

Sol: Method I:

Let A=(9,1), B=(7,9), C=(–2,12), D=(6,10)

Let $S(x_1,y_1)$ be the centre of the circle $\Rightarrow$ SA=SB=SC

Now, $SA = SB \Rightarrow SA^2 = SB^2 \Rightarrow (x_1-9)^2+(y_1-1)^2=(x_1-7)^2+(y_1-9)^2$

$\Rightarrow (x_1^2-18x_1+\cancel{81})+(y_1^2-2y_1+1)=(x_1^2-14x_1+49)+(y_1^2-18y_1+\cancel{81})$

$\Rightarrow -4x_1+16y_1-48=0 \Rightarrow x_1-4y_1+12=0$ ....(1)

Also, $SB = SC \Rightarrow SB^2 = SC^2 \Rightarrow (x_1-7)^2+(y_1-9)^2=(x_1+2)^2+(y_1-12)^2$

$\Rightarrow (x_1^2-14x_1+49)+(y_1^2-18y_1+81)=(x_1^2+4x_1+4)+(y_1^2-24y_1+144)$

$\Rightarrow -18x_1+6y_1-18=0 \Rightarrow 3x_1-y_1+3=0$ ......(2)

Solving (1) & (2) we get the centre $S(x_1,y_1)$

From (2), $y_1=3x_1+3$

Put in (1) $\Rightarrow x-4(3x_1+3)+12=0 \Rightarrow x-12x_1-12+12=0 \Rightarrow -11x_1=0 \Rightarrow x_1=0$

Put $x_1=0$ in (2) $3x_1-y_1+3=0 \Rightarrow 0-y_1+3=0 \Rightarrow y_1=3$

$\therefore$ Centre $S(x_1,y_1)=(0,3)$

Now radius $r = SA = \sqrt{(9-0)^2+(1-3)^2} = \sqrt{81+4} = \sqrt{85}$

$\therefore$ the equation of the circle with centre (0,3) and radius $\sqrt{85}$ is $(x-a)^2+(y-b)^2=r^2$

$\Rightarrow (x-0)^2+(y-3)^2=(\sqrt{85})^2 \Rightarrow x^2+y^2-6y+9=85 \Rightarrow x^2+y^2-6y-76=0$

Now, substituting D(6,10) in the above equation, we have

$(6)^2+(10)^2+6(10)-76=36+100-60-76=136-136=0$

$\therefore$ D(6,10) lies on the circle

Hence, the given 4 points are concyclic.

19. Show that four common tangents can be drawn for the circles given by $x^2+y^2-14x+6y+33=0$ and $x^2+y^2+30x-2y+1=0$ and find the internal and external centres of similitude.

Sol: For the circle $x^2+y^2-14x+6y+33=0$, centre $C_1=(7,-3)$, radius $r_1=\sqrt{49+9-33}=\sqrt{25}=5$

For the circle $x^2+y^2+30x-2y+1=0$, centre $C_2=(-15,1)$, radius $r_2=\sqrt{225+1-1}=\sqrt{225}=15$

Now, $C_1C_2=\sqrt{(7+15)^2+(-3-1)^2}=10\sqrt{5}$. Also $r_1+r_2=20$

Here, $C_1C_2>r_1+r_2$ $(\because C_1C_2=10\sqrt{5}=\sqrt{500};\ r_1+r_2=20=\sqrt{400})$

$\therefore$ the two circles are such that one lies entirely outside the other

$\Rightarrow$ Four common tangents exist for the given circles.

Now, $r_1:r_2=5:15=1:3$

The internal centre of simultude $I=\left(\frac{(3)(7)+(1)(-15)}{3+1},\frac{(3)(-3)+(1)(1)}{3+1}\right)=\left(\frac{3}{2},-2\right)$

The external centre of simultude $E=\left(\frac{(3)(7)-(1)(-15)}{3-1},\frac{(3)(-3)-(1)(1)}{3-1}\right)=(18,-5)$

20. From an external point P, tangents are drawn to the parabola $y^2 = 4ax$ and these tangents make angles θ_1, θ_2 with its axis, such that $\cot\theta_1 + \cot\theta_2$ is a constant d. Then show that all such P lie on a horizontal line.

Sol: Let the external point $P=(x_1,y_1)$

The equation of the tangent with slope m to the parabola $y^2=4ax$ is $y = mx + \frac{a}{m}$

If this tangent passes through $P(x_1,y_1)$ then $y_1 = mx_1 + \frac{a}{m} \Rightarrow m^2x_1 - my_1 + a = 0$

The above equation is a quadratic in m and its roots be taken as m_1, m_2.

Here, $m_1 = \tan\theta_1$ and $m_2=\tan\theta_2$.

Then $m_1 + m_2 = \frac{y_1}{x_1} \Rightarrow \tan\theta_1 + \tan\theta_2 = \frac{y_1}{x_1}$

$m_1m_2 = \frac{a}{x_1}$

Given locus condition $\cot\theta_1 + \cot\theta_2 = d \Rightarrow \frac{1}{\tan\theta_1} + \frac{1}{\tan\theta_2} = d$

$\Rightarrow \frac{1}{m_1} + \frac{1}{m_2} = d \Rightarrow \frac{m_1 + m_2}{m_1m_2} = d \Rightarrow m_1 + m_2 = dm_1m_2$

$\Rightarrow \frac{y_1}{x_1} = d\frac{a}{x_1} \Rightarrow y_1 = ad$ is in the form $y = k$

$\therefore$ Locus of $P(x_1,y_1)$ is a horizontal line y=ad

21. Evaluate $\int e^{ax}\sin(bx+c)dx$

Proof: As per ILATE rule, we take u=sin(bx+c) and v= e^{ax}.

Then $v_1=\frac{e^{ax}}{a}$ and $u' = b\cos(bx+c)$

From 'Integration by parts Rule', we have $\int uv=uv_1-\int v_1u'$

$$I=\int \sin(bx+c)\,(e^{ax})\,dx$$

$$=\sin(bx+c)\left(\frac{e^{ax}}{a}\right)-\int\left(\frac{e^{ax}}{a}\right)b\cos(bx+c)dx=\frac{1}{a}e^{ax}\sin(bx+c)-\frac{b}{a}\int\cos(bx+c)\left(e^{ax}\right)dx$$

$$=\frac{1}{a}e^{ax}\sin(bx+c)-\frac{b}{a}\left[\cos(bx+c)\left(\frac{e^{ax}}{a}\right)-\int\frac{e^{ax}}{a}(-b\sin(bx+c))dx\right]$$

$$=\frac{1}{a}e^{ax}\sin(bx+c)-\frac{b}{a^2}e^{ax}\cos(bx+c)-\frac{b^2}{a^2}\int e^{ax}\sin(bx+c)dx$$

$$=\frac{1}{a}e^{ax}\sin(bx+c)-\frac{b}{a^2}e^{ax}\cos(bx+c)-\frac{b^2}{a^2}I$$

$$\Rightarrow I\left(1+\frac{b^2}{a^2}\right)=\frac{1}{a^2}e^{ax}[a\sin(bx+c)-b\cos(bx+c)]$$

$$\Rightarrow I\left(\frac{a^2+b^2}{a^2}\right)=\frac{e^{ax}}{a^2}[a\sin(bx+c)-b\cos(bx+c)]$$

$$\therefore I=\frac{e^{ax}}{a^2+b^2}[a\sin(bx+c)-b\cos(bx+c)]+c$$

22. Obtain reduction formula for $I_n = \int \cot^n x\,dx$, n being a positive integer, n≥2 and deduce the value of $\int \cot^4 x\,dx$

Sol: Let $I_n = \int \cot^n x dx = \int (\cot^{n-2} x)\cot^2 x dx$

$$= \int (\cot^{n-2} x)(\csc^2 x - 1)dx$$

$$= \int \cot^{n-2} x .\csc^2 x dx - I_{n-2}$$

$$= \frac{\cot^{n-1} x}{n-1} - I_{n-2} \,.........(1)$$

Put n=4, 2,0 successively in (1), we get

$$I_4 = -\frac{\cot^3 x}{3} - I_2 \;= -\frac{\cot^3 x}{3} - (-\cot x - I_0) \;= -\frac{\cot^3 x}{3} - (-\cot x - x)$$

$$= -\frac{\cot^3 x}{3} + \cot x + x + c$$

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23. Evaluate $\int_0^{\pi} x\sin^7 x.\cos^6 x\,dx$

Sol: We know, $\int_0^a f(x)dx = \int_0^a f(a-x)dx.$

$$\therefore I = \int_0^{\pi} x\sin^7 x\cos^6 x dx = \int_0^{\pi} (\pi - x)\sin^7(\pi - x)\cos^6(\pi - x)dx$$

$$= \int_0^{\pi} (\pi - x)\sin^7 x\cos^6 x\,dx = \pi\int_0^{\pi} \sin^7 x\cos^6 x\,dx - I$$

$$\therefore 2I = \pi\int_0^{\pi} \sin^7 x\cos^6 x dx = 2\pi\int_0^{\pi/2} \sin^7 x\cos^6 x dx = 2\pi.\frac{[(6)(4)(2)][(5)(3)(1)]}{(13)(11)(9)(7)(5)(3)(1)} = \frac{32\pi}{3003} \Rightarrow I = \frac{16\pi}{3003}$$

24. Solve the differential equation $\frac{dy}{dx}=\frac{2y+x+1}{2x+4y+3}$

Sol: Given D.E is $\frac{dy}{dx}=\frac{(2y+x)+1}{2(x+2y)+3}$

Let x + 2y = z, then $1+2\frac{dy}{dx}=\frac{dz}{dx}$

$$\Rightarrow 2\frac{dy}{dx}=\frac{dz}{dx}-1$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2}\left(\frac{dz}{dx}-1\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{dz}{dx}-1\right)=\frac{z+1}{2z+3}$$

$$\Rightarrow \frac{dz}{dx}-1=\frac{2z+2}{2z+3} \Rightarrow \frac{dz}{dx}=\frac{2z+2}{2z+3}+1=\frac{2z+2+2z+3}{2z+3}$$

$$\Rightarrow \frac{dz}{dx}=\frac{4z+5}{2z+3}$$

$$\Rightarrow \frac{2z+3}{4z+5}dz=dx$$

$$\frac{2z+3}{4z+5}dz=dx \Rightarrow \frac{1}{2}\left(\frac{4z+6}{4z+5}\right)dz=dx$$

$$\Rightarrow \frac{1}{2}\left[\frac{(4z+5)+1}{4z+5}\right]dz=dx$$

$$\Rightarrow \frac{1}{2}\left[1+\frac{1}{4z+5}\right]dz=dx$$

$$\Rightarrow \frac{1}{2}+\frac{1}{2}\left(\frac{1}{4z+5}\right)dz=dx$$

$$\int\left(\frac{1}{2}+\frac{1}{2}\left(\frac{1}{4z+5}\right)dz\right)=\int dx$$

$$\Rightarrow \frac{z}{2}+\frac{1}{8}\log(4z+5)=x+c$$

$$\Rightarrow \frac{x+2y}{2}+\frac{1}{8}\log[4(x+2y)+5]=x+c \quad [\because z=x+2y]$$

$$\Rightarrow 4x+8y+\log(4x+8y+5)=8x+8c$$

$$\Rightarrow 8y-4x+\log(4x+8y+5)=8c$$