Previous IPE

# SOLVED PAPERS

# MARCH -2019 (TS)

PREVIOUS PAPERS

# IPE: MARCH-2019(TS)

## MATHS-2A

Time : 3 Hours Max.Marks : 75

### SECTION-A

**I. Answer ALL the following VSAQ:** **10 × 2 = 20**

1. If z=(cosθ, sinθ), find $\left(z-\frac{1}{z}\right)$
2. If z=x+iy, |z|=1, find the locus of z.
3. If 1,ω,$\omega^2$ are the cube roots of unity, find the value of $(1-\omega+\omega^2)^3$
4. Find the set of solutions of $x^2+x-12\le 0$ by algebraic method
5. If α,β,γ are the roots of $4x^3-6x^2+7x+3=0$ then find αβ+βγ+γα
6. Find the number of different chains that can be prepared using 6 different coloured beads.
7. If ${}^nC_4=210$ find n
8. Find the number of terms in the expansion of $(2x+3y+z)^7$.
9. Find the variance and standard deviation for the discrete data : 5,12,3,18,6,8,2,10
10. The mean and variance of a binomial distribution are 4 and 3 respectively. Find the distribution and find P(X≥1)

### SECTION-B

**II. Answer any FIVE of the following SAQs:** **5 × 4 = 20**

11. If $x+iy=\frac{1}{1+\cos\theta+i\sin\theta}$ then, show that $4x^2-1=0$
12. If $c^2\neq ab$ and the roots of the equation $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ are equal, then show that $a^3+b^3+c^3=3abc$ or $a=0$
13. If the letters of the word 'MASTER' are permutted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "MASTER"
14. Prove that $\frac{{}^{4n}C_{2n}}{{}^{2n}C_n}=\frac{1.3.5.....(4n-1)}{[1.3.5.....(2n-1)]^2}$
15. Resolve $\frac{x^3}{(x-a)(x-b)(x-c)}$ into Partial fractions
16. The probability for a contractor to get a road contract is 2/3 and to get a building contract is 5/9. The probability to get atleast one contract is 4/5. Find the probability that he gets both the contracts.
17. A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.

### SECTION-C

**III. Answer any FIVE of the following Long Answer Questions:** **5 × 7 = 35**

18. In n is an integer and z=cisθ then show that $\frac{z^{2n}-1}{z^{2n}+1}=i\tan n\theta$.
19. Solve $x^4+x^3-16x^2-4x+48=0$,given that the product of two of the roots is 6.
20. If 36, 84, 126 are three successive binomial coefficients in the expansion of $(1+x)^n$, then find n.
21. If $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+...\infty$, then find $3x^2+6x$
22. Find the mean deviation about mean for the following data:

| | | | | | | |
|---|---|---|---|---|---|---|
| $x_i$ | 2 | 5 | 7 | 8 | 10 | 35 |
| $f_i$ | 6 | 8 | 10 | 6 | 8 | 2 |

23. Three boxes numbered I, II, III contain the balls as follows :

| | White | Black | Red |
|---|---|---|---|
| I | 1 | 2 | 3 |
| II | 2 | 1 | 1 |
| III | 4 | 5 | 3 |

One box is randomly selected and a ball is drawn from i=t. If the ball is red, then find the probability that it is from BOX - II

24. A cubical die is thrown. Find the mean and variance of X, giving the number on the face that shows up.

# IPE TS MARCH-2019
## SOLUTIONS

## SECTION-A

**1. If z=(cosθ, sinθ), find $\left(z-\frac{1}{z}\right)$**

**Sol:** Given that z=(cosθ, sinθ) = cosθ+isinθ

$$\Rightarrow \frac{1}{z}=\frac{1}{\cos\theta+i\sin\theta}=\frac{\cos\theta-i\sin\theta}{(\cos\theta+i\sin\theta)(\cos\theta-i\sin\theta)}$$

$$=\frac{\cos\theta-i\sin\theta}{(\cos^2\theta+\sin^2\theta)}=\frac{\cos\theta-i\sin\theta}{1}=\cos\theta-i\sin\theta$$

$$\therefore z-\frac{1}{z}=(\cos\theta+i\sin\theta)-(\cos\theta-i\sin\theta)=2i\sin\theta$$

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**2. If z=x+iy, |z|=1, find the locus of z.**

**Sol:** $|z|=1 \Rightarrow \sqrt{x^2+y^2}=1 \Rightarrow x^2+y^2=1$

Locus of z is $x^2+y^2=1$

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**3. If $1,\omega,\omega^2$ are the cube roots of unity, find the value of $(1-\omega+\omega^2)^3$**

**Sol:** $(1-\omega+\omega^2)^3=(1+\omega^2-\omega)^3=(-\omega-\omega)^3$ $[\because 1+\omega+\omega^2=0 \Rightarrow 1+\omega^2=-\omega]$

$=(-2\omega)^3=-8\omega^3=-8(1)=-8$

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**4. Find the set of solutions of $x^2+x-12\le 0$ by algebraic method**

**Sol:** $x^2+x-12\le 0 \Rightarrow x^2+4x-3x-12\le 0$

$\Rightarrow x(x+4)-3(x+4)\le 0 \Rightarrow (x-3)(x+4)\le 0 \Rightarrow x\in[-4,3]$

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**5. If $\alpha,\beta,\gamma$ are the roots of $4x^3-6x^2+7x+3=0$ then find $\alpha\beta+\beta\gamma+\gamma\alpha$**

**Sol:** Here, $a_0=4, a_1=-6, a_2=7, a_3=3$

$\therefore \alpha\beta+\beta\gamma+\gamma\alpha = S_2 = a_2/a_0 = 7/4$

**6. Find the number of different chains that can be prepared using 6 different coloured beads.**

**Sol:** Number of circular permuations from n things $=\frac{1}{2}(n-1)!$

Hence the number of chains $=\frac{1}{2}(6-1)!=\frac{1}{2}(5!)=\frac{1}{2}(120)=60$

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**7. If $^{n}C_4=210$ find n**

**Sol:** $^{n}C_4=210 \Rightarrow \frac{n(n-1)(n-2)(n-3)}{1.2.3.4}=10\times 21$

$\Rightarrow n(n-1)(n-2)(n-3)=21\times 10\times 1\times 2\times 3\times 4=10\times 7\times 3\times 2\times 3\times 4=10\times 9\times 8\times 7$

$\therefore n=10$

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**8. Find the number of terms in the expansion of $(2x+3y+z)^7$.**

**Sol:** The number of terms in the trinomial expansion of $(x+y+z)^n=\frac{(n+1)(n+2)}{2}$

$\therefore$ The number of terms in the expansion of $(2x+3y+z)^7=\frac{(7+1)(7+2)}{2}=\frac{8\times 9}{2}=36$

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**9. Find the variance and standard deviation for discrete data: 5,12,3,18,6,8,2,10**

**Sol:** Mean $\bar{x}=\frac{\sum x_i}{n}=\frac{5+12+3+18+6+8+2+10}{8}=\frac{64}{8}=8$

**Deviations from the mean:**

5–8= –3; 12–8=4; 3–8= –5; 18–8=10;

6–8= –2;8–8=0; 2–8= –6; 10–8=2

**Absolute values of these deviations:** 3, 4, 5, 10, 2, 0, 6, 2

Variance $\sigma^2=\frac{\Sigma(x_i-\bar{x})^2}{n}=\frac{3^2+4^2+5^2+10^2+2^2+0^2+6^2+2^2}{8}$

$=\frac{9+16+25+100+4+0+36+4}{8}=\frac{194}{8}=24.25$

Standard deviation $\sigma=\sqrt{24.25}\cong 4.95$

**10. The mean and variance of a binomial distribution are 4 and 3 respectively. Find the distribution and find $P(X\geq1)$**

**Sol:** Given mean np = 4 ,variance npq = 3

Now, $(np)q = 3 \Rightarrow (4)q = 3 \Rightarrow q = \frac{3}{4} \Rightarrow p = 1 - q = 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}$

Take $np = 4 \Rightarrow n\left(\frac{1}{4}\right) = 4 \Rightarrow n = 4(4) = 16$

$\therefore$ n=16 , q=3/4 and p=1/4

Binomial distribution is $P(X = r) = {}^{n}C_r q^{n-r}.p^r = {}^{16}C_r \left(\frac{3}{4}\right)^{16-r}.\left(\frac{1}{4}\right)^r$

$\therefore P(X\geq1) = 1 - P(X=0) = 1 - q^n = 1 - \left(\frac{3}{4}\right)^n = 1 - \left(\frac{3}{4}\right)^{16}$

# SECTION-B

**11. If $x+iy=\dfrac{1}{1+\cos\theta+i\sin\theta}$ then, show that $4x^2-1=0$**

**Sol:** Given that $x+iy=\dfrac{1}{(1+\cos\theta)+i\sin\theta}=\dfrac{1}{(2\cos^2\frac{\theta}{2})+i(2\sin\frac{\theta}{2}\cos\frac{\theta}{2})}$

$$\Rightarrow x+iy=\frac{1}{(2\cos\frac{\theta}{2})(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})}=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{(2\cos\frac{\theta}{2})(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2})}$$

$$=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\left(2\cos\frac{\theta}{2}\right)\left(\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\right)}=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\left(2\cos\frac{\theta}{2}\right)(1)}=\frac{\cos\frac{\theta}{2}}{2\cos\frac{\theta}{2}}-\frac{i\sin\frac{\theta}{2}}{2\cos\frac{\theta}{2}}=\frac{1}{2}-\frac{1}{2}i\tan\frac{\theta}{2}$$

Equating the real parts, we get $x=\frac{1}{2}\Rightarrow 2x=1\Rightarrow(2x)^2=1^2\Rightarrow 4x^2=1\Rightarrow 4x^2-1=0$

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**12. If $c^2\neq ab$ and the roots of the equation $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$ are equal, then show that $a^3+b^3+c^3=3abc$ (or) $a=0$**

**Sol:** Given equation is $(c^2-ab)x^2-2(a^2-bc)x+(b^2-ac)=0$

Comparing with $Ax^2+Bx+C=0$, we get $A=(c^2-ab)$; $B=-2(a^2-bc)$; $C=(b^2-ac)$

Given that the roots of the given equation are equal.

$\therefore$ Discriminant $\Delta=B^2-4AC=0$

$\Rightarrow[-2(a^2-bc)]^2-4(c^2-ab)(b^2-ac)=0$

$\Rightarrow 4(a^2-bc)^2-4(c^2-ab)(b^2-ac)=0$

$\Rightarrow a^4+b^2c^2-2a^2bc-b^2c^2+c^3a+b^3a-a^2bc=0$

$\Rightarrow a^4+ab^3+ac^3-3abc=0\Rightarrow a(a^3+b^3+c^3-3abc)=0$

$\therefore a^3+b^3+c^3-3abc=0$ (or) $a=0$

$\Rightarrow a^3+b^3+c^3=3abc$ (or) $a=0$

**13. If the letters of the word MASTER are permutted in all possible ways and the words thus formed are arranged in dictionary order, then find the rank of the word MASTER.**

**Sol:** The alphabetical order of the letters of the word MASTER is

**A,E,M,R,S,T**

The number of words that begin with A – – – – – = 5! = 120

The number of words that begin with E – – – – – = 5! = 120

The number of words that begin with MAE – – – = 3! = 6

The number of words that begin with MAR – – – = 3! = 6

The number of words that begin with MASE – – = 2! = 2

The number of words that begin with MASR – – = 2! = 2

The next word is MASTER = 1! = 1

$\therefore$ Rank of the word MASTER = 2(120) + 2(6) + 2(2) + 1

= 240 + 12 + 4 + 1 = 257

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**14. Prove that** $\dfrac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \dfrac{1.3.5.....(4n-1)}{[1.3.5.....(2n-1)]^2}$

**Sol:** $$\text{L.H.S} = \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{\dfrac{4n!}{2n!.2n!}}{\dfrac{2n!}{n!.n!}} = \frac{(4n)!}{(2n!)^2}\times\frac{(n!)^2}{(2n)!} \qquad \left[\text{Since } {}^nC_r = \frac{n!}{r!(n-r)!}\right]$$

$$= \frac{(4n)(4n-1)(4n-2)(4n-3)(4n-4)..........6.5.4.3.2.1}{[(2n)(2n-1)(2n-2)(2n-3).........4.3.2.1]^2}\times\frac{(n!)^2}{(2n)!}$$

$$= \frac{[(4n)(4n-2)(4n-4)..........(6)(4)(2)][(4n-1)(4n-3).....5.3.1]}{[(2n)(2n-2)......4.2]^2[(2n-1)(2n-3).......(3)(1)]^2}\times\frac{(n!)^2}{(2n)!}$$

$$= \frac{[2^{2n}(2n)(2n-1)(2n-2).......(3)(2)(1)][(4n-1)(4n-3).....5.3.1]}{[2^n(n)(n-1)........(2)(1)]^2[(2n-1)(2n-3).......(3)(1)]^2}\times\frac{(n!)^2}{(2n)!}$$

$$= \frac{[2^{2n}(2n!)][(4n-1)(4n-3).....5.3.1]}{2^{2n}(n!)^2[(2n-1)(2n-3).......(3)(1)]^2}\times\frac{(n!)^2}{(2n)!}\ \frac{1.3.5......(4n-3)(4n-1)}{[1.3.5......(2n-3)(2n-1)]^2} = \text{R.H.S}$$

**15. Resolve $\dfrac{x^3}{(x-a)(x-b)(x-c)}$ into partial fractions.**

**Sol :** Let $\dfrac{x^3}{(x-a)(x-b)(x-c)} = 1 + \dfrac{A}{x-a} + \dfrac{B}{x-b} + \dfrac{C}{x-c}$ Here deg(Nr)=deg(Dr)

$$= \frac{(x-a)(x-b)(x-c) + A(x-b)(x-c) + B(x-a)(x-c) + c(x-a)(x-b)}{(x-a)(x-b)(x-c)}$$

$$\Rightarrow (x-a)(x-b)(x-c) + A(x-b)(x-c) + B(x-a)(x-c) + C(x-a)(x-b) = x^3 .....(1)$$

Putting x=a in (1), we get 0+A(a–b)(a–c)+0+0 = $a^3 \Rightarrow A = \dfrac{a^3}{(a-b)(a-c)}$

Similarly by putting x=b and x=c we get, $B = \dfrac{b^3}{(b-a)(b-c)}$, $C = \dfrac{c^3}{(c-a)(c-b)}$

$$\therefore \frac{x^3}{(x-a)(x-b)(x-c)} = 1 + \frac{a^3}{(a-b)(a-c)(x-a)} + \frac{b^3}{(b-c)(b-a)(x-b)} + \frac{c^3}{(c-a)(c-b)(x-c)}$$

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**16. The probability for a contractor to get a road contract is 2/3 and to get a building contract is 5/9. The probability to get atleast one contract is 4/5. Find the probability that he gets both the contracts.**

**Sol:** Let A be the event of getting road contract and B be the event of getting building contract.

Given that $P(A) = \dfrac{2}{3}$, $P(B) = \dfrac{5}{9}$, $P(A \cup B) = \dfrac{4}{5}$

$\therefore$ Probability that the contractor will get both the contracts is

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{2}{3} + \frac{5}{9} - \frac{4}{5} = \frac{30 + 25 - 36}{45} = \frac{19}{45}$$

**17. A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.**

**Sol:** Let A,B denote the events of speaking truth by A,B respectively

$$P(A)=\frac{75}{100}=\frac{3}{4};\ P(B)=\frac{80}{100}=\frac{4}{5}$$

$$\therefore P(\bar{A})=1-P(A)=1-\frac{3}{4}=\frac{1}{4};\quad P(\bar{B})=1-P(B)=1-\frac{4}{5}=\frac{1}{5}$$

Let E be the event that A and B contradict to each other

$$\Rightarrow P(E)=P\left[(A\cap\bar{B})\cup(\bar{A}\cap B)\right]=P(A\cap\bar{B})+P(\bar{A}\cap B)$$

$$=P(A)P(\bar{B})+P(\bar{A})P(B)\ [\because A,B \text{ are independent}]=\frac{3}{4}.\frac{1}{5}+\frac{1}{4}.\frac{4}{5}=\frac{7}{20}$$

## SECTION-C

**18. If n is an integer and z=cisθ then show that $\frac{z^{2n}-1}{z^{2n}+1}=i\tan n\theta$.**

**Sol:** Given z=cisθ=cosθ+isinθ

$$\text{L.H.S}=\frac{z^{2n}-1}{z^{2n}+1}=\frac{(\cos\theta+i\sin\theta)^{2n}-1}{(\cos\theta+i\sin\theta)^{2n}+1}$$

$$=\frac{\cos(2n)\theta+i\sin(2n)\theta-1}{\cos(2n)\theta+i\sin(2n)\theta+1}=\frac{-(1-\cos2n\theta)+i\sin2n\theta}{(1+\cos2n\theta)+i\sin2n\theta}$$

$$=\frac{-(2\sin^2 n\theta)+i(2\sin n\theta\cos n\theta)}{2\cos^2 n\theta+i(2\sin n\theta\cos n\theta)}=\frac{(2i^2\sin^2 n\theta)+i(2\sin n\theta\cos n\theta)}{2\cos^2 n\theta+i(2\sin n\theta\cos n\theta)}$$

$$=\frac{2i\sin n\theta\,(i\sin n\theta+\cos n\theta)}{2\cos n\theta\,(\cos n\theta+i\sin n\theta)}=i\tan n\theta=\text{R.H.S}$$

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**19. Solve $x^4+x^3-16x^2-4x+48=0$, given that the product of two of the roots is 6.**

**Sol:** Let α,β,γ,δ be the roots of $x^4+x^3-16x^2-4x+48=0$ ........ (1)

$\Rightarrow S_1=\alpha+\beta+\gamma+\delta=-1$ and $S_4=\alpha\beta\gamma\delta=48$

Given that product of two roots is 6. Let αβ=6

$\therefore\ \alpha\beta\gamma\delta=48 \Rightarrow (6)(\gamma\delta)=48 \Rightarrow \gamma\delta=8$

Let us take α+β=p and γ+δ=q

Now, the equation with roots α,β is $x^2-(\alpha+\beta)x+\alpha\beta=0 \Rightarrow x^2-px+6=0$ ....... (2)

The equation with roots γ,δ is $x^2-(\gamma+\delta)x+\gamma\delta=0 \Rightarrow x^2-qx+8=0$ ....... (3)

from (1),(2),(3), we have $x^4+x^3-16x^2-4x+48=(x^2-px+6)(x^2-qx+8)$

$=x^4-(p+q)x^3+(pq+14)x^2-(8p+6q)x+48$

Comparing the coefficient of $x^3$, we get $p+q=-1 \Rightarrow q=-1-p$

Comparing the coefficient of x, we get $8p+6q=4 \Rightarrow 4p+3q=2$

Hence $4p+3(-1-p)=2 \Rightarrow 4p-3-3p=2 \Rightarrow p=5$

Also $q=-1-p=-1-5=-6$

Now, (2) $\Rightarrow x^2-px+6=0 \Rightarrow x^2-5x+6=0 \Rightarrow (x-2)(x-3)=0 \Rightarrow x=2$ or 3

(3) $\Rightarrow x^2-qx+8=0 \Rightarrow x^2+6x+8=0 \Rightarrow (x+2)(x+4)=0 \Rightarrow x=-2$ or $-4$

∴ The roots of the given equation are 2,3,–2,–4

**20. If 36, 84, 126 are three successive binomial coefficients in the expansion of $(1+x)^n$, then find n.**

**Sol :** Let the 3 successive coefficients of $(1+x)^n$ be taken as

$^nC_{r-1} = 36$ .....(1); $^nC_r = 84$ ......(2); $^nC_{r+1} = 126$ ......(3)

Now, $\frac{(2)}{(1)} \Rightarrow \frac{^nC_r}{^nC_{r-1}} = \frac{84}{36} \Rightarrow \frac{n-r+1}{r} = \frac{7}{3} \Rightarrow 3n - 3r + 3 = 7r \Rightarrow 3n - 10r = -3$....(4)

$\frac{(3)}{(2)} \Rightarrow \frac{^nC_{r+1}}{^nC_r} = \frac{126}{84} \Rightarrow \frac{n-r}{r+1} = \frac{3}{2} \Rightarrow 2n - 2r = 3r + 3 \Rightarrow 2n - 5r = 3$....(5)

Solving (4) & (5) we get n

$2 \times (5) \Rightarrow 4n - 10r = 6$ ......(6)

Now (6) – (4) $\Rightarrow$ n = 9

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**21. If $x = \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + ...\infty$, then find $3x^2 + 6x$**

**Sol:** Given $\mathbf{x} = \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + ...\infty$

Adding 1 on both sides, we get $1 + x = 1 + \frac{1}{5} + \frac{1.3}{5.10} + \frac{1.3.5}{5.10.15} + ...\infty$

$$= 1 + \frac{1}{1!}\left(\frac{1}{5}\right) + \frac{1.3}{2!}\left(\frac{1}{5}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{5}\right)^3 + ...\infty$$

Comparing the above series with $1 + \frac{p}{1!}\left(\frac{y}{q}\right) + \frac{p(p+q)}{2!}\left(\frac{y}{q}\right)^2 + ....... = (1-y)^{-p/q}$

we get p =1, p+q =3 $\Rightarrow$q =2 and $\frac{y}{q} = \frac{1}{5} \Rightarrow y = \frac{q}{5} = \frac{2}{5}$

$$\therefore 1 + x = (1-y)^{\frac{-p}{q}} = \left(1 - \frac{2}{5}\right)^{\frac{-1}{2}} = \left(\frac{3}{5}\right)^{\frac{-1}{2}} = \left(\frac{5}{3}\right)^{\frac{1}{2}} = \sqrt{\frac{5}{3}}$$

$$\Rightarrow (1+x)^2 = \frac{5}{3} \Rightarrow 1 + 2x + x^2 = \frac{5}{3} \Rightarrow 3 + 6x + 3x^2 = 5 \Rightarrow 3x^2 + 6x = 2.$$

**22. Find the mean deviation about mean for the following data:**

| $x_i$ | 2 | 5 | 7 | 8 | 10 | 35 |
|---|---|---|---|---|---|---|
| $f_i$ | 6 | 8 | 10 | 6 | 8 | 2 |

**Sol:** We form the following table from the given data.

| $x_i$ | $f_i$ | $f_i x_i$ | $\lvert x_i - \bar{x} \rvert$ | $f_i \lvert x_i - \bar{x} \rvert$ |
|---|---|---|---|---|
| 2 | 6 | 12 | 6 | 36 |
| 5 | 8 | 40 | 3 | 24 |
| 7 | 10 | 70 | 1 | 10 |
| 8 | 6 | 48 | 0 | 0 |
| 10 | 8 | 80 | 2 | 16 |
| 35 | 2 | 70 | 27 | 54 |
| | **40** | **320** | | **140** |

Here, $N = \Sigma f_i = 40$; $\Sigma f_i x_i = 320$ $\Rightarrow$ Mean $\bar{x} = \dfrac{\Sigma f_i x_i}{N} = \dfrac{320}{40} = 8$

From the table $\Sigma f_i \lvert x_i - \bar{x} \rvert = 140$

$\therefore$ Mean deviation about mean is $M.D = \dfrac{\Sigma f_i \lvert x_i - \bar{x} \rvert}{N} = \dfrac{140}{40} = 3.5$

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**23. Three boxes numbered I, II, III contains the balls as follows:**

| | White | Black | Red |
|---|---|---|---|
| I | 1 | 2 | 3 |
| II | 2 | 1 | 1 |
| III | 4 | 5 | 3 |

**One box is randomly selected and a ball is drawn from it. If the ball is red, then find the probability that it is from box II.**

**Sol:** Let $B_1, B_2, B_3$ be the events of selecting boxes $B_1, B_2, B_3$ and R be the event of getting drawing a red ball

$$\therefore P(B_1) = \frac{1}{3}, P(B_2) = \frac{1}{3}, P(B_3) = \frac{1}{3} \text{ and } P\left(\frac{R}{B_1}\right) = \frac{3}{6} = \frac{1}{2}, P\left(\frac{R}{B_2}\right) = \frac{1}{4}, P\left(\frac{R}{B_3}\right) = \frac{3}{12} = \frac{1}{4}$$

$\therefore$ by Baye's theorem, the required probability is

$$P\left(\frac{B_2}{R}\right) = \frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right) + P(B_2)P\left(\frac{R}{B_2}\right) + P(B_3)P\left(\frac{R}{B_3}\right)} = \frac{\cancel{\frac{1}{3}} \times \frac{1}{4}}{\cancel{\frac{1}{3}}\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)} = \frac{\frac{1}{4}}{\frac{1}{2} + \frac{1}{4} + \frac{1}{4}} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{4}{4}\right)} = \frac{1}{4}$$

**24. A cubical die is thrown. Find the mean and variance of X, giving the number on the face that shows up.**

**Sol:** Let S be the sample space of throwing a die and X be the random variable.

Then P(X) is given by the following table.

| $X = x_i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(X=x_i)$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ | $\frac{1}{6}$ |

Mean of X is $\mu = \sum_{i=1}^{6} X_i . P(X = x_i)$

$$= 1.\frac{1}{6} + 2.\frac{1}{6} + 3.\frac{1}{6} + 4.\frac{1}{6} + 5.\frac{1}{6} + 6.\frac{1}{6} = \frac{1}{6}(1+2+3+4+5+6) = \frac{21}{6} = \frac{7}{2}$$

Variance of X is $\sigma^2 = \sum_{i=1}^{6} x_i^2 . P(X = x_i) - \mu^2$

$$= 1^2.\frac{1}{6} + 2^2.\frac{1}{6} + 3^2.\frac{1}{6} + 4^2.\frac{1}{6} + 5^2.\frac{1}{6} + 6^2.\frac{1}{6} - \left(\frac{7}{2}\right)^2$$

$$= \frac{1}{6}(1+4+9+16+25+36) - \frac{49}{4} = \frac{91}{6} - \frac{49}{4} = \frac{182-147}{12} = \frac{35}{12}$$