Previous IPE

SOLVED PAPERS

MARCH -2019 (TS)

PREVIOUS PAPERS

IPE: MARCH-2019(TS)

Time : 3 Hours **MATHS-1A** Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **10 × 2 = 20**

1. If f(x)=2x–1, $g(x)=\frac{x+1}{2}$ for all $x\in R$, find (gof)(x) and (fog)(x)
2. Find the domain of the real valued function : $f(x)=\frac{1}{6x-x^2-5}$
3. If $A=\begin{bmatrix}1&2&3\\3&2&1\end{bmatrix}$ and $B=\begin{bmatrix}3&2&1\\1&2&3\end{bmatrix}$, find 3B–2A **4.** If $A=\begin{bmatrix}2&0&1\\-1&1&5\end{bmatrix}$, $B=\begin{bmatrix}-1&1&0\\0&1&-2\end{bmatrix}$ then find $(AB^T)^T$
5. If $\overline{OA}=\bar{i}+\bar{j}+\bar{k}$, $\overline{AB}=3\bar{i}-2\bar{j}+\bar{k}$, $\overline{BC}=\bar{i}+2\bar{j}-2\bar{k}$, $\overline{CD}=2\bar{i}+\bar{j}+3\bar{k}$ then find the vector $\overline{OD}$
6. Let $\bar{a}=2\bar{i}+4\bar{j}-5\bar{k}, \bar{b}=\bar{i}+\bar{j}+\bar{k}, \bar{c}=\bar{j}+2\bar{k}$. Find the unit vector in the opposite direction of $\bar{a}+\bar{b}+\bar{c}$
7. Find the equation of the plane passing through the point (3, – 2, 1) andperpendicular to the vector (4, 7, –4)
8. If $\sin\theta=-\frac{1}{3}$ and θ does not lie in the 3rd quadrant, find the value of cosθ and cot θ
9. Find $\sin^2 82\frac{1}{2}^o-\sin^2 22\frac{1}{2}^o$ **10.** If coshx=5/2, then find the values of (i) cosh(2x) and (ii) sinh(2x)

SECTION-B

II. Answer any FIVE of the following SAQs: **5 × 4 = 20**

11. If $A=\begin{bmatrix}1&-2&1\\0&1&-1\\3&-1&1\end{bmatrix}$ then show that A^3-3A^2-A-3I, where I is unit matrix of order 3
12. If $\bar{a},\bar{b},\bar{c}$ are noncoplanar, find the point of intersection of the line passing through the points $2\bar{a}+3\bar{b}-\bar{c}, 3\bar{a}+4\bar{b}-2\bar{c}$ with the line joining the points $\bar{a}-2\bar{b}+3\bar{c}, \bar{a}-6\bar{b}+6\bar{c}$
13. If $\bar{a}=2\bar{i}+3\bar{j}+4\bar{k}, \bar{b}=\bar{i}+\bar{j}-\bar{k}, \bar{c}=\bar{i}-\bar{j}+\bar{k}$, compute $\bar{a}\times(\bar{b}\times\bar{c})$ and verify that it is perpendicular to $\bar{a}$ **14.** Prove that $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)=\frac{1}{16}$
15. If θ_1,θ_2 are solutions of the equation : acos2θ+bsin2θ= c , $\tan\theta_1\neq\tan\theta_2$ and $a+c\neq 0$. then find the values of (i) $\tan\theta_1+\tan\theta_2$ (ii) $\tan\theta_1.\tan\theta_2$
16. Prove that $Sin^{-1}\frac{3}{5}+Sin^{-1}\frac{8}{17}=Cos^{-1}\frac{36}{85}$ **17.** In ΔABC, if $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$ then show that C=60°

SECTION-C

III. Answer any FIVE of the following LAQs: **5 × 7 = 35**

18. If f:A→B is a bijective function then prove that (i) $fof^{-1}=I_B$ (ii) $f^{-1}of=I_A$
19. Using Mathematical Induction, prove that statement for all n∈N $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\cdots\cdots\left(1+\frac{2n+1}{n^2}\right)=(n+1)^2$
20. Show that $\begin{vmatrix}a+b+2c&a&b\\c&b+c+2a&b\\c&a&c+a+2b\end{vmatrix}=2(a+b+c)^3$
21. Solve the equations 3x + 4y + 5z = 18, 2x – y + 8z = 13, 5x – 2y + 7z = 20 by matrix inversion method
22. If A=(1,–2,–1),B=(4,0,–3), C=(1,2,–1), D=(2,–4,–5) then find the distance between $\overline{AB}$ and $\overline{CD}$.
23. If A, B, C are angles in a triangle, then prove that $\cos A+\cos B-\cos C=-1+4\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2}$
24. Show that $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^2}$

IPE TS MARCH-2019
SOLUTIONS

SECTION-A

1. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$ for all $x \in R$, find (i) (gof)(x) (ii) (fog)(x)

A: (i) $(gof)(x) = g(f(x)) = g(2x-1) = \frac{(2x-1)+1}{2} = \frac{2x}{2} = x$

(ii) $(fog)(x) = f(g(x)) = f\left(\frac{x+1}{2}\right) = \cancel{2}\left(\frac{x+1}{\cancel{2}}\right) - 1 = x + 1 - 1 = x$

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2. Find the domain of $f(x) = \frac{1}{6x - x^2 - 5}$

A: Given f(x) is 'defined' when $6x - x^2 - 5 \neq 0$

$\Rightarrow -(x^2 - 6x + 5) \neq 0 \Rightarrow -(x-1)(x-5) \neq 0 \Rightarrow x \neq 1, 5$

$\therefore$ Domain is $R - \{1, 5\}$

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3. If $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, find 3B–2A

A: Given $B = \begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$; $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$

$$\therefore 3B - 2A = 3\begin{bmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 6 & 3 \\ 3 & 6 & 9 \end{bmatrix} - \begin{bmatrix} 2 & 4 & 6 \\ 6 & 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-2 & 6-4 & 3-6 \\ 3-6 & 6-4 & 9-2 \end{bmatrix} = \begin{bmatrix} 7 & 2 & -3 \\ -3 & 2 & 7 \end{bmatrix}$$

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4. If $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 5 \end{bmatrix}, B = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -2 \end{bmatrix}$ then find (AB')'

A: $AB' = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 5 \end{bmatrix}_{2\times3} \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & -2 \end{bmatrix}_{3\times2} = \begin{bmatrix} 2(-1)+0(1)+1(0) & 2(0)+0(1)+1(-2) \\ -1(-1)+1(1)+5(0) & -1(0)+1(1)+5(-2) \end{bmatrix}_{2\times2}$

$$= \begin{bmatrix} -2+0+0 & 0+0-2 \\ 1+1+0 & 0+1-10 \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ 2 & -9 \end{bmatrix}$$

$$\therefore (AB')' = \begin{bmatrix} -2 & 2 \\ -2 & -9 \end{bmatrix}$$

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5. If $\overline{OA} = \bar{i} + \bar{j} + \bar{k},\ \overline{AB} = 3\bar{i} - 2\bar{j} + \bar{k},\ \overline{BC} = \bar{i} + 2\bar{j} - 2\bar{k},\ \overline{CD} = 2\bar{i} + \bar{j} + 3\bar{k}$ then find the vector $\overline{OD}$

A: Consider $\overline{OA} + \overline{AB} + \overline{BC} + \overline{CD} = (\overline{OA} + \overline{AB}) + \overline{BC} + \overline{CD} = (\overline{OB} + \overline{BC}) + \overline{CD} = \overline{OC} + \overline{CD} = \overline{OD}$

Hence, $\overline{OD} = \overline{OA} + \overline{AB} + \overline{BC} + \overline{CD} = (\bar{i} + \bar{j} + \bar{k}) + (3\bar{i} - 2\bar{j} + \bar{k}) + (\bar{i} + 2\bar{j} - 2\bar{k}) + (2\bar{i} + \bar{j} + 3\bar{k})$

$= 7\bar{i} + 2\bar{j} + 3\bar{k}$

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6. Let $\bar{a} = 2\bar{i} + 4\bar{j} - 5\bar{k}, \bar{b} = \bar{i} + \bar{j} + \bar{k}, \bar{c} = \bar{j} + 2\bar{k}$. Find the unit vector in the opposite direction of $\bar{a} + \bar{b} + \bar{c}$

A: Given $\bar{a} = 2\bar{i} + 4\bar{j} - 5\bar{k}$, $\bar{b} = \bar{i} + \bar{j} + \bar{k}$, $\bar{c} = 0\,\bar{i} + \bar{j} + 2\,\bar{k}$, then $\bar{a} + \bar{b} + \bar{c}$

$$= (2\bar{i} + 4\bar{j} - 5\bar{k}) + (\bar{i} + \bar{j} + \bar{k}) + (0\bar{i} + \bar{j} + 2\bar{k})$$

$$= 3\bar{i} + 6\bar{j} - 2\bar{k}$$

$$\Rightarrow |\bar{a} + \bar{b} + \bar{c}| = \sqrt{3^2 + 6^2 + (-2)^2} = \sqrt{9 + 36 + 4} = \sqrt{49} = 7$$

$$\therefore \text{Opposite Unit vector } = \frac{-(\bar{a} + \bar{b} + \bar{c})}{|\bar{a} + \bar{b} + \bar{c}|} = \frac{-(3\bar{i} + 6\bar{j} - 2\bar{k})}{7}$$

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7. Find the equation of the plane through the point (3,–2,1) and perpendicular to the vector (4,7,–4).

A: The equation of the plane passing through the point $\bar{a} = 3\bar{i} - 2\bar{j} + \bar{k}$ and perpendicular to the vector $\bar{n} = 4\bar{i} + 7\bar{j} - 4\bar{k}$ is $\bar{r}.\bar{n} = \bar{a}.\bar{n}$

$\Rightarrow \bar{r}.(4\bar{i} + 7\bar{j} - 4\bar{k}) = (3\bar{i} - 2\bar{j} + \bar{k}).(4\bar{i} + 7\bar{j} - 4\bar{k}) = 12 - 14 - 4 = -6$

$\Rightarrow \bar{r}.(4\bar{i} + 7\bar{j} - 4\bar{k}) = -6 \quad \Rightarrow \bar{r}.(-4\bar{i} - 7\bar{j} + 4\bar{k}) = 6$

8. If $\sin\theta = \frac{-1}{3}$ and θ does not lie in 3rd quadrant, find the value of $\cos\theta$

A: Given $\sin\theta$ is negative and θ is not in Q_3 $\quad \therefore \theta \in Q_4$

In Q_4, $\cos\theta$ is positive .

$$\therefore \cos\theta = +\sqrt{1-\sin^2\theta} = \sqrt{1-\left(-\frac{1}{3}\right)^2} = \sqrt{1-\frac{1}{9}} = \frac{2\sqrt{2}}{3}$$

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9. Find the value of $\sin^2 82\frac{1}{2}^{o} - \sin^2 22\frac{1}{2}^{o}$

A: We know $\sin^2A - \sin^2B = \sin(A+B)\sin(A-B)$

$$\therefore \sin^2 82\frac{1}{2}^{o} - \sin^2 22\frac{1}{2}^{o} = \sin(82\frac{1}{2}^{o} + 22\frac{1}{2}^{o}).\sin(82\frac{1}{2}^{o} - 22\frac{1}{2}^{o})$$

$$= \sin(105^o).\sin 60^o = \sin(60^o+45^o).\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}(\sin 60^o \cos 45^o + \cos 60^o \sin 45^o)$$

$$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right) = \frac{\sqrt{3}(\sqrt{3}+1)}{4\sqrt{2}}$$

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10. If coshx=5/2, find cosh(2x), sinh(2x)

A: Given coshx=5/2, then $\sinh x = \sqrt{\cosh^2 x - 1}$

$$= \sqrt{\left(\frac{5}{2}\right)^2 - 1} = \sqrt{\frac{25}{4} - 1} = \sqrt{\frac{25-4}{4}} = \sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$$

(i) $\cosh(2x) = \cosh^2 x + \sinh^2 x = \left(\frac{5}{2}\right)^2 + \left(\frac{\sqrt{21}}{2}\right)^2 = \frac{25}{4} + \frac{21}{4} = \frac{\cancel{46}}{\cancel{4}} = \frac{23}{2}$

(ii) $\sinh(2x) = 2\sinh x \cosh x = \cancel{2} \times \frac{\sqrt{21}}{\cancel{2}} \times \frac{5}{2} = \frac{5\sqrt{21}}{2}$

SECTION-B

11. If $A=\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}$ **then show that** $A^3-3A^2-A-3I=O$

A : $$A^2=A\times A=\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}=\begin{bmatrix}1+0+3 & -2-2-1 & 1+2+1\\ 0+0-3 & 0+1+1 & 0-1-1\\ 3-0+3 & -6-1-1 & 3+1+1\end{bmatrix}=\begin{bmatrix}4 & -5 & 4\\ -3 & 2 & -2\\ 6 & -8 & 5\end{bmatrix}$$

$$A^3=A^2.A=\begin{bmatrix}4 & -5 & 4\\ -3 & 2 & -2\\ 6 & -8 & 5\end{bmatrix}\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}=\begin{bmatrix}4+0+12 & -8-5-4 & 4+5+4\\ -3+0-6 & 6+2+2 & -3-2-2\\ 6+0+15 & -12-8-5 & 6+8+5\end{bmatrix}=\begin{bmatrix}16 & -17 & 13\\ -9 & 10 & -7\\ 21 & -25 & 19\end{bmatrix}$$

$$\therefore A^3-3A^2-A-3I=\begin{bmatrix}16 & -17 & 13\\ -9 & 10 & -7\\ 21 & -25 & 19\end{bmatrix}-3\begin{bmatrix}4 & -5 & 4\\ -3 & 2 & -2\\ 6 & -8 & 5\end{bmatrix}-\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}-3\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}16 & -17 & 13\\ -9 & 10 & -7\\ 21 & -25 & 19\end{bmatrix}+\begin{bmatrix}-12 & 15 & -12\\ 9 & -6 & 6\\ -18 & 24 & -15\end{bmatrix}-\begin{bmatrix}1 & -2 & 1\\ 0 & 1 & -1\\ 3 & -1 & 1\end{bmatrix}-\begin{bmatrix}3 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 3\end{bmatrix}$$

$$=\begin{bmatrix}16-12-1-3 & -17+15+2-0 & 13-12-1-0\\ -9+9+0-0 & 10-6-1-3 & -7+6+1+0\\ 21-18-3+0 & -25+24+1+0 & 19-15-1-3\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}=O$$

12. If $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar,then prove that the points with position vectors $2\bar{a}+3\bar{b}-\bar{c}, \bar{a}-2\bar{b}+3\bar{c}, 3\bar{a}+4\bar{b}-2\bar{c}, \bar{a}-6\bar{b}+6\bar{c}$ are coplanar.

A: We take $\overline{OP} = 2\bar{a}+3\bar{b}-\bar{c}, \overline{OQ} = \bar{a}-2\bar{b}+3\bar{c}$,

$\overline{OR} = 3\bar{a}+4\bar{b}-2\bar{c}, \overline{OS} = \bar{a}-6\bar{b}+6\bar{c}$ where 'O' is the origin.

$\overline{PQ} = \overline{OQ}-\overline{OP} = (\bar{a}-2\bar{b}+3\bar{c})-(2\bar{a}+3\bar{b}-\bar{c}) = -\bar{a}-5\bar{b}+4\bar{c}$

$\overline{PR} = \overline{OR}-\overline{OP} = (3\bar{a}+4\bar{b}-2\bar{c})-(2\bar{a}+3\bar{b}-\bar{c}) = \bar{a}+\bar{b}-\bar{c}$

$\overline{PS} = \overline{OS}-\overline{OP} = (\bar{a}-6\bar{b}+6\bar{c})-(2\bar{a}+3\bar{b}-\bar{c}) = -\bar{a}-9\bar{b}+7\bar{c}$

$$[\overline{PQ}\ \overline{PR}\ \overline{PS}] = \begin{vmatrix} -1 & -5 & 4 \\ 1 & 1 & -1 \\ -1 & -9 & 7 \end{vmatrix} [\bar{a}\ \bar{b}\ \bar{c}]$$

$$=[-1(7-9)+5(7-1)+4(-9+1)][\bar{a}\ \bar{b}\ \bar{c}]$$

$$=[-1(-2)+5(6)+4(-8)]\ [\bar{a}\ \bar{b}\ \bar{c}]$$

$$=[2+30-32][\bar{a}\ \bar{b}\ \bar{c}] = 0\times[\bar{a}\ \bar{b}\ \bar{c}]=0$$

So, $\overline{PQ}, \overline{PR}, \overline{PS}$ are coplanar.

Hence proved that the four points P,Q,R,S are coplanar.

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13. If $\bar{a} = 2\bar{i}+3\bar{j}+4\bar{k}, \bar{b} = \bar{i}+\bar{j}-\bar{k}, \bar{c} = \bar{i}-\bar{j}+\bar{k}$, compute $\bar{a}\times(\bar{b}\times\bar{c})$ and verify that it is perpendicular to $\bar{a}$

A: Given $\bar{a} = 2\bar{i}+3\bar{j}+4\bar{k}, \bar{b} = \bar{i}+\bar{j}-\bar{k}, \bar{c} = \bar{i}-\bar{j}+\bar{k}$

To find $\bar{a}\times(\bar{b}\times\bar{c})$, first we have to find $\bar{b}\times\bar{c}$ (the term in the bracket)

$$\bar{b}\times\bar{c} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = \bar{i}(1-1)-\bar{j}(1+1)+\bar{k}(-1-1) = -2\bar{j}-2\bar{k}$$

$$\therefore \bar{a}\times(\bar{b}\times\bar{c}) = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 3 & 4 \\ 0 & -2 & -2 \end{vmatrix} = \bar{i}(-6+8)-\bar{j}(-4-0)+\bar{k}(-4-0) = 2\bar{i}+4\bar{j}-4\bar{k}$$

Now $\bar{a}\times(\bar{b}\times\bar{c}).\bar{a} = (2\bar{i}+4\bar{j}-4\bar{k}).(2\bar{i}+3\bar{j}+4\bar{k})$

$= 2(2)+4(3)-4(4) = 4+12-16=0$

Since, Dot product is zero, $\bar{a}\times(\bar{b}\times\bar{c})$ is perpendicular to $\bar{a}$.

14. Prove that $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1+\cos\frac{7\pi}{10}\right)\left(1+\cos\frac{9\pi}{10}\right)=\frac{1}{16}$

A: $\cos\frac{\pi}{10}=\cos\frac{180^\circ}{10}=\cos 18^\circ$; $\cos\frac{3\pi}{10}=\cos\frac{3(180^\circ)}{10}=\cos 54^\circ$

$$\cos\frac{7\pi}{10}=\cos\frac{7(180^\circ)}{10}=\cos 126^\circ=\cos(180^\circ-54^\circ)=-\cos 54^\circ$$

$$\cos\frac{9\pi}{10}=\cos\frac{9(180^\circ)}{10}=\cos 162^\circ=\cos(180^\circ-18^\circ)=-\cos 18^\circ$$

$\therefore$ L.H.S= $(1+\cos 18^\circ)(1+\cos 54^\circ)(1-\cos 54^\circ)(1-\cos 18^\circ)=(1-\cos^2 18^\circ)(1-\cos^2 54^\circ)$

$$=\sin^2 18^\circ\sin^2 54^\circ=\left(\frac{\sqrt{5}-1}{4}\right)^2\left(\frac{\sqrt{5}+1}{4}\right)^2=\left(\frac{5-1}{16}\right)^2=\left(\frac{4}{16}\right)^2=\left(\frac{1}{4}\right)^2=\frac{1}{16}=\text{R.H.S}$$

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15. If $a\cos2\theta+b\sin2\theta=c$ has θ_1,θ_2 as its solutions then show that

(i) $\tan\theta_1+\tan\theta_2=\frac{2b}{c+a}$, (ii) $\tan\theta_1.\tan\theta_2=\frac{c-a}{c+a}$ and hence show that $\tan(\theta_1+\theta_2)=b/a$.

Sol: Given equation is $a\cos2\theta+b\sin2\theta=c \Rightarrow a\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)+b\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)=c$

Upon Cross Multiplying, we get $a(1-\tan^2\theta)+b(2\tan\theta)=c(1+\tan^2\theta)$

$\Rightarrow a-a\tan^2\theta+2b\tan\theta=c+c\tan^2\theta$

$\Rightarrow c\tan^2\theta+a\tan^2\theta-2b\tan\theta+(c-a)=0$

$\Rightarrow (c+a)\tan^2\theta-2b\tan\theta+(c-a)=0$

This is a quadratic equation in $\tan\theta$ with roots $\tan\theta_1, \tan\theta_2$

$\therefore \tan\theta_1+\tan\theta_2=\frac{2b}{c+a}$ [$\because$ Sum of the roots of $ax^2+bx+c=0$ is $\frac{-b}{a}$]

$\tan\theta_1.\tan\theta_2=\frac{c-a}{c+a}$, [$\because$ Product of roots of $ax^2+bx+c=0$ is $\frac{c}{a}$]

$$\therefore \tan(\theta_1+\theta_2)=\frac{\tan\theta_1+\tan\theta_2}{1-\tan\theta_1\tan\theta_2}=\frac{\frac{2b}{c+a}}{1-\left(\frac{c-a}{c+a}\right)}=\frac{\frac{2b}{\cancel{c+a}}}{\frac{\cancel{c}+a-\cancel{c}+a}{\cancel{c+a}}}=\frac{\cancel{2}b}{\cancel{2}a}=\frac{b}{a}$$

Hence proved.

16. Prove that $\text{Sin}^{-1}\frac{3}{5}+\text{Sin}^{-1}\frac{8}{17}=\text{Cos}^{-1}\frac{36}{85}$

A: Take $\text{Sin}^{-1}\frac{3}{5}=\alpha$ and $\text{Sin}^{-1}\frac{8}{17}=\beta$

Required To Prove (RTP): $\alpha+\beta=\text{Cos}^{-1}\frac{36}{85}\Rightarrow\cos(\alpha+\beta)=\frac{36}{85}$

$$\text{Sin}^{-1}\frac{3}{5}=\alpha\Rightarrow\sin\alpha=\frac{3}{5}\Rightarrow\cos\alpha=\frac{4}{5}$$

$$\text{Sin}^{-1}\frac{8}{17}=\beta\Rightarrow\sin\beta=\frac{8}{17}\Rightarrow\cos\beta=\frac{15}{17}$$

$$\therefore\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta=\frac{4}{5}\times\frac{15}{17}-\frac{3}{5}\times\frac{8}{17}=\frac{60-24}{85}=\frac{36}{85}$$

Hence proved.

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17. In ΔABC, if $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$ then show that C=60°

A: Given that $\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$

$$\Rightarrow\frac{b+c+a+c}{(a+c)(b+c)}=\frac{3}{a+b+c}\Rightarrow\frac{(a+b+c)+c}{(a+c)(b+c)}=\frac{3}{(a+b+c)}$$

$$\Rightarrow(a+b+c)^2+c(a+b+c)=3(a+c)(b+c)$$

$$\Rightarrow(a^2+b^2+c^2+2ab+2bc+2ca)+ca+cb+c^2=3ab+3ac+3cb+3c^2$$

$$\Rightarrow a^2+b^2-c^2=ab\Rightarrow\frac{a^2+b^2-c^2}{ab}=1$$

$$\Rightarrow\frac{a^2+b^2-c^2}{2ab}=\frac{1}{2}\Rightarrow\cos C=\frac{1}{2}=\cos 60^\circ\Rightarrow C=60^\circ$$

SECTION-C

18. If $f:A\rightarrow B$ is a bijective function then prove that (i) $fof^{-1}=I_B$ (ii) $f^{-1}of=I_A$

A: (i) **To prove that $fof^{-1}=I_B$**

Part–1: Given $f:A\rightarrow B$ is a bijective function, then $f^{-1}: B\rightarrow A$ is also a bijection

$\therefore fof^{-1}: B\rightarrow B$

We know, $I_B: B\rightarrow B$

So, fof^{-1} and I_B, both have same domain B

Part–2: For $b\in B$, $(fof^{-1})(b)=f[f^{-1}(b)]$

$=f(a)$ [$\because f:A\rightarrow B$ is bijection $\Rightarrow f(a)=b \Rightarrow f^{-1}(b)=a$, for $a\in A$]

$=b=I_B(b)$ [$\because I_B(b)=b$, for $b\in B$]

Hence we proved that $fof^{-1}=I_B$

(ii) **To prove that $f^{-1}of=I_A$**

Part–1: Given $f:A\rightarrow B$ is a bijective function, then $f^{-1}:B\rightarrow A$ is also a bijection

$\therefore f^{-1}of : A\rightarrow A$

We know $I_A: A\rightarrow A$

So, $f^{-1}of$ and I_A, both have same domain A

Part–2: for $a\in A$, $(f^{-1}of)(a)=f^{-1}[f(a)]$

$= f^{-1}(b)= a$ [$\because f:A\rightarrow B$ is a bijection $\Rightarrow f(a)=b \Rightarrow f^{-1}(b)=a$]

$= I_A(a)$[$\because I_A(a)=a$, for $a\in A$]

Hence we proved that $f^{-1}of=I_A$

19. Using Mathematical Induction, prove that statement for all $n \in N$

$$\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\ldots\ldots\left(1+\frac{2n+1}{n^2}\right)=(n+1)^2$$

A: Let S(n): $\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\ldots\ldots\left(1+\frac{2n+1}{n^2}\right)=(n+1)^2$

Step 1: L.H.S of S(1)= $1+\frac{3}{1}=4$, R.H.S.of $S(1)=(1+1)^2=4$

$\therefore$ L.H.S of S(1) = R.H.S of S(1) $\Rightarrow$ S(1) is true

Step-2: Assume that S(k) is true for $k \in N$

$$S(k)=\left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\ldots\ldots\left(1+\frac{2k+1}{k^2}\right)=(k+1)^2 \ldots\ldots\ldots(1)$$

Step-3: We show that S(k+1) is true

Multiplying boths sides of (1) by $1+\frac{2k+3}{(k+1)^2}$ we get

$$\text{L.H.S of S(k+1)} = \left(1+\frac{3}{1}\right)\left(1+\frac{5}{4}\right)\left(1+\frac{7}{9}\right)\ldots\ldots\ldots\left(1+\frac{2k+1}{k^2}\right)\left(1+\frac{2k+3}{(k+1)^2}\right)$$

$$=(k+1)^2\left(1+\frac{2k+3}{(k+1)^2}\right) \text{ From(1)}$$

$$=\cancel{(k+1)^2}\left(\frac{(k+1)^2+2k+3}{\cancel{(k+1)^2}}\right)=(k+1)^2+2k+3=k^2+2k+1+2k+3$$

$$=k^2+4k+4=(k+2)^2$$

=R.H.S of S(k+1)

$\therefore$ L.H.S of S(k+1)=R.H.S of S(k+1) $\Rightarrow$ S(k+1) is true whenever S(k) is true

Hence, by the principle of Mathematical induction, the given statement is true $\forall\ n \in N$

20. Show that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

A: $\text{L.H.S} = \begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$

$$= \begin{vmatrix} 2a+2b+2c & a & b \\ 2a+2b+2c & b+c+2a & b \\ 2a+2b+2c & a & c+a+2b \end{vmatrix} (\because C_1 \to C_1 + C_2 + C_3)$$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 1 & b+c+2a & b \\ 1 & a & c+a+2b \end{vmatrix}$$

$$= 2(a+b+c)\begin{vmatrix} 1 & a & b \\ 0 & a+b+c & 0 \\ 0 & 0 & a+b+c \end{vmatrix} \begin{matrix} (\because R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1) \end{matrix}$$

$$= 2(a+b+c)1[(a+b+c)^2 - 0] = 2(a+b+c)^3 = \text{R.H.S}$$

21. By using Matrix inversion method, solve 3x+4y+5z=18, 2x–y+8z=13, 5x–2y+7z=20.

A : The matrix equation corresponding to the given system of equations be AX=D, where

$$A=\begin{bmatrix}3 & 4 & 5\\ 2 & -1 & 8\\ 5 & -2 & 7\end{bmatrix};\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix};\ D=\begin{bmatrix}18\\ 13\\ 20\end{bmatrix}\qquad \therefore \text{ The solution of } AX=D \text{ is } X=A^{-1}D$$

First, we find A^{-1}.

$$|A|=\begin{vmatrix}3 & 4 & 5\\ 2 & -1 & 8\\ 5 & -2 & 7\end{vmatrix}=3(-7+16)-4(14-40)+5(-4+5)=27+104+5=136\neq 0$$

The cofactor matrix of A is

$$\begin{bmatrix}+\begin{vmatrix}-1 & 8\\ -2 & 7\end{vmatrix} & -\begin{vmatrix}2 & 8\\ 5 & 7\end{vmatrix} & +\begin{vmatrix}2 & -1\\ 5 & -2\end{vmatrix}\\ -\begin{vmatrix}4 & 5\\ -2 & 7\end{vmatrix} & +\begin{vmatrix}3 & 5\\ 5 & 7\end{vmatrix} & -\begin{vmatrix}3 & 4\\ 5 & -2\end{vmatrix}\\ +\begin{vmatrix}4 & 5\\ -1 & 8\end{vmatrix} & -\begin{vmatrix}3 & 5\\ 2 & 8\end{vmatrix} & +\begin{vmatrix}3 & 4\\ 2 & -1\end{vmatrix}\end{bmatrix}=\begin{bmatrix}-7+16 & -(14-40) & -4+5\\ -(28+10) & (21-25) & -(-6-20)\\ (32+5) & -(24-10) & (-3-8)\end{bmatrix}=\begin{bmatrix}9 & 26 & 1\\ -38 & -4 & 26\\ 37 & -14 & -11\end{bmatrix}$$

$$\Rightarrow \text{Adj } A=\begin{bmatrix}9 & -38 & 37\\ 26 & -4 & -14\\ 1 & 26 & -11\end{bmatrix}\qquad \therefore A^{-1}=\frac{1}{|A|}(\text{adj } A)=\frac{1}{136}\begin{bmatrix}9 & -38 & 37\\ 26 & -4 & -14\\ 1 & 26 & -11\end{bmatrix}$$

$$\text{Now, } X=A^{-1}D=\frac{1}{136}\begin{bmatrix}9 & -38 & 37\\ 26 & -4 & -14\\ 1 & 26 & -11\end{bmatrix}\begin{bmatrix}18\\ 13\\ 20\end{bmatrix}=\frac{1}{136}\begin{bmatrix}162-494+740\\ 468-52-280\\ 18+338-220\end{bmatrix}=\frac{1}{136}\begin{bmatrix}408\\ 136\\ 136\end{bmatrix}=\begin{bmatrix}3\\ 1\\ 1\end{bmatrix}$$

$$\therefore X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3\\ 1\\ 1\end{bmatrix}$$

$\therefore$ the solution is x=3, y=1, z=1

22. If A=(1,–2,–1),B=(4,0,–3), C=(1,2,–1), D=(2,–4,–5) then find the distance between $\overline{AB}$ and $\overline{CD}$.

A: Given points A=(1,–2,–1),B=(4,0,–3), C=(1,2,–1), D=(2,–4,–5)

$\overline{OA}=\bar{i}-2\bar{j}-\bar{k}, \overline{OB}=4\bar{i}-3\bar{k}, \overline{OC}=\bar{i}+2\bar{j}-\bar{k}, \overline{OD}=2\bar{i}-4\bar{j}-5\bar{k}$

(i) Vector equation of the line $\overline{AB}$ is $\bar{r}=\bar{a}+t\bar{b}, t\in R$, where

$\bar{a}=\overline{OA}=\bar{i}-2\bar{j}-\bar{k}$ & $\bar{b}=\overrightarrow{AB}=\overline{OB}-\overline{OA}=(4\bar{i}-3\bar{k})-(\bar{i}-2\bar{j}-\bar{k})=3\bar{i}+2\bar{j}-2\bar{k}$

(ii) Vector equation of the line $\overline{CD}$ is $\bar{r}=\bar{c}+s\bar{d}, s\in R$, where

$\bar{c}=\overline{OC}=\bar{i}+2\bar{j}-\bar{k}$ and $\bar{d}=\overline{CD}=\overline{OD}-\overline{OC}=(2\bar{i}-4\bar{j}-5\bar{k})-(\bar{i}+2\bar{j}-\bar{k})=\bar{i}-6\bar{j}-4\bar{k}$

So, $\bar{a}-\bar{c}=(\bar{i}-2\bar{j}-\bar{k})-(\bar{i}+2\bar{j}-\bar{k})=-4\bar{j}$

$$\bar{b}\times\bar{d}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 3 & 2 & -2\\ 1 & -6 & -4\end{vmatrix}=\bar{i}(-8-12)-\bar{j}(-12+2)+\bar{k}(-18-2)=-20\bar{i}+10\bar{j}-20\bar{k}$$

Now, $(\bar{a}-\bar{c}).(\bar{b}\times\bar{d})=(-4\bar{j}).(-20\bar{i}+10\bar{j}-20\bar{k})=-4(10)=-40$

Also, $|\bar{b}\times\bar{d}|=\sqrt{(-20)^2+10^2+(-20)^2}=\sqrt{400+100+400}=\sqrt{900}=30$

$\therefore$ Shortest distance(SD) $=\dfrac{|(\bar{a}-\bar{c}).(\bar{b}\times\bar{d})|}{|\bar{b}\times\bar{d}|}=\dfrac{|-40|}{30}=\dfrac{\cancel{40}}{\cancel{30}}=\dfrac{4}{3}$ units

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23. If A, B, C are angles in a triangle, then prove that

$\cos A+\cos B-\cos C=-1+4\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2}$

A: Given A,B,C are angles of a triangle, then A+B+C=180° $\Rightarrow\dfrac{A+B+C}{2}=90^o\Rightarrow\dfrac{A+B}{2}=90^o-\dfrac{C}{2}$

L.H.S $=(\cos A+\cos B)-\cos C=2\cos\dfrac{A+B}{2}.\cos\dfrac{A-B}{2}-\cos C$ [From cosC+cosD formula]

$=2\cos\left(90^o-\dfrac{C}{2}\right).\cos\dfrac{A-B}{2}-\cos C=2\sin\dfrac{C}{2}.\cos\dfrac{A-B}{2}-\left(1-2\sin^2\dfrac{C}{2}\right)$ $\left[\because\cos\theta=1-2\sin^2\dfrac{\theta}{2}\right]$

$=-1+2\sin\dfrac{C}{2}.\cos\dfrac{A-B}{2}+2\sin^2\dfrac{C}{2}=-1+2\sin\dfrac{C}{2}\left(\cos\dfrac{A-B}{2}+\sin\dfrac{C}{2}\right)$

$=-1+2\sin\dfrac{C}{2}\left[\cos\dfrac{A-B}{2}+\sin\left(90^o-\dfrac{A+B}{2}\right)\right]=-1+2\sin\dfrac{C}{2}\left(\cos\dfrac{A-B}{2}+\cos\dfrac{A+B}{2}\right)$

$=-1+2\sin\dfrac{C}{2}\left(2\cos\dfrac{A}{2}.\cos\dfrac{B}{2}\right)$ $[\because\cos(A+B)+\cos(A-B)=2\cos A\cos B]$

$=-1+4\cos\dfrac{A}{2}.\cos\dfrac{B}{2}.\sin\dfrac{C}{2}=$ R.H.S

24. Show that $\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\frac{a^2+b^2+c^2}{\Delta^2}$

A: $\text{L.H.S}=\frac{1}{r^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\left(\frac{s}{\Delta}\right)^2+\left(\frac{s-a}{\Delta}\right)^2+\left(\frac{s-b}{\Delta}\right)^2+\left(\frac{s-c}{\Delta}\right)^2$

$$=\frac{1}{\Delta^2}\left(s^2+(s-a)^2+(s-b)^2+(s-c)^2\right)=\frac{1}{\Delta^2}(s^2+(s^2-2as+a^2)+(s^2-2bs+b^2)+(s^2-2sc+c^2))$$

$$=\frac{1}{\Delta^2}(4s^2-2s(a+b+c)+a^2+b^2+c^2)=\frac{1}{\Delta^2}(4s^2-2s(2s)+a^2+b^2+c^2)$$

$$=\frac{(a^2+b^2+c^2)}{\Delta^2}=\text{R.H.S}$$
