Previous IPE

SOLVED PAPERS

MARCH-2023 (AP)

PREVIOUS PAPERS

IPE: MARCH-2023(AP)

MATHS-2A

Time : 3 Hours Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: $10 \times 2 = 20$

1. Find a square root of the complex number 7+24i.
2. If $z_1 = -1, z_2 = i$ find $Arg\left(\frac{z_1}{z_2}\right)$
3. If $1, \omega, \omega^2$ are the cube roots of unity, find the value of $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$.
4. Form a quadratic equation, whose roots are $(-3\pm5i)$.
5. If the product of the roots of $4x^3+16x^2-9x-a=0$ is 9 then find a.
6. Find the number of different chains that can be prepared using 6 different coloured beads.
7. If ${}^nC_5 = {}^nC_6$ then find ${}^{13}C_n$
8. Find the middle term (s) in the expansion of $\left(\frac{3x}{7}-2y\right)^{10}$.
9. Find the mean deviation about median for the data 4, 6, 9, 3, 10, 13, 2
10. A poisson variable satisfies P(X=1)=P(X=2). Find P(X=5).

SECTION-B

II. Answer any FIVE of the following SAQs: $5 \times 4 = 20$

11. If $x+iy = \frac{1}{1+\cos\theta+i\sin\theta}$ then show that $4x^2-1=0$
12. Show that $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4, if x is real.
13. If the 6 letters of the word 'PRISON' are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then find the rank of the word "PRISON"
14. P.T $\frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{1.3.5.....(4n-1)}{[1.3.5.....(2n-1)]^2}$
15. Resolve $\frac{x^2-3}{(x+2)(x^2+1)}$ into PF
16. Find the probability that a non-leap year contains i) 53 Sundays ii) 52 Sundays only.
17. A problem in calculus is given to two students A and B whose chances of solving it are 1/3 and 1/4. What is the probability that the problem will be solved if both of them try independently?

SECTION-C

III. Answer any FIVE of the following LAQs: $5 \times 7 = 35$

18. If α,β are the roots of the equation $x^2-2x+4=0$, then show that $\alpha^n+\beta^n = 2^{n+1}\cos\left(\frac{n\pi}{3}\right)$
19. Solve the equation $x^5-5x^4+9x^3-9x^2+5x-1=0$
20. If n is a positive integer & x is any non zero real number, then P.T $C_0+C_1.\frac{x}{2}+C_2.\frac{x^2}{3}+C_3.\frac{x^3}{4}+.....+C_n.\frac{x^n}{n+1} = \frac{(1+x)^{n+1}-1}{(n+1)x}$.
21. If $t = \frac{4}{5}+\frac{4.6}{5.10}+\frac{4.6.8}{5.10.15}+....$ then prove that $9t=16$.
22. Find the variance and standard deviation of the following frequency distribution.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

23. State and Prove Baye's theorem on Probability.
24. A random variable X has the following probability distribution

X=x	0	1	2	3	4	5	6	7
P(X=x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) the mean and (iii) P(0<x<5)

IPE AP MARCH-2023
SOLUTIONS

SECTION-A

1. Find the square root of 7+24i.

Sol: Let $7+24i = a+bi \Rightarrow a=7, b=24$

$$\therefore r = \sqrt{a^2+b^2} = \sqrt{7^2+24^2} = \sqrt{49+576} = \sqrt{625} = 25$$

Formula: $\sqrt{a+ib} = \pm\left(\sqrt{\frac{r+a}{2}} + i\sqrt{\frac{r-a}{2}}\right)$

$$\therefore \sqrt{7+24i} = \pm\left(\sqrt{\frac{25+7}{2}} + i\sqrt{\frac{25-7}{2}}\right) = \pm\left(\sqrt{\frac{32}{2}} + i\sqrt{\frac{18}{2}}\right) = \pm(\sqrt{16}+i\sqrt{9}) = \pm(4+3i)$$

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2. If $z_1 = -1, z_2 = i$, find $Arg\left(\frac{z_1}{z_2}\right)$

Sol: We know that $Arg(-1)=\pi$, $Arg\, i = \frac{\pi}{2}$

$$\therefore Arg\left(\frac{z_1}{z_2}\right) = Argz_1 - Argz_2 = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

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3. If $1,\omega,\omega^2$ are the cube roots of unity, find $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$.

Sol: $G.E = (1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$

$= (1+\omega^2-\omega)^5+(1+\omega-\omega^2)^5 = (-\omega-\omega)^5+(-\omega^2-\omega^2)^5$

$= (-2\omega)^5+(-2\omega^2)^5 = -2^5[\omega^2+\omega] = -32(-1)=32$

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4. Form a quadratic equation, whose roots are $(-3\pm5i)$

Sol: Let $\alpha = -3+5i$ and $\beta = -3-5i$

$\Rightarrow \alpha+\beta = -6$

$\alpha\beta = (-3+5i)(-3-5i) = 3^2+5^2=9+25=34$

The quadratic equation with roots α,β is $x^2-(\alpha+\beta)x+\alpha\beta=0 \Rightarrow x^2+6x+34=0$

5. If the product of the roots of $4x^3+16x^2-9x-a=0$ is 9, then find a.

Sol: From the given equation we get, $a_0=4, a_1=16, a_2=-9, a_3=-a$

Product of the roots is $9 \Rightarrow S_3 = -\frac{a_3}{a_0} = 9 \Rightarrow \frac{a}{4} = 9 \Rightarrow a = 4\times 9 = 36$

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6. Find the number of different chains that can be prepared using 6 different coloured beads.

Sol: Number of circular permuations from n things $= \frac{1}{2}(n-1)!$

Hence the number of chains $= \frac{1}{2}(6-1)! = \frac{1}{2}(5!) = \frac{1}{2}(120) = 60$

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7. If ${}^nC_5 = {}^nC_6$, then find ${}^{13}C_n$

Sol : Formula: ${}^nC_r = {}^nC_s \Rightarrow r+s=n$ (or) $r=s$

$\therefore {}^nC_5 = {}^nC_6 \Rightarrow n = 5+6 = 11$

$\therefore {}^{13}C_n = {}^{13}C_{11} = {}^{13}C_{13-11} = {}^{13}C_2 = \frac{13\times 12}{1\times 2} = 13\times 6 = 78$

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8. Find the middle term (s) in the expansion of $\left(\frac{3x}{7} - 2y\right)^{10}$

Sol: The Binomial exponent n=10 is even

$\therefore$ the middle term is $T_{\frac{10}{2}+1} = T_{5+1} = T_6$

$\therefore T_6 = T_{5+1} = {}^{10}C_5\left(\frac{3x}{7}\right)^{10-5}(-2y)^5 = -{}^{10}C_5\left(\frac{3}{7}\right)^5.x^5.2^5.y^5 = -{}^{10}C_5\left(\frac{6}{7}\right)^5.x^5.y^5$

9. Find the mean deviation about median for the data 4, 6, 9, 3, 10, 13, 2

Sol: **Given data:** 4, 6, 9, 3, 10, 13, 2.

Its ascending order : 2,3,4,<u>6</u>,9,10,13.

Number of observations n = 7 is odd .

$\therefore$ Median is the middle most term $\Rightarrow M=6$

Deviations from the median:

2–6= –4; 3–6= –3; 4–6= –2; 6–6=0;

9–6=3;10–6=4; 13–6=7

Absolute values of these deviations:

4, 3, 2, 0, 3, 4, 7

$\therefore$ M.D from Median is $M.D=\frac{\Sigma|x_i-M|}{7}=\frac{4+3+2+0+3+4+7}{7}=\frac{23}{7}=3.29$

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10. A Poisson variable satisfies P(X=1)=P(X=2), find P(X=5)

Sol: We have $P(X=r)=\frac{e^{-\lambda}\lambda^r}{r!}, \lambda>0$

Given that P(X=1)=P(X=2)

$\Rightarrow \frac{\lambda e^{-\lambda}}{1!}=\frac{\lambda^2 e^{-\lambda}}{2!} \Rightarrow \frac{\lambda}{1}=\frac{\lambda^2}{2} \Rightarrow \lambda^2=2\lambda$

$\Rightarrow \lambda(\lambda-2)=0 \Rightarrow \lambda=2 \ (\because \lambda>0)$

$\therefore P(X=5)=\frac{e^{-2}2^5}{5!}$

SECTION-B

11. If $x+iy=\dfrac{1}{1+\cos\theta+i\sin\theta}$ then, show that $4x^2-1=0$

Sol: Given that $x+iy=\dfrac{1}{(1+\cos\theta)+i\sin\theta}=\dfrac{1}{(2\cos^2\frac{\theta}{2})+i(2\sin\frac{\theta}{2}\cos\frac{\theta}{2})}$

$$\Rightarrow x+iy=\frac{1}{(2\cos\frac{\theta}{2})(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})}=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{(2\cos\frac{\theta}{2})(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2})}$$

$$=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\left(2\cos\frac{\theta}{2}\right)\left(\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\right)}=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\left(2\cos\frac{\theta}{2}\right)(1)}=\frac{\cos\frac{\theta}{2}}{2\cos\frac{\theta}{2}}-\frac{i\sin\frac{\theta}{2}}{2\cos\frac{\theta}{2}}=\frac{1}{2}-\frac{1}{2}i\tan\frac{\theta}{2}$$

Equating the real parts, we get $x=\frac{1}{2}\Rightarrow 2x=1\Rightarrow(2x)^2=1^2\Rightarrow 4x^2=1\Rightarrow 4x^2-1=0$

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12. Show that $\dfrac{1}{3x+1}+\dfrac{1}{x+1}-\dfrac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4, if x is real.

Sol: $G.E=\dfrac{1}{3x+1}+\dfrac{1}{x+1}-\dfrac{1}{(3x+1)(x+1)}=\dfrac{x+1+3x+1-1}{(3x+1)(x+1)}=\dfrac{4x+1}{3x^2+4x+1}$

Let $y=\dfrac{4x+1}{3x^2+4x+1}\Rightarrow y(3x^2+4x+1)=4x+1$

$\Rightarrow 3yx^2+4yx+y=4x+1$

$\Rightarrow 3yx^2+(4y-4)x+(y-1)=0$............(1)

(1) is a quadratic in x and its roots are reals. $\therefore\ \Delta=b^2-4ac\ge 0$

$(4y-4)^2-4(3y)(y-1)\ge 0$

$\Rightarrow 16y^2+16-32y-12y^2+12y\ge 0\Rightarrow 4y^2-20y+16\ge 0$

$\Rightarrow 4(y^2-5y+4)\ge 0 \quad \Rightarrow y^2-5y+4\ge 0 \quad \Rightarrow (y-1)(y-4)\ge 0 \quad \Rightarrow y\le 1$ or $y\ge 4$

$\Rightarrow$ y does not lie between 1 and 4

Hence the given expression does not lie between 1 and 4.

13. If the letters of the word PRISON are permutted in all possible ways and the words thus formed are arranged in dictionary order, then find the rank of the word PRISON.

Sol: The alphabetical order of the letters of the word PRISON is

I,N,O,P,R,S

The number of words that begin with I – – – – – = 5! = 120

The number of words that begin with N – – – – – = 5! = 120

The number of words that begin with O – – – – – = 5! = 120

The number of words that begin with P I – – – – – = 4! = 24

The number of words that begin with P N – – – – – = 4! = 24

The number of words that begin with P O – – – – – = 4! = 24

The number of words that begin with PRIN – – – = 2! = 2

The number of words that begin with PRIO – – – = 2! = 2

The number of words that begin with PRISN – – = 1! = 1

The next word is PRISON = 1! = 1

$\therefore$ Rank of the word PRISON = 3(120) + 3(24)+2(2)+1 + 1= 360 + 72 + 4 + 1+1 =438.

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14. Prove that $\dfrac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \dfrac{1.3.5.....(4n-1)}{[1.3.5.....(2n-1)]^2}$

Sol: $$\text{L.H.S} = \frac{{}^{4n}C_{2n}}{{}^{2n}C_n} = \frac{\frac{4n!}{2n!.2n!}}{\frac{2n!}{n!.n!}} = \frac{(4n)!}{(2n!)^2}\times\frac{(n!)^2}{(2n)!} \qquad \left[\text{Since } {}^nC_r = \frac{n!}{r!(n-r)!}\right]$$

$$= \frac{(4n)(4n-1)(4n-2)(4n-3)(4n-4)..........6.5.4.3.2.1}{[(2n)(2n-1)(2n-2)(2n-3).........4.3.2.1]^2}\times\frac{(n!)^2}{(2n)!}$$

$$= \frac{[(4n)(4n-2)(4n-4)..........(6)(4)(2)][(4n-1)(4n-3).....5.3.1]}{[(2n)(2n-2)......4.2]^2[(2n-1)(2n-3).......(3)(1)]^2}\times\frac{(n!)^2}{(2n)!}$$

$$= \frac{[2^{2n}(2n)(2n-1)(2n-2).......(3)(2)(1)][(4n-1)(4n-3).....5.3.1]}{[2^n(n)(n-1)........(2)(1)]^2[(2n-1)(2n-3).......(3)(1)]^2}\times\frac{(n!)^2}{(2n)!}$$

$$= \frac{[2^{2n}(2n!)][(4n-1)(4n-3).....5.3.1]}{2^{2n}(n!)^2[(2n-1)(2n-3).......(3)(1)]^2}\times\frac{(n!)^2}{(2n)!}\;\frac{1.3.5......(4n-3)(4n-1)}{[1.3.5......(2n-3)(2n-1)]^2} = \text{R.H.S}$$

15. Resolve $\frac{x^2-3}{(x+2)(x^2+1)}$ into partial fractions.

Sol: Let $\frac{x^2-3}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$

$\Rightarrow A(x^2+1)+(Bx+C)(x+2) = x^2-3$ ....(1)

Putting $x=-2$ in (1) we get $A(4+1)+(Bx+C)(0)=4-3 \Rightarrow 5A=1 \Rightarrow A=1/5$

Putting $x=0$ in (1) we get $A+2C=-3 \Rightarrow C=-8/5$

Comparing the coefficients of x^2, we get $A+B=1 \Rightarrow B=1-A=1-\frac{1}{5}=\frac{4}{5}$

$\therefore \frac{x^2-3}{(x+2)(x^2+1)} = \frac{1}{5(x+2)} + \frac{4x-8}{5(x^2+1)}$

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16. Find the probability that a non-leap year contains i) 53 Sundays ii) 52 Sundays only.

Sol: i) Let E be the event of containing 53 Sundays in a non-leap year and S be the sample space.

A non-leap year contains 365 days i.e., 52 weeks and 1 day extra.

The extra day may be any one of Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday. Thus, $n(S)=7, n(E)=1$

$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{7}$

(ii) If the remaining day is a non-Sunday, then we get 52 Sundays $\therefore P(\bar{E}) = 1-\frac{1}{7} = \frac{6}{7}$

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17. A problem in calculus is given to two students A and B whose chances of solving it are 1/3, 1/4 respectively. Find the probability of the problem being solved if both of them try independently.

Sol: Let A,B denote the events of solving the problem by A, B respectively $\Rightarrow P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$

$\therefore P(\bar{A}) = 1-P(A) = 1-\frac{1}{3} = \frac{2}{3}$; $P(\bar{B}) = 1-P(B) = 1-\frac{1}{4} = \frac{3}{4}$

$\therefore P(A \cup B) = 1-P(\bar{A} \cap \bar{B}) = 1-P(\bar{A}).P(\bar{B}) = 1-\left(\frac{2}{\cancel{3}}\right)\left(\frac{\cancel{3}}{4}\right) = 1-\frac{1}{2} = \frac{1}{2}$

SECTION-C

18. If α,β are roots of the equation $x^2-2x+4=0$, then show that $\alpha^n+\beta^n=2^{n+1}\cos\left(\frac{n\pi}{3}\right)$

Sol: Given $x^2-2x+4=0 \Rightarrow x=\frac{2\pm\sqrt{4-4(1)(4)}}{2(1)}=\frac{2\pm\sqrt{-12}}{2}=\frac{2\pm 2i\sqrt{3}}{2}=1\pm i\sqrt{3}$

Now, we find the mod-amp form of $1+i\sqrt{3}$

Let $x+iy=1+i\sqrt{3} \Rightarrow x=1,\ y=\sqrt{3}$

$\therefore\ r=\sqrt{x^2+y^2}=\sqrt{1^2+(\sqrt{3})^2}=\sqrt{4}=2.$

Also, $\tan\theta=\frac{y}{x}=\frac{\sqrt{3}}{1}=\tan\frac{\pi}{3}\Rightarrow\theta=\frac{\pi}{3}$

$\therefore$ mod– Amp form of $1+i\sqrt{3}$ is $r(\cos\theta+i\sin\theta)=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

$\Rightarrow\ (1+i\sqrt{3})^n=\left(2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right)^n$

$=\ (2)^n\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^n=2^n\left(\cos n\frac{\pi}{3}+i\sin n\frac{\pi}{3}\right)$ ..... (1) (by Demoivre's theorem)

Similarly, $(1-i\sqrt{3})^n=2^n\left(\cos n\frac{\pi}{3}-i\sin n\frac{\pi}{3}\right)$ ..... (2)

Adding (1) & (2), we get $\alpha^n+\beta^n=(1+i\sqrt{3})^n+(1-i\sqrt{3})^n$

$=2^n\left(\left(\cos n\frac{\pi}{3}+\cancel{i\sin n\frac{\pi}{3}}\right)+\left(\cos n\frac{\pi}{3}-\cancel{i\sin n\frac{\pi}{3}}\right)\right)=2^n\left(2\cos\frac{n\pi}{3}\right)=2^{n+1}.\cos\frac{n\pi}{3}$

19. Solve the equation $x^5-5x^4+9x^3-9x^2+5x-1=0$

Sol: The degree of the given equation is n=5, which is odd. Also $a_k=-a_{n-k}\ \forall\ k=0,1,2,3,4,5$

Hence the given equation is a reciprocal equation of class II of odd degree

$\therefore$ 1 is a root of $x^5-5x^4+9x^3-9x^2+5x-1=0$

On dividing the expression by (x–1), we have

1	1	–5	9	–9	5	–1
	0	1	–4	5	–4	1
	1	–4	5	–4	1	0

Now, we solve the S.R.E $x^4-4x^3+5x^2-4x+1=0$.

On dividing this equation by x^2, we get

$$x^2-4x+5-4\frac{1}{x}+\frac{1}{x^2}=0\Rightarrow\left(x^2+\frac{1}{x^2}\right)-4\left(x+\frac{1}{x}\right)+5=0\ ......(1)$$

Put $x+\frac{1}{x}=y$, so that $x^2+\frac{1}{x^2}=\left(x+\frac{1}{x}\right)^2-2=y^2-2$

$\therefore$ (1) $\Rightarrow (y^2-2)-4y+5=0\Rightarrow y^2-4y+3=0$

$y^2-4y+3=0\Leftrightarrow(y-3)(y-1)=0\Leftrightarrow y=3$ or 1.

If y=3 then $x+\frac{1}{x}=3\Rightarrow\frac{x^2+1}{x}=3\Rightarrow x^2+1=3x\Rightarrow x^2-3x+1=0$

$$\Rightarrow x=\frac{3\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}=\frac{3\pm\sqrt{9-4}}{2}=\frac{3\pm\sqrt{5}}{2}$$

If y=1 then $x+\frac{1}{x}=1\Rightarrow\frac{x^2+1}{x}=1\Rightarrow x^2+1=x\Rightarrow x^2-x+1=0$

$$\Rightarrow x=\frac{1\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}=\frac{1\pm\sqrt{1-4}}{2}=\frac{1\pm\sqrt{3}}{2}=\frac{1\pm i\sqrt{3}}{2}$$

Hence all the five roots of the given equation are 1, $\frac{3\pm\sqrt{5}}{2}$ and $\frac{1\pm i\sqrt{3}}{2}$

20. If n is a positive integer and x is any nonzero real number, then prove that

$$C_0 + C_1\frac{x}{2} + C_2\frac{x^2}{3} + C_3\frac{x^3}{4} + \ldots\ldots + C_n\frac{x^n}{n+1} = \frac{(1+x)^{n+1}-1}{(n+1)x}$$

Also deduce that (i) $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots.\frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$

Sol: Let $S = C_0 + C_1.\frac{x}{2} + C_2.\frac{x^2}{3} + \ldots. + C_n.\frac{x^n}{n+1} = {}^nC_0 + {}^nC_1\frac{x}{2} + {}^nC_2\frac{x^2}{3} + \ldots\ldots + {}^nC_n.\frac{x^n}{n+1}$

$$\Rightarrow x.S = {}^nC_0.x + {}^nC_1\frac{x^2}{2} + {}^nC_2.\frac{x^3}{3} + \ldots. + {}^nC_n.\frac{x^{n+1}}{n+1}$$

$$\Rightarrow (n+1)xS = \frac{n+1}{1}\cdot {}^nC_0.x + \frac{n+1}{2}.{}^nC_1.x^2 + \frac{n+1}{3}.{}^nC_2x^3 + \ldots. + \frac{n+1}{n+1}\,{}^nC_n.x^{n+1}$$

$$= {}^{n+1}C_1.x + {}^{n+1}C_2x^2 + {}^{n+1}C_3x^3 + \ldots. + {}^{n+1}C_{n+1}.x^{n+1} \qquad \left(\because \left(\frac{n+1}{r+1}\right).{}^nC_r = {}^{(n+1)}C_{r+1}\right)$$

$$\Rightarrow (n+1)xS = (1+x)^{n+1} - 1 \qquad (\because {}^nC_1x + {}^nC_2x^2 + \ldots.. + {}^nC_nx^n = (1+x)^n - 1)$$

$$\therefore S = \frac{(1+x)^{n+1}-1}{(n+1).x}$$

Corollary: **Prove that** $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots.\frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$

Proof: $S = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \ldots.\frac{C_n}{n+1}$

$$\Rightarrow S = {}^nC_0 + \frac{1}{2}.{}^nC_1 + \frac{1}{3}.{}^nC_2 + \ldots.. + \frac{1}{n+1}.C_n$$

$$\Rightarrow (n+1)S = \frac{n+1}{1}.\,{}^nC_0 + \frac{n+1}{2}.{}^nC_1 + \frac{n+1}{3}.{}^nC_2 + \ldots. + \frac{n+1}{n+1}\,{}^nC_n$$

$$= {}^{n+1}C_1 + {}^{n+1}C_2 + {}^{n+1}C_3 + \ldots. + {}^{n+1}C_{n+1} \qquad \left(\text{Since } \frac{n+1}{r+1}.{}^nC_r = {}^{n+1}C_{r+1}\right)$$

$$= 2^{n+1} - 1 \qquad (\because {}^nC_1 + {}^nC_2 + \ldots\ldots + {}^nC_n = 2^n - 1)$$

$$\therefore S = \frac{2^{n+1}-1}{n+1}$$

21. If $t=\frac{4}{5}+\frac{4.6}{5.10}+\frac{4.6.8}{5.10.15}+....$ then prove that $9t=16$.

Sol: Given that $t=\frac{4}{5}+\frac{4.6}{5.10}+\frac{4.6.8}{5.10.15}+....$

Adding 1 on both sides, we get

$$1+t=1+\frac{4}{1!}\left(\frac{1}{5}\right)+\frac{4.6}{2!}\left(\frac{1}{5}\right)^2+\frac{4.6.8}{3!}\left(\frac{1}{5}\right)^3+......$$

Comparing the above series with $1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+.......=(1-x)^{-p/q}$

we get $p=4, p+q=6 \Rightarrow 4+q=6 \Rightarrow q=2$. Also $\frac{x}{q}=\frac{1}{5} \Rightarrow x=\frac{q}{5}=\frac{2}{5}$

$$\therefore 1+t=(1-x)^{-p/q}=(1-\frac{2}{5})^{-4/2}=\left(\frac{3}{5}\right)^{-2}=\left(\frac{5}{3}\right)^2=\frac{25}{9}$$

$$\Rightarrow 1+t=\frac{25}{9} \Rightarrow 9(1+t)=25 \Rightarrow 9+9t=25 \Rightarrow 9t=16$$

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22. Find the variance and standard deviation of the following frequency distribution.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Sol: Here $N=\Sigma f_i=3+5+9+5+4+3+1=30$

Also $\Sigma f_i x_i=4(3)+8(5)+11(9)+17(5)+20(4)+24(3)+32(1)=420$ $\quad\therefore \bar{x}=\frac{\Sigma f_i x_i}{N}=\frac{420}{30}=14$

x_i	f_i	$f_i x_i$	$x_i-\bar{x}$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
4	3	12	–10	100	300
8	5	40	–6	36	180
11	9	99	–3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
		$\Sigma f_i x_i=420$			$\Sigma f_i(x_i-\bar{x})^2=1374$

Variance $(\sigma^2)=\frac{1}{N}\Sigma f_i(x_i-\bar{x})^2=\frac{1}{30}(1374)=45.8$. $\quad$ Standard Deviation $\sigma=\sqrt{45.8}=6.77$

23. State and Prove Baye's theorem on Probability.

Sol: ***Statement***: If $E_1, E_2 ... E_n$ are mutually exclusive and exhaustive events in a sample space S and A is any event intersecting with any E_i such that $P(A) \neq 0$ then $P(E_k \mid A) = \dfrac{P(E_k)P(A/E_k)}{\sum_{i=1}^{n} P(Ei).P(A/E_i)}$

Proof: From the definition of conditional probability: $P(E_k \mid A) = \dfrac{P(E_k \cap A)}{P(A)} = \dfrac{P(E_k).P(A \mid E_k)}{P(A)}$ ...(1)

Given that $E_1, E_2 .... E_n$ are mutually exclusive and exhaustive events in a sample space S

$\Rightarrow \bigcup_{i=1}^{n} E_i = S$ and $A \cap E_1, A \cap E_2, ... A \cap E_n$ are mutually disjoint $\Rightarrow A \cap E_i = \phi$

Now, $P(A) = P(S \cap A) = P\left(\left(\bigcup_{i=1}^{n} E_i\right) \cap A\right) = P\left(\bigcup_{i=1}^{n} (E_i \cap A)\right) = \sum_{i=1}^{n} P(E_i \cap A) = \sum_{i=1}^{n} P(E_i)P(A \mid E_i)$

$\therefore$ From (1), $P(E_k \mid A) = \dfrac{P(E_k)P(A \mid E_k)}{\sum_{i=1}^{n} P(E_i)P(A \mid E_i)}$

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24. A random variable x has the following probability distribution

$X=x_i$	0	1	2	3	4	5	6	7
$P(X=x_i)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Find (i) k (ii) the mean (iii) P(0<X<5)

Sol: We know $\Sigma P(X=x_i)=1$

$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1 \Rightarrow 10k^2+9k=1 \Rightarrow 10k^2+9k-1=0$

$\Rightarrow 10k^2+10k-k-1=0 \Rightarrow 10k(k+1)-1(k+1)=0 \Rightarrow (10k-1)(k+1)=0 \Rightarrow k=1/10$, (since $k>0$)

(i) $k=1/10$

(ii) Mean $\mu = \sum_{i=1}^{n} x_i.P(X=x_i) = 0(0)+1(k)+2(2k)+3(2k)+4(3k)+5(k^2)+6(2k^2)+7(7k^2+k)$

$=0+k+4k+6k+12k+5k^2+12k^2+49k^2+7k=66k^2+30k$

$= 66\left(\frac{1}{100}\right) + 30\left(\frac{1}{10}\right) = 0.66 + 3 = 3.66$

(iii) $P(0<X<5)=P(X=1)+P(X=2)+P(X=3)+P(X=4) = k+2k+2k+3k=8k = 8\left(\frac{1}{10}\right) = \frac{8}{10} = \frac{4}{5}$