Previous IPE

SOLVED PAPERS

MARCH -2020 (AP)

PREVIOUS PAPERS

IPE: MARCH-2020(AP)

Time : 3 Hours **MATHS-2A** Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **10 × 2 = 20**

1. Find the complex conjugate of (3+4i)(2–3i)
2. Write $z = -\sqrt{3} + i$ in the modulus- amplitude form.
3. If α,β are the roots of the equation $x^2+x+1=0$, then prove that $\alpha^4+\beta^4+\alpha^{-1}\beta^{-1}=0$
4. Find the quadratic equation, the sum of whose roots is 7 and the sum of the squares of the roots is 25.
5. If the product of the roots of $4x^3+16x^2-9x-a=0$ is 9, then find a.
6. If ${}^nP_7 = 42.\,{}^nP_5$ then find n.
7. Find the number of positive divisors of 1080.
8. If ${}^{22}C_r$ is the largest binomial coefficient in the expansion of $(1+x)^{22}$, find the value of ${}^{13}C_r$.
9. Find the mean deviation about mean for the data 3,6,10,4,9,10.
10. For a binomial distribution with mean 6 and variance 2. Find the first two terms of the distribution.

SECTION-B

II. Answer any FIVE of the following SAQs: **5 × 4 = 20**

11. Show that the points in the Argand plane represented by the complex numbers $-2+7i, \frac{-3}{2}+\frac{1}{2}i, 4-3i, \frac{7}{2}(1+i)$ are the vertices of a rhombus.
12. Find the range of $\frac{x^2+x+1}{x^2-x+1}$
13. If the letters of the word MASTER are permutted in all possible ways then find rank of MASTER.
14. Prove that for $3 \le r \le n;\ {}^{(n-3)}C_r + 3.{}^{(n-3)}C_{r-1} + 3.{}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3} = {}^nC_r$
15. Resolve $\frac{x^2-3}{(x+2)(x^2+1)}$ into partial fractions.
16. The probability for a contractor to get a road contract is 2/3 and to get a building contract is 5/9. The probability to get atleast one contract is 4/5. Find the probability that he gets both the contracts.
17. (a) Define conditional probability. (b) If A and B are independent events with P(A) = 0.2, P(B) = 0.5, find (i) P(A/B) (ii) P(B/A) (iii) $P(A\cap B)$

SECTION-C

III. Answer any FIVE of the following LAQs: **5 × 7 = 35**

18. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$, then show that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=3/2=\sin^2\alpha+\sin^2\beta+\sin^2\gamma$.
19. Solve $4x^3-24x^2+23x+18=0$, given that the roots of this equation are in arithmetic progression.
20. If I, n are positive integers, $0<f<1$ and if $(7+4\sqrt{3})^n = I+f$, then S.T (i) I is an odd integer (ii) $(I+f)(1-f)=1$
21. Find the sum of the infinite series $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+....\infty$
22. Find the variance and standard deviation of the following frequency distribution.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

23. State and Prove Baye's theorem on Probability.
24. A random variable X has its range{0,1,2} and the probabilities are $P(X=0)=3c^3$, $P(X=1)=4c-10c^2$, $P(X=2)=5c-1$ where 'c' is a constant, find (i) c (ii) $P(0<x<3)$ (iii) $P(1<x\le2)$ (iv) $P(x<1)$

IPE AP MARCH-2020
SOLUTIONS

SECTION-A

1. Write the conjugate of (3+4i)(2–3i)

Sol: $(3+4i)(2-3i) = 3(2)-3(3i)+4i(2)-4i(3i) = 6-9i+8i+12$ $[\because i^2=-1]$

$= 18-i$

$\therefore$ the conjugate of $18-i$ is $18+i$

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2. Write $z=-\sqrt{3}+i$ in modulus-amplitude form.

Sol: Let $-\sqrt{3}+i = x+iy$. $\Rightarrow x=-\sqrt{3}, y=1$

$\therefore r=\sqrt{x^2+y^2}=\sqrt{(-\sqrt{3})^2+(1)^2}=\sqrt{3+1}=\sqrt{4}=2$

Now $\theta=\mathrm{Tan}^{-1}\left(\frac{y}{x}\right)=\mathrm{Tan}^{-1}\left(-\frac{1}{\sqrt{3}}\right)=\frac{5\pi}{6}[\because(-\sqrt{3},1)\in Q_2]$

$\therefore$ Modulus-amplitude form is $r(\cos\theta+i\sin\theta)=2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$]

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3. If α,β are the roots of the equation $x^2+x+1=0$, then prove that $\alpha^4+\beta^4+\alpha^{-1}\beta^{-1}=0$

Sol: Since α,β are the complex cube roots of unity, we may take $\alpha=\omega$, $\beta=\omega^2$.

$\therefore \alpha^4+\beta^4+\alpha^{-1}\beta^{-1}=\omega^4+\omega^8+\omega^{-1}.\omega^{-2}$

$=\omega^3.\omega+(\omega^3)^2\omega^2+\frac{1}{\omega^3}=\omega+\omega^2+1=0$

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4. Find the quadratic equation, the sum of whose roots is 7 and the sum of the squares of the roots is 25.

Sol: Let α,β be the roots of the required equation.

Given that $\alpha+\beta=7$ and $\alpha^2+\beta^2=25$

Now, $\alpha+\beta=7\Rightarrow(\alpha+\beta)^2=7^2 \Rightarrow(\alpha^2+\beta^2)+2\alpha\beta=7^2\Rightarrow 25+2\alpha\beta=49$

$\Rightarrow 2\alpha\beta=24\Rightarrow\alpha\beta=12$

$\therefore$ The quadratic equation with roots α,β is $x^2-(\alpha+\beta)x+\alpha\beta=0$

$\Rightarrow x^2-7(x)+12=0\Rightarrow x^2-7x+12=0$

5. If the product of the roots of $4x^3+16x^2-9x-a=0$ is 9, then find a.

Sol: From the given equation we get, $a_0=4, a_1=16, a_2=-9, a_3=-a$

Product of the roots is $9 \Rightarrow S_3=-\frac{a_3}{a_0}=9 \Rightarrow \frac{a}{4}=9 \Rightarrow a=4\times 9=36$

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6. If ${}^nP_7 = 42.\ {}^nP_5$ then find n.

Sol: Given that ${}^nP_7 = 42.\ {}^nP_5 \Rightarrow \frac{{}^nP_7}{{}^nP_5}=\frac{42}{1}$

$$\Rightarrow \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)}{n(n-1)(n-2)(n-3)(n-4)}=\frac{42}{1}$$

$\Rightarrow (n-5)(n-6)=42 \Rightarrow (n-5)(n-6)=7\times 6$

$\Rightarrow n-5=7 \Rightarrow n=12$

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7. Find the number of positive divisors of 1080

Sol : $1080=108\times 10=(2\times 54)\times(2\times 5)$

$=(2\times 2\times 27)\times(2\times 5)$

$=(2\times 2\times 3\times 9)\times(2\times 5)=2^3\times 3^3\times 5^1$

$\therefore$ the number of positive divisors

$=(3+1)(3+1)(1+1)=4\times 4\times 2=32.$

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8. If ${}^{22}C_r$ is the largest binomial coefficient in the expansion of $(1+x)^{22}$, find the value of ${}^{13}C_r$.

Sol : The Binomial exponent n=22 is even.

$\therefore$ the largest binomial coefficient is ${}^nC_{\frac{n}{2}}={}^{22}C_{\frac{22}{2}}={}^{22}C_{11}$

Now, ${}^{22}C_r={}^{22}C_{11} \Rightarrow r=11$

$\therefore {}^{13}C_r={}^{13}C_{11}={}^{13}C_2=\frac{13\times 12}{2\times 1}=78$

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9. Find the mean deviation about mean for the data 3,6,10,4,9,10.

Sol: **Given data:** 3,6,10,4,9,10. Here n= 6

Mean $\bar{x}=\frac{3+6+10+4+9+10}{6}=\frac{42}{6}=7$

Deviations from the mean:

3–7 = –4; 6–7 = –1; 10–7=3; 4 –7= –3; 9–7=2; 10–7=3

Absolute values of these deviations:

4, 1, 3, 3, 2 3

$\therefore$ M.D from Mean is $\text{M.D.} = \frac{\Sigma|x_i - \bar{x}|}{6} = \frac{4+1+3+3+2+3}{6} = \frac{16}{6} = 2.67$

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10. For a binomial distribution with mean 6 and variance 2. Find the first two terms of the distribution.

Sol: Given mean np=6, variance npq=2

$\therefore (np)q = 2 \Rightarrow 6(q) = 2 \Rightarrow q = \frac{2}{6} = \frac{1}{3} \Rightarrow p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$

Take $np = 6 \Rightarrow n.\frac{2}{3} = 6 \Rightarrow n = \frac{18}{2} = 9$ $\quad \therefore n = 9, q = \frac{1}{3}$ and $p = \frac{2}{3}$

(i) $P(X=0) = {}^9C_0\left(\frac{1}{3}\right)^9 = \frac{1}{3^9}$;

(ii) $P(X=1) = {}^9C_1\left(\frac{1}{3}\right)^8\left(\frac{2}{3}\right) = \frac{2}{3^7}$

SECTION-B

11. Show that the points in the Argand plane represented by the complex numbers $-2+7i, \frac{-3}{2}+\frac{1}{2}i, 4-3i, \frac{7}{2}(1+i)$ are the vertices of a rhombus.

Sol: Given complex numbers are taken as $A(-2,7), B\left(-\frac{3}{2},\frac{1}{2}\right), C(4,-3); D\left(\frac{7}{2},\frac{7}{2}\right)$

$$AB=\sqrt{\left(-2+\frac{3}{2}\right)^2+\left(7-\frac{1}{2}\right)^2}=\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{13}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{169}{4}}=\frac{\sqrt{170}}{2}$$

$$BC=\sqrt{\left(4+\frac{3}{2}\right)^2+\left(-3-\frac{1}{2}\right)^2}=\sqrt{\left(\frac{11}{2}\right)^2+\left(\frac{-7}{2}\right)^2}=\sqrt{\frac{121}{4}+\frac{49}{4}}=\frac{\sqrt{170}}{2}$$

$$CD=\sqrt{\left(4-\frac{7}{2}\right)^2+\left(-3-\frac{7}{2}\right)^2}=\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{-13}{2}\right)^2}=\sqrt{\frac{1}{4}+\frac{169}{4}}=\frac{\sqrt{170}}{2}$$

$$DA=\sqrt{\left(\frac{7}{2}+2\right)^2+\left(\frac{7}{2}-7\right)^2}=\sqrt{\left(\frac{11}{2}\right)^2+\left(\frac{-7}{2}\right)^2}=\sqrt{\frac{121}{4}+\frac{49}{4}}=\frac{\sqrt{170}}{2}$$

$$AC=\sqrt{(4+2)^2+(-3-7)^2}=\sqrt{6^2+(-10)^2}=\sqrt{36+100}=\sqrt{136}$$

$$BD=\sqrt{\left(\frac{7}{2}+\frac{3}{2}\right)^2+\left(\frac{7}{2}-\frac{1}{2}\right)^2}=\sqrt{\left(\frac{10}{2}\right)^2+\left(\frac{6}{2}\right)^2}=\sqrt{\frac{100}{4}+\frac{36}{4}}=\frac{\sqrt{136}}{2}$$

Hence, the four sides AB, BC, CD, DA are equal.

The two diagonals AC, BD are unequal.

$\therefore$ A, B, C, D form a Rhombus.

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12. Find the range of $\frac{x^2+x+1}{x^2-x+1}$ for $x \in R$.

Sol: Let $y=\frac{x^2+x+1}{x^2-x+1}$

$\Rightarrow y(x^2-x+1)=x^2+x+1$

$\Rightarrow yx^2-yx+y=x^2+x+1$

$\Rightarrow yx^2-x^2-yx-x+y-1=0$

$\Rightarrow x^2(y-1)-x(y+1)+(y-1)=0$

$\Rightarrow (y-1)x^2-(y+1)x+(y-1)=0$ ....(1)

(1) is a quadratic in x and its roots are reals.

$\therefore \Delta = b^2-4ac \geq 0$

$\Rightarrow (y+1)^2-4(y-1)^2 \geq 0$

$\Rightarrow (y+1)^2-(2y-2)^2 \geq 0$

$\Rightarrow (y+1+2y-2)(y+1)-(2y-2) \geq 0$ $\quad [\because a^2-b^2=(a+b)(a-b)]$

$\Rightarrow (3y-1)(3-y) \geq 0 \Rightarrow (3y-1)(y-3) \leq 0$

$\Rightarrow y \in \left[\frac{1}{3}, 3\right]$ $\quad \therefore$ Range$=\left[\frac{1}{3}, 3\right]$

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13. If the letters of the word MASTER are permuted in all possible ways and the words thus formed are arranged in dictionary order, then find the rank of the word MASTER.

Sol: The alphabetical order of the letters of the word MASTER is

A,E,M,R,S,T

The number of words that begin with A – – – – – = 5! = 120

The number of words that begin with E – – – – – = 5! = 120

The number of words that begin with MAE – – – = 3! = 6

The number of words that begin with MAR – – – = 3! = 6

The number of words that begin with MASE – – = 2! = 2

The number of words that begin with MASR – – = 2! = 2

The next word is MASTER = 1! = 1

$\therefore$ Rank of the word MASTER = 2(120) + 2(6) + 2(2) + 1

= 240 + 12 + 4 + 1 = 257

14. Prove that for $3 \le r \le n$; ${}^{(n-3)}C_r + 3.{}^{(n-3)}C_{r-1} + 3.{}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3} = {}^{n}C_r$

Sol: We know that ${}^{n}C_r + {}^{n}C_{r-1} = {}^{(n+1)}C_r$

$$L.H.S = {}^{(n-3)}C_r + 3.{}^{(n-3)}C_{r-1} + 3.{}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3} \quad \text{[On rewriting the terms]}$$

$$= \left[{}^{(n-3)}C_r + {}^{(n-3)}C_{r-1}\right] + 2\left[{}^{(n-3)}C_{r-1} + {}^{(n-3)}C_{r-2}\right] + \left[{}^{(n-3)}C_{r-2} + {}^{(n-3)}C_{r-3}\right]$$

$$= {}^{(n-3+1)}C_r + 2.{}^{(n-3+1)}C_{r-1} + {}^{(n-3+1)}C_{r-2} = {}^{(n-2)}C_r + 2.{}^{(n-2)}C_{r-1} + {}^{(n-2)}C_{r-2}$$

$$= \left[{}^{(n-2)}C_r + {}^{(n-2)}C_{r-1}\right] + \left[{}^{(n-2)}C_{r-1} + {}^{(n-2)}C_{r-2}\right]$$

$$= {}^{(n-2+1)}C_r + {}^{(n-2+1)}C_{r-1} = {}^{(n-1)}C_r + {}^{(n-1)}C_{r-1} = {}^{(n-1+1)}C_r = {}^{n}C_r = R.H.S$$

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15. Resolve $\frac{x^2-3}{(x+2)(x^2+1)}$ into partial fractions.

Sol: Let $\frac{x^2-3}{(x+2)(x^2+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2+1} = \frac{A(x^2+1)+(Bx+C)(x+2)}{(x+2)(x^2+1)}$

$\Rightarrow A(x^2+1)+(Bx+C)(x+2) = x^2-3$ ....(1)

Putting $x=-2$ in (1) we get $A(4+1)+(Bx+C)(0) = 4-3 \Rightarrow 5A = 1 \Rightarrow A = 1/5$

Putting $x=0$ in (1) we get $A+2C = -3 \Rightarrow C = -8/5$

Comparing the coefficients of x^2, we get $A+B=1 \Rightarrow B = 1-A = 1-\frac{1}{5} = \frac{4}{5}$

$$\therefore \frac{x^2-3}{(x+2)(x^2+1)} = \frac{1}{5(x+2)} + \frac{4x-8}{5(x^2+1)}$$

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16. The probability for a contractor to get a road contract is 2/3 and to get a building contract is 5/9. The probability to get atleast one contract is 4/5. Find the probability that he gets both the contracts.

Sol: Let A be the event of getting road contract and B be the event of getting building contract.

Given that $P(A) = \frac{2}{3}$, $P(B) = \frac{5}{9}$, $P(A \cup B) = \frac{4}{5}$

$\therefore$ Probability that the contractor will get both the contracts is

$$P(A \cap B) = P(A) + P(B) - P(A \cup B) = \frac{2}{3} + \frac{5}{9} - \frac{4}{5} = \frac{30+25-36}{45} = \frac{19}{45}$$

17. (a) Define conditional probability.

(b) If A and B are independent events with P(A) = 0.2, P(B) = 0.5,

find (i) P(A/B) (ii) P(B/A) (iii) P(A∩B)

Sol: (a) Conditional Probability: Let A,B be two events of a random experiment, then P(B/A) represents the conditional probability of occurrence of B given A (probability of occurrence of B (from A) when A occurs or conditional probability of occurrence of B relative to A).

The conditional probability of B given A, is defined as $P\left(\frac{B}{A}\right) = \frac{P(B \cap A)}{P(A)}$, where $P(A) \neq 0$.

(b) Given that A,B are independent, hence

(i) $P(A/B)=P(A)=0.2$

(ii) $P(B/A)=P(B)=0.5$

(iii) $P(A \cap B)=P(A).P(B)=0.2 \times 0.5 =0.1$

(iv) $P(A \cup B) = P(A)+P(B)-P(A \cap B) = 0.2 + 0.5 - 0.1 = 0.6$

SECTION-C

18. If $\cos\alpha+\cos\beta+\cos\gamma=0=\sin\alpha+\sin\beta+\sin\gamma$, then show that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=3/2=\sin^2\alpha+\sin^2\beta+\sin^2\gamma$.

Sol: Let $a=\cos\alpha+i\sin\alpha=\text{cis}\alpha$ $b=\cos\beta+i\sin\beta=\text{cis}\beta, c=\cos\gamma+i\sin\gamma=\text{cis}\gamma$ then we have

$a+b+c=(\cos\alpha+i\sin\alpha)+(\cos\beta+i\sin\beta)+(\cos\gamma+i\sin\gamma)$

$=(\cos\alpha+\cos\beta+\cos\gamma)+i.(\sin\alpha+\sin\beta+\sin\gamma)$

$=0+i(0)=0$ [$\because$ From the given conditions]

$\therefore a+b+c=0 \Rightarrow (a+b+c)^2=0$

$\Rightarrow a^2+b^2+c^2+2(ab+bc+ca)=0$

$\Rightarrow a^2+b^2+c^2+2abc\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)=0$

$\Rightarrow a^2+b^2+c^2+2abc\left(\frac{1}{\text{cis}\alpha}+\frac{1}{\text{cis}\beta}+\frac{1}{\text{cis}\gamma}\right)=0$

$\Rightarrow a^2+b^2+c^2+2abc\left(\text{cis}(-\alpha)+\text{cis}(-\beta)+\text{cis}(-\gamma)\right)=0$

$\Rightarrow a^2+b^2+c^2+2abc[(\cos\alpha-i\sin\alpha)+(\cos\beta-i\sin\beta)+(\cos\gamma-i\sin\gamma)]=0$

$\Rightarrow a^2+b^2+c^2+2abc[(\cos\alpha+\cos\beta+\cos\gamma)-i(\sin\alpha+\sin\beta+\sin\gamma)]=0$

$\Rightarrow a^2+b^2+c^2+2abc(0-i(0))=0$

$\Rightarrow a^2+b^2+c^2+2abc(0)=0$

$\Rightarrow a^2+b^2+c^2=0$

$\Rightarrow (\text{cis}\alpha)^2+(\text{cis}\beta)^2+(\text{cis}\gamma)^2=0$ [$\because$ $a=\text{cis}\alpha$, $b=\text{cis}\beta$, $c=\text{cis}\gamma$]

$\Rightarrow \text{cis}2\alpha+\text{cis}2\beta+\text{cis}2\gamma=0$

$\Rightarrow (\cos2\alpha+i\sin2\alpha)+(\cos2\beta+i\sin2\beta)+(\cos2\gamma+i\sin2\gamma)=0$

$\Rightarrow (\cos2\alpha+\cos2\beta+\cos2\gamma)+i(\sin2\alpha+\sin2\beta+\sin2\gamma)=0+i(0)$

Equating the real parts, we get $\cos2\alpha+\cos2\beta+\cos2\gamma=0$ .... (1)

| $\cos2\theta = 2\cos^2\theta-1$

Now, (1) $\Rightarrow (2\cos^2\alpha-1)+(2\cos^2\beta-1)+(2\cos^2\gamma-1)=0$

$\Rightarrow 2\cos^2\alpha+2\cos^2\beta+2\cos^2\gamma=1+1+1$

$\Rightarrow 2(\cos^2\alpha+\cos^2\beta+\cos^2\gamma)=3$

$\Rightarrow \cos^2\alpha+\cos^2\beta+\cos^2\gamma=3/2$

| $\cos2\alpha = 1-2\sin^2\alpha$

Again (1) $\Rightarrow (1-2\sin^2\alpha)+(1-2\sin^2\beta)+(1-2\sin^2\gamma)=0$

$\Rightarrow 2\sin^2\alpha+2\sin^2\beta+2\sin^2\gamma=1+1+1$

$\Rightarrow 2(\sin^2\alpha+\sin^2\beta+\sin^2\gamma)=3$

$\Rightarrow \sin^2\alpha+\sin^2\beta+\sin^2\gamma=3/2$

19. Solve the equation $4x^3-24x^2+23x+18=0$ the roots being in A.P.

Sol: Let the roots of $4x^3-24x^2+23x+18=0$ in A.P be taken as a–d, a, a+d

$$\text{Now, } s_1=(a-d)+a+(a+d)=\frac{24}{4}\Rightarrow 3a=6\Rightarrow a=2$$

$$s_3=(a-d)a(a+d)=\frac{-18}{4}\Rightarrow a(a^2-d^2)=\frac{-9}{2}\Rightarrow 2(4-d^2)=\frac{-9}{2}$$

$$\Rightarrow 4(4-d^2)=-9\Rightarrow 16-4d^2=-9\Rightarrow 4d^2=25\Rightarrow d^2=\frac{25}{4}\Rightarrow d=\pm\frac{5}{2}$$

$$\therefore \text{The roots are } a-d,\ a,\ a+d=2-\frac{5}{2},\ 2,\ 2+\frac{5}{2}=-\frac{1}{2},\ 2,\ \frac{9}{2}$$

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20. Suppose that n is a natural number and $(7+4\sqrt{3})^n$ = I+F where I, F are respectively the integral part and fractional part then show that
(i) I is an odd integer (ii) (I+F)(1–F)=1

Sol: Given that, I is integral part, hence its is an integer

F is fractional point hence 0<F<1

Given, $(7+4\sqrt{3})^n=I+F$

Let $(7-4\sqrt{3})^n=G\Rightarrow 0<G<1\ (\because 36<48<49\Rightarrow 6<4\sqrt{3}<7)$

Now, $(I+F)+G=(7+4\sqrt{3})^n+(7-4\sqrt{3})^n$

$$={}^nC_0 7^n+{}^nC_1 7^{n-1}(4\sqrt{3})^1+{}^nC_2 7^{n-2}(4\sqrt{3})^2+\text{..........})+$$

$$({}^nC_0 7^n-{}^nC_1 7^{n-1}(4\sqrt{3})^1+{}^nC_2 7^{n-2}(4\sqrt{3})^2+\text{..........})$$

$$=2(7^n+{}^nC_2 7^{n-2}(4\sqrt{3})^2+{}^nC_4 7^{n-4}(4\sqrt{3})^4+\text{..........})$$

= 2(an integer) = an even integer

$\therefore$ I+F+G is an even integer $\Rightarrow$ F+G is an integer ($\because$ I is an Integer)

Given that 0<F<1, also we have 0<G<1

$\Rightarrow$ 0<F+G<2 $\Rightarrow$ F+G=1 ($\because$ the only integer between 0 and 2 is 1)

Now I+(F+G) = an even integer $\Rightarrow$ I+1=an even integer

$\Rightarrow$ I = (an even integer –1)= an odd integer

Now F+G=1 $\Rightarrow$ G=1–F

$\therefore$ (I+F)(1–F)=(I+F)(G)= $(7+4\sqrt{3})^n(7-4\sqrt{3})^n=(49-48)^n=1^n=1$

21. Find the sum of the infinite series $1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+..........$

Sol: Let $S=1+\frac{1}{3}+\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+.......\text{upto }\infty$

$$=1+\frac{1}{1!}\left(\frac{1}{3}\right)+\frac{1.3}{2!}\left(\frac{1}{3}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3+.......$$

Comparing the above series with $1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+....=(1-x)^{-\frac{p}{q}}$

we get $p = 1$, $p + q = 3 \Rightarrow 1+q = 3 \Rightarrow q = 2$

Also, we have $\frac{x}{q}=\frac{1}{3}\Rightarrow x=\frac{q}{3}=\frac{2}{3}$

$$\therefore S=(1-x)^{\frac{-p}{q}}=\left(1-\frac{2}{3}\right)^{\frac{-1}{2}}=\left(\frac{1}{3}\right)^{\frac{-1}{2}}=\left(\frac{3}{1}\right)^{\frac{1}{2}}=3^{\frac{1}{2}}=\sqrt{3}$$

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22. Find the variance and standard deviation of the following frequency distribution.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Sol: Here $N=\Sigma f_i=3+5+9+5+4+3+1=30$

Also $\Sigma f_i x_i=4(3)+8(5)+11(9)+17(5)+20(4)+24(3)+32(1)=420$ $\therefore \bar{x}=\frac{\Sigma f_i x_i}{N}=\frac{420}{30}=14$

x_i	f_i	$f_i x_i$	$x_i-\bar{x}$	$(x_i-\bar{x})^2$	$f_i(x_i-\bar{x})^2$
4	3	12	–10	100	300
8	5	40	–6	36	180
11	9	99	–3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
		$\Sigma f_i x_i=420$			$\Sigma f_i(x_i-\bar{x})^2=1374$

Variance $(\sigma^2)=\frac{1}{N}\Sigma f_i(x_i-\bar{x})^2=\frac{1}{30}(1374)=45.8$. Standard Deviation $\sigma=\sqrt{45.8}=6.77$

23. State and Prove Baye's theorem on Probability.

Sol: ***Statement*:** If $E_1, E_2 \ldots E_n$ are mutually exclusive and exhaustive events in a sample space S and A is any event intersecting with any E_i such that $P(A) \neq 0$ then $P(E_k | A) = \dfrac{P(E_k)P(A/E_k)}{\sum_{i=1}^{n} P(Ei).P(A/E_i)}$

Proof: From the definition of conditional probability: $P(E_k | A) = \dfrac{P(E_k \cap A)}{P(A)} = \dfrac{P(E_k).P(A | E_k)}{P(A)}$ ...(1)

Given that $E_1, E_2 \ldots E_n$ are mutually exclusive and exhaustive events in a sample space S

$$\Rightarrow \bigcup_{i=1}^{n} E_i = S \text{ and } A \cap E_1, A \cap E_2, \ldots A \cap E_n \text{ are mutually disjoint} \Rightarrow A \cap E_i = \phi$$

$$\text{Now, } P(A) = P(S \cap A) = P\left(\left(\bigcup_{i=1}^{n} E_i\right) \cap A\right) = P\left(\bigcup_{i=1}^{n} (E_i \cap A)\right) = \sum_{i=1}^{n} P(E_i \cap A) = \sum_{i=1}^{n} P(E_i)P(A | E_i)$$

$$\therefore \text{ From (1), } P(E_k | A) = \frac{P(E_k)P(A | E_k)}{\sum_{i=1}^{n} P(E_i)P(A | E_i)}$$

24. A random variable X has its range{0,1,2} and the probabilities are $P(X=0)=3c^3$, $P(X=1)=4c-10c^2$, $P(X=2)=5c-1$ where 'c' is a constant, find (i) c (ii) $P(0<X<3)$ (iii) $P(1<X\le2)$ (iv) $P(X<1)$

Sol: (i) We know $\Sigma P(X=xi)=1$

$$\Rightarrow 3c^3+4c-10c^2+5c-1=1 \Rightarrow 3c^3-10c^2+9c-1=1 \Rightarrow 3c^3-10c^2+9c-2=0$$

Here, the sum of the coefficients is 3–10+9–2=0. Hence 1 is a root of the above equation.

∴ By synthetic division, we have

1	3	–10	9	–2
	0	3	–7	2
	3	–7	2	0

$$\therefore 3c^3-10c^2+9c-2=(c-1)(3c^2-7c+2)=(c-1)(c-2)(3c-1)$$

Now, $(c-1)(c-2)(3c-1)=0 \Rightarrow c=1,2,\frac{1}{3}$

∴ c=1/3 is the only possible value. $[\because 0\le p\le 1]$

(ii) $P(0<X<3)=P(X=1)+P(X=2)=(4c-10c^2)+(5c-1)=9c-10c^2-1$

$$=9\left(\frac{1}{3}\right)-10\left(\frac{1}{3}\right)^2-1=\frac{9}{3}-\frac{10}{9}-1=3-\frac{10}{9}-1=2-\frac{10}{9}=\frac{8}{9}$$

(iii) $P(1<X\le 2)=P(X=2)=5c-1=5\left(\frac{1}{3}\right)-1=\frac{5}{3}-1=\frac{2}{3}$

(iv) $P(X<1)=P(X=0)=3c^3=3\left(\frac{1}{3}\right)^3=3.\frac{1}{27}=\frac{1}{9}$