Previous IPE

SOLVED PAPERS

MARCH-2020 (AP)

PREVIOUS PAPERS

IPE: MARCH-2020(AP)

MATHS-1A

ime : 3 Hours Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **$10 \times 2 = 20$**

1. If $f(x)=2x-1$, $g(x)=\dfrac{x+1}{2}$ for all $x\in R$, find (gof)(x) **2.** If $f=\{(1,2),(2,-3),(3,-1)\}$ then find (i) 2f (ii) f^2

3. Find the trace of $\begin{bmatrix}1 & 3 & -5\\ 2 & -1 & 5\\ 2 & 0 & 1\end{bmatrix}$ **4.** Find the rank of $\begin{bmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 2 & 1\end{bmatrix}$

5. If $\bar{a}=2\bar{i}+5\bar{j}+\bar{k}$, $\bar{b}=4\bar{i}+m\bar{j}+n\bar{k}$ are collinear vectors then find m,n

6. Find the vector equation of the plane passing through the points (0,0,0), (0,5,0) and (2,0,1)

7. Find the angle between the planes $\bar{r}.(2\bar{i}-\bar{j}+2\bar{k})=3$, $\bar{r}.(3\bar{i}+6\bar{j}+\bar{k})=4$

8. If $\tan 20^\circ=\lambda$ then show that $\dfrac{\tan160^\circ-\tan110^\circ}{1+\tan160^\circ.\tan110^\circ}=\dfrac{1-\lambda^2}{2\lambda}$

9. Find the range of $7\cos x-24\sin x+5$ 10. Prove that $(\cosh x-\sinh x)^n=\cosh(nx)-\sinh(nx)$

SECTION-B

II. Answer any FIVE of the following SAQs: **$5 \times 4 = 20$**

11. If $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$, then show that AA'=A'A=I

12. If the points whose position vectors are $3\bar{i}-2\bar{j}-\bar{k}$, $2\bar{i}+3\bar{j}-4\bar{k}$, $-\bar{i}+\bar{j}+2\bar{k}$, $4\bar{i}+5\bar{j}+\lambda\bar{k}$ are coplanar, then show that $\lambda=-146/17$

13. Find the vector area and area of the parallelogram having $\bar{a}=\bar{i}+2\bar{j}-\bar{k}$, $\bar{b}=2\bar{i}-\bar{j}+2\bar{k}$ as adjacent sides.

14. Show that $\sin^4\frac{\pi}{8}+\sin^4\frac{3\pi}{8}+\sin^4\frac{5\pi}{8}+\sin^4\frac{7\pi}{8}=\frac{3}{2}$ **15.** Solve $7\sin^2\theta+3\cos^2\theta=4$

16. Prove that $\mathrm{Tan}^{-1}\frac{1}{2}+\mathrm{Tan}^{-1}\frac{1}{5}+\mathrm{Tan}^{-1}\frac{1}{8}=\frac{\pi}{4}$ **17.** If $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A.P., then P.T a,b,c are in A.P

SECTION-C

III. Answer any FIVE of the following LAQs: **$5 \times 7 = 35$**

18. (a) If $f(x)=\dfrac{x+1}{x-1}$, $x\neq\pm1$ then find (fofof)(x)

(b) If f:A→B, g:B→C and h:C→D are three functions then prove that ho(gof)=(hog)of.

19. By Mathematical Induction, show that $49^n+16n-1$ is divisible by 64 for all positive Integer n.

20. Show that $\begin{vmatrix}a^2+2a & 2a+1 & 1\\ 2a+1 & a+2 & 1\\ 3 & 3 & 1\end{vmatrix}=(a-1)^3$

21. By using Cramer's solve $x+y+z=1$, $2x+2y+3z=6$, $x+4y+9z=3$

22. Find the shortest distance between the skew lines $\bar{r}=(6\bar{i}+2\bar{j}+2\bar{k})+t(\bar{i}-2\bar{j}+2\bar{k})$ and $\bar{r}=(-4\bar{i}-\bar{k})+s(3\bar{i}-2\bar{j}-2\bar{k})$

23. If $A+B+C=\dfrac{\pi}{2}$, then prove that $\cos2A+\cos2B+\cos2C=1+4\sin A\sin B\sin C$.

24. Show that $r+r_3+r_1-r_2=4R\cos B$.

IPE AP MARCH-2020

SOLUTIONS

SECTION-A

1. If $f(x) = 2x - 1$, $g(x) = \frac{x+1}{2}$ for all $x \in R$, find (i) (gof)(x) (ii) (fog)(x)

A: (i) $(gof)(x) = g(f(x)) = g(2x-1) = \frac{(2x-1)+1}{2} = \frac{2x}{2} = x$

(ii) $(fog)(x) = f(g(x)) = f\left(\frac{x+1}{2}\right) = 2\left(\frac{x+1}{2}\right) - 1 = x + 1 - 1 = x$

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2. If $f = \{(1,2),(2,-3),(3,-1)\}$ then find (i) 2f (ii) f^2

A: Given that $f = \{(1, 2), (2, -3), (3, -1)\}$

(i) $2f = \{(1,2(2)),(2,2(-3)),(3,2(-1)\} = \{(1,4),(2,-6),(3,-2)\}$

(ii) $f^2 = \{(1,2^2),(2,3^2),(3,1^2)\} = \{(1,4),(2,9),(3,1)\}$

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3. Define Trace of a Matrix. Find the trace of $\begin{bmatrix} 1 & 3 & -5 \\ 2 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$

A: Trace: The trace of a square matrix is the **sum of elements in the principal diagonal.**

$A = \begin{bmatrix} 1 & 3 & -5 \\ 2 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix} \Rightarrow Tr(A) = 1 + (-1) + 1 = 1$

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4. Find the rank of $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$

A: We take $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 2 & 1 \end{vmatrix} = 0(2-5) - 1(1-9) + 2(2-6)$

$= 0 - 1(-8) + 2(-4)$

$= 8 - 8$

$\therefore |A| = 0.$

Take a 2×2 minor, $\begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 2 - 6 = -4 \neq 0$

$\therefore$ Rank (A) = 2.

5. If $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$, $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear vectors then find m, n.

A: Given vectors $\bar{a} = 2\bar{i} + 5\bar{j} + \bar{k}$, $\bar{b} = 4\bar{i} + m\bar{j} + n\bar{k}$ are collinear.

$$\therefore \frac{2}{4} = \frac{5}{m} = \frac{1}{n}$$

$$\Rightarrow \frac{1}{2} = \frac{5}{m} \Rightarrow m = 2 \times 5 = 10 \text{ and } \frac{1}{n} = \frac{1}{2} \Rightarrow n = 2$$

$\therefore$ m=10, n=2

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6. Find the vector equation of the plane passing through the points (0,0,0), (0,5,0) and (2,0,1)

A: Given $A(\bar{a}) = \bar{0}$, $B(\bar{b}) = 5\bar{j}$,

$C(\bar{c}) = 2\bar{i} + \bar{k}$

Vector equation of the plane is $\bar{r} = (1-s-t)\bar{a} + s\bar{b} + t\bar{c}$, s, t$\in$R

$\bar{r} = (1-s-t)\bar{0} + s(5\bar{j}) + t(2\bar{i} + \bar{k})$

$\therefore \bar{r} = (5\bar{j})s + t(2\bar{i} + \bar{k})$, s, t $\in$ R

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7. Find the angle between the planes $\bar{r}.(2\bar{i} - \bar{j} + 2\bar{k}) = 3$, $\bar{r}.(3\bar{i} + 6\bar{j} + \bar{k}) = 4$

A: Comparing the given planes with $\bar{r}.\bar{n}_1 = p_1$, $\bar{r}.\bar{n}_2 = p_2$, we get

$\bar{n}_1 = 2\bar{i} - \bar{j} + 2\bar{k}$, $\bar{n}_2 = 3\bar{i} + 6\bar{j} + \bar{k}$

$$\therefore \cos\theta = \frac{\bar{n}_1.\bar{n}_2}{|\bar{n}_1||\bar{n}_2|}$$

$$= \frac{(2\bar{i} - \bar{j} + 2\bar{k}).(3\bar{i} + 6\bar{j} + \bar{k})}{\sqrt{4+1+4}.\sqrt{9+36+1}}$$

$$= \frac{2(3) - 1(6) + 2(1)}{\sqrt{9}.\sqrt{46}} = \frac{6-6+2}{\sqrt{9}.\sqrt{46}} = \frac{2}{3\sqrt{46}}$$

$$\therefore \cos\theta = \frac{2}{3\sqrt{46}} \Rightarrow \theta = \cos^{-1}\frac{2}{3\sqrt{46}}$$

8. If $\tan 20^\circ = \lambda$ then show that $\dfrac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ . \tan 110^\circ} = \dfrac{1-\lambda^2}{2\lambda}$

A: $\tan 160^\circ = \tan(180^\circ - 20^\circ) = -\tan 20^\circ = -\lambda$;

$\tan 110^\circ = \tan(90^\circ + 20^\circ) = -\cot 20^\circ = -\dfrac{1}{\tan 20^\circ} = \dfrac{-1}{\lambda}$

$$\therefore \text{L.H.S} = \frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \tan 110^\circ} = \frac{-\lambda - \left(-\frac{1}{\lambda}\right)}{1 + (-\lambda)\left(-\frac{1}{\lambda}\right)} = \frac{\frac{1}{\lambda} - \lambda}{1+1} = \frac{1-\lambda^2}{\lambda(2)} = \frac{1-\lambda^2}{2\lambda} = \text{R.H.S}$$

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9. Find the range of 7cosx–24sinx+5

A: Comparing 7cosx–24sinx+5 with acosx+bsinx+c, we get a= 7, b= –24, c = 5

$$\text{Max. Value} = (\sqrt{a^2 + b^2}) + c = (\sqrt{7^2 + 24^2}) + 5 = (\sqrt{49 + 576}) + 5 = \sqrt{625} + 5 = 25 + 5 = 30$$

$$\text{Min. Value} = (-\sqrt{a^2 + b^2}) + c = (-\sqrt{7^2 + 24^2}) + 5 = (-\sqrt{49 + 576}) + 5 = -\sqrt{625} + 5 = -25 + 5 = -20$$

Range = [Min. Value, Max.Value] = [–20, 30]

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10. Prove that $(\cosh x - \sinh x)^n = \cosh(nx) - \sinh(nx)$

A: $\text{L.H.S} = (\cosh x - \sinh x)^n = \left[\dfrac{e^x + e^{-x}}{2} - \dfrac{(e^x - e^{-x})}{2}\right]^n$

$$= \left(\frac{\cancel{e^x} + e^{-x} \cancel{- e^x} + e^{-x}}{2}\right)^n = \left(\frac{\cancel{2}e^{-x}}{\cancel{2}}\right)^n = e^{-nx} \text{ ....(1)}$$

$\text{R.H.S} = \cosh nx - \sinh nx$

$$= \left(\frac{e^{nx} + e^{-nx}}{2}\right) - \left(\frac{e^{nx} - e^{-nx}}{2}\right) = \frac{\cancel{e^{nx}} + e^{-nx} \cancel{- e^{nx}} + e^{-nx}}{2} = \frac{\cancel{2}e^{-nx}}{\cancel{2}} = e^{-nx} \text{ ....(2)}$$

From (1) & (2), L.H.S = R.H.S

Hence proved.

SECTION-B

11. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then show that AA'=A'A

A: $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$AA' = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos^2\alpha+\sin^2\alpha & -\cos\alpha\sin\alpha+\sin\alpha\cos\alpha \\ -\sin\alpha\cos\alpha+\cos\alpha\sin\alpha & \sin^2\alpha+\cos^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ......(1)

$A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} \cos^2\alpha+\sin^2\alpha & \cos\alpha\sin\alpha-\sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha-\cos\alpha\sin\alpha & \sin^2\alpha+\cos^2\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ......(2)

From (1) and (2), AA' = A'A

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12. If the points whose position vectors are $3\bar{i}-2\bar{j}-\bar{k}, 2\bar{i}+3\bar{j}-4\bar{k}, -\bar{i}+\bar{j}+2\bar{k}, 4\bar{i}+5\bar{j}+\lambda\bar{k}$ are coplanar, then show that $\lambda = -146/17$

A: We take $\overline{OP} = 3\bar{i}-2\bar{j}-\bar{k}, \overline{OQ} = 2\bar{i}+3\bar{j}-4\bar{k}$,

$\overline{OR} = -\bar{i}+\bar{j}+2\bar{k}, \overline{OS} = 4\bar{i}+5\bar{j}+\lambda\bar{k}$, where 'O' is the origin.

$\therefore \overline{PQ} = \overline{OQ}-\overline{OP} = (2\bar{i}+3\bar{j}-4\bar{k})-(3\bar{i}-2\bar{j}-\bar{k}) = -\bar{i}+5\bar{j}-3\bar{k}$

$\overline{PR} = \overline{OR}-\overline{OP} = (-\bar{i}+\bar{j}+2\bar{k})-(3\bar{i}-2\bar{j}-\bar{k}) = -4\bar{i}+3\bar{j}+3\bar{k}$

$\overline{PS} = \overline{OS}-\overline{OP} = (4\bar{i}+5\bar{j}+\lambda\bar{k})-(3\bar{i}-2\bar{j}-\bar{k}) = \bar{i}+7\bar{j}+(\lambda+1)\bar{k}$

But $[\overline{PQ}\ \overline{PR}\ \overline{PS}] = 0$ [Since P,Q,R,S are coplanar]

$\Rightarrow \begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & \lambda+1 \end{vmatrix} = 0 \Rightarrow (-1)\,[3(\lambda+1)-21] - 5\,[-4(\lambda+1)-3] - 3[(-28)-3] = 0$

$\Rightarrow -1(3\lambda-18) - 5(-4\lambda-7) - 3(-31) = 0$

$\Rightarrow -3\lambda + 18 + 20\lambda + 35 + 93 = 0$

$\Rightarrow -3\lambda + 20\lambda + 35 + 93 + 18 = 0$

$\Rightarrow 17\lambda + 146 = 0$

$\Rightarrow 17\lambda = -146$

$\Rightarrow \lambda = -146/17$

13. Find the vector area and area of the parallelogram having $\bar{a} = \bar{i} + 2\bar{j} - \bar{k}$, $\bar{b} = 2\bar{i} - \bar{j} + 2\bar{k}$ as adjacent sides.

A: Given that $\bar{a} = \bar{i} + 2\bar{j} - \bar{k},\ \bar{b} = 2\bar{i} - \bar{j} + 2\bar{k}$

$$\therefore \bar{a} \times \bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & -1 \\ 2 & -1 & 2 \end{vmatrix} = \bar{i}(4-1) - \bar{j}(2+2) + \bar{k}(-1-4) = 3\bar{i} - 4\bar{j} - 5\bar{k}$$

(i) Vector area of the parallelogram with $\bar{a}, \bar{b}$ as adjacent sides is $\bar{a} \times \bar{b} = 3\bar{i} - 4\bar{j} - 5\bar{k}$

(ii) Also $|\bar{a} \times \bar{b}| = |3\bar{i} - 4\bar{j} - 5\bar{k}| = \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}$

$\therefore$ Area of the parallelogram is $|\bar{a} \times \bar{b}| = 5\sqrt{2}$ sq.units

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14. Show that $\sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8} = \frac{3}{2}$

A: $\sin\frac{3\pi}{8} = \sin\left(\frac{4\pi - \pi}{8}\right) = \sin\left(\frac{\pi}{2} - \frac{\pi}{8}\right) = \cos\frac{\pi}{8}$

$\sin\frac{5\pi}{8} = \sin\left(\frac{4\pi + \pi}{8}\right) = \sin\left(\frac{\pi}{2} + \frac{\pi}{8}\right) = \cos\frac{\pi}{8}$

$\sin\frac{7\pi}{8} = \sin\left(\frac{8\pi - \pi}{8}\right) = \sin\left(\pi - \frac{\pi}{8}\right) = \sin\frac{\pi}{8}$

L.H.S $= \sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$

$= \sin^4\frac{\pi}{8} + \left(\cos\frac{\pi}{8}\right)^4 + \left(\cos\frac{\pi}{8}\right)^4 + \left(\sin\frac{\pi}{8}\right)^4$

$= \sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^4\frac{\pi}{8} = 2\left[\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8}\right]$

$= 2\left[\left(\sin^2\frac{\pi}{8} + \cos^2\frac{\pi}{8}\right)^2 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right]$, $\quad [\because a^2 + b^2 = (a+b)^2 - 2ab]$

$= 2\left[1 - 2\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right] = 2 - 4\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$

$= 2 - \left[2\sin\frac{\pi}{8}\cos\frac{\pi}{8}\right]^2 = 2 - \left[\sin\frac{2\pi}{8}\right]^2$

$= 2 - \left[\sin\frac{\pi}{4}\right]^2 = 2 - \left(\frac{1}{\sqrt{2}}\right)^2 = 2 - \frac{1}{2} = \frac{3}{2} = $ R.H.S

15. Solve $7\sin^2\theta+3\cos^2\theta=4$

A: Given equation is $7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow 7\sin^2\theta + 3(1 - \sin^2\theta) = 4$

$\Rightarrow 7\sin^2\theta + 3 - 3\sin^2\theta = 4$

$\Rightarrow 4\sin^2\theta = 1$

$\Rightarrow \sin^2\theta = (1/2)^2 = \sin^2(\pi/6)$

Here Principal value is $\alpha = \pi/6$

$\therefore$ General solution is $\theta = n\pi \pm \frac{\pi}{6}, n \in Z$

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16. Pove that $\text{Tan}^{-1}\frac{1}{2} + \text{Tan}^{-1}\frac{1}{5} + \text{Tan}^{-1}\frac{1}{8} = \frac{\pi}{4}$

A: We know, $\text{Tan}^{-1}x + \text{Tan}^{-1}y = \text{Tan}^{-1}\left(\frac{x+y}{1-xy}\right)$

$$\therefore \text{Tan}^{-1}\frac{1}{2} + \text{Tan}^{-1}\frac{1}{5} = \text{Tan}^{-1}\left(\frac{\frac{1}{2}+\frac{1}{5}}{1-\frac{1}{2}.\frac{1}{5}}\right) = \text{Tan}^{-1}\left(\frac{\frac{5+2}{\cancel{10}}}{\frac{10-1}{\cancel{10}}}\right) = \text{Tan}^{-1}\frac{7}{9}$$

$$\therefore \text{L.H.S} = \text{Tan}^{-1}\frac{7}{9} + \text{Tan}^{-1}\frac{1}{8}$$

$$= \text{Tan}^{-1}\left(\frac{\frac{7}{9}+\frac{1}{8}}{1-\frac{7}{9}.\frac{1}{8}}\right) = \text{Tan}^{-1}\left(\frac{\frac{56+9}{\cancel{72}}}{\frac{72-7}{\cancel{72}}}\right) = \text{Tan}^{-1}\left(\frac{65}{65}\right) = \text{Tan}^{-1}1 = \frac{\pi}{4} = \text{R.H.S}$$

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17. If $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A.P., then prove that a,b,c are in A.P

A: Given that $\cot\frac{A}{2}, \cot\frac{B}{2}, \cot\frac{C}{2}$ are in A.P

$\Rightarrow \frac{(s)(s-a)}{\Delta}, \frac{(s)(s-b)}{\Delta}, \frac{(s)(s-c)}{\Delta}$ are in A.P

$\Rightarrow$ s – a, s – b, s – c are in A.P

$\Rightarrow$ –a, –b, –c are in A.P

$\Rightarrow$ a, b, c are in A.P

a, b, c are in AP
$\Leftrightarrow$ ka, kb, kc are in AP
$\Leftrightarrow$ $a \pm k, b \pm k, c \pm k$ are in AP

SECTION-C

18. (a) If $f(x) = \frac{x+1}{x-1}$, $x \neq 1$ then find (fofof)(x)

(b) If f:A→B, g : B→C and h:C→D are functions then prove that ho(gof)=(hog)of

A: (a) $(fof)(x) = f(f(x)) = f\left(\frac{x+1}{x-1}\right) = \frac{\left(\frac{x+1}{x-1}\right)+1}{\left(\frac{x+1}{x-1}\right)-1} = \frac{\frac{x+1+x-1}{\cancel{x-1}}}{\frac{x+1-(x-1)}{\cancel{x-1}}} = \frac{\cancel{2}x}{\cancel{2}} = x$

$\therefore (fofof)(x) = f[fof(x)] = f(x)$

(b) Proof: Part – 1: Given that $f : A \to B$ and $g : B \to C \Rightarrow gof : A \to C$

Now, $gof : A \to C$ and $h : C \to D \Rightarrow ho(gof) : A \to D$

Also, $g : B \to C$ and $h : C \to D \Rightarrow hog : B \to D$

Now, $f : A \to B$ and $hog : B \to D \Rightarrow (hog)of : A \to D$

Thus ho(gof) and (hog)of have the same domain A

Part–2: Let $a \in A$

Now, [ho(gof)](a) = h[(gof)(a)] = h[g(f(a))] = (hog)[f(a)] = [(hog)of](a)

Hence, from Part–1, Part–2 we conclude that ho(gof) = (hog)of

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19. By Mathematical Induction, show that $49^n + 16n - 1$ is divisible by 64 for all positive Integer n.

A: Given $S(n) : 49^n + 16n - 1 = 64q,\ q \in Z$

Step 1: L.H.S of $S(1) = 49^{(1)} + 16(1) - 1 = 49 + 16 - 1 = 64 = 64(1)$

$\Rightarrow$ 64 is divisible by 64

So, S(1) is true

Step 2: Assume that S(k) is true for $k \in N$

$S(k): 49^k + 16k - 1 = 64q$ ....(1), $q \in Z$

Step 3: We show that S(k+1) is true

Writing $(k+1)^{th}$ term from (1), we get

$L.H.S = 49^{k+1} + 16(k+1) - 1 = 49^k.49 + 16k + 16 - 1$

$= (64q - 16k + 1).49 + 16k + 15$ (from (1))

$= 64q.49 - 16k.49 + 1.49 + 16k + 15 = 64q.49 - 16k.(49-1) + (49+15)$

$= 64q.49 - 16k.(48) + 64 = 64q.49 - 16k.(4.12) + 64$

$= 64q.49 - 64k(12) + 64 = 64(49q - 12k + 1) = 64$(an integer)

So, S(k+1) is true whenever S(k) is true

Hence, by P.M.I the given statement is true, for all $n \in N$

20. Show that $\begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$

A: $\text{L.H.S} = \begin{vmatrix} a^2+2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = \begin{vmatrix} a^2-1 & a-1 & 0 \\ 2a-2 & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix} \begin{matrix} R_1 \to R_1 - R_2 \\ R_2 \to R_2 - R_3 \end{matrix} = \begin{vmatrix} (a-1)(a+1) & a-1 & 0 \\ 2(a-1) & a-1 & 0 \\ 3 & 3 & 1 \end{vmatrix}$

$= (a-1)(a-1)\begin{vmatrix} a+1 & 1 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^2[0(6-3)-0[3(a+1)-3]+1(a+1-2)]$

$= (a-1)^2\ (a-1) = (a-1)^3 = \text{R.H.S}$

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21. By using Cramer's rule solve x + y + z = 1, 2x + 2y + 3z = 6, x + 4y + 9z = 3

A: Given equations in the matrix equation form: $AX = D$, where

$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, D = \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix}$

Now, $\Delta = \det A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 4 & 9 \end{vmatrix}$

$= 1(18-12) - 1(18-3) + 1(8-2)$

$= 1(6) - 1(15) + 1(6) = 6 - 15 + 6 = -3$

$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 6 & 2 & 3 \\ 3 & 4 & 9 \end{vmatrix}$

$= 1(18-12) - 1(54-9) + 1(24-6)$

$= 1(6) - 1(45) + 1(18) = 6 - 45 + 18 = -21$

$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 6 & 3 \\ 1 & 3 & 9 \end{vmatrix}$

$= 1(54-9) - 1(18-3) + 1(6-6)$

$= 1(45) - 1(15) + 1(0) = 45 - 15 + 0 = 30$

$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 2 & 6 \\ 1 & 4 & 3 \end{vmatrix}$

$= 1(6-24) - 1(6-6) + 1(8-2)$

$= 1(-18) - 1(0) + 1(6) = -18 - 0 + 6 = -12$

$\therefore$ By Cramer's rule, $x=\dfrac{\Delta_1}{\Delta}=\dfrac{\cancel{-21}}{\cancel{3}}=7;\ y=\dfrac{\Delta_2}{\Delta}=\dfrac{\cancel{30}}{\cancel{-3}}=10;z=\dfrac{\Delta_3}{\Delta}=\dfrac{\cancel{-12}}{\cancel{-3}}=4$

$\therefore$ The solution is x=7, y=–10, z=4

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22. Find the shortest distance between the skew lines

$\bar{r}=(6\bar{i}+2\bar{j}+2\bar{k})+t(\bar{i}-2\bar{j}+2\bar{k})$ and $\bar{r}=(-4\bar{i}-\bar{k})+s(3\bar{i}-2\bar{j}-2\bar{k})$

A: Given skew lines $\bar{r}=(6\bar{i}+2\bar{j}+2\bar{k})+t(\bar{i}-2\bar{j}+2\bar{k})$; $\bar{r}=(-4\bar{i}-\bar{k})+s(3\bar{i}-2\bar{j}-2\bar{k})$

Formula: For the skew lines $\bar{r}=\bar{a}+t\bar{b},\ \bar{r}=\bar{c}+s\bar{d}$ shortest distance(SD) $=\dfrac{|(\bar{a}-\bar{c}).(\bar{b}\times\bar{d})|}{|\bar{b}\times\bar{d}|}$

On comparing the given skew lines with $\bar{r}=\bar{a}+t\bar{b};\ \bar{r}=\bar{c}+s\bar{d}$ we get

$\bar{a}=6\bar{i}+2\bar{j}+2\bar{k}$ and $\bar{b}=\bar{i}-2\bar{j}+2\bar{k}$

Also, $\bar{c}=-4\bar{i}-\bar{k}$ and $\bar{d}=3\bar{i}-2\bar{j}-2\bar{k}$

So, $\bar{a}-\bar{c}=(6\bar{i}+2\bar{j}+2\bar{k})-(-4\bar{i}-\bar{k})=10\bar{i}+2\bar{j}+3\bar{k}$

$$\bar{b}\times\bar{d}=\begin{vmatrix}\bar{i} & \bar{j} & \bar{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix}=\bar{i}(4+4)-\bar{j}(-2-6)+\bar{k}(-2+6)=8\bar{i}+8\bar{j}+4\bar{k}$$

$\therefore(\bar{a}-\bar{c}).(\bar{b}\times\bar{d})=(10\bar{i}+2\bar{j}+3\bar{k}).(8\bar{i}+8\bar{j}+4\bar{k})=80+16+12=108$

Also, $|\bar{b}\times\bar{d}|=\sqrt{8^2+8^2+4^2}=\sqrt{64+64+16}=\sqrt{144}=12$

$\therefore$ Shortest distance(SD) $=\dfrac{|(\bar{a}-\bar{c}).(\bar{b}\times\bar{d})|}{|\bar{b}\times\bar{d}|}=\dfrac{\cancel{108}}{\cancel{12}}=9$

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23. If $A+B+C=\frac{\pi}{2}$ then prove that cos2A+cos2B+cos2C=1+4sinAsinBsinC.

A: L.H.S $=\cos2A+\cos2B+\cos2C=2\cos\left(\dfrac{2A+2B}{2}\right)\cos\left(\dfrac{2A-2B}{2}\right)+\cos 2C$

$=2\cos(A+B)\cos(A-B)+\cos 2C=2\cos\left(\dfrac{\pi}{2}-C\right)\cos(A-B)+\cos 2C$

$=2\sin C\cos(A-B)+(1-2\sin^2C)$ $[\because \cos2\theta=1-2\sin^2\theta]$

$=1+2\sin C[\cos(A-B)-\sin C]=1+2\sin C[\cos(A-B)-\sin\left(\dfrac{\pi}{2}-(A+B)\right)]$

$=1+2\sin C[\cos(A-B)-\cos(A+B)]=1+2\sin C[2\sin A\sin B]$

$=1+4\sin A\sin B\sin C=$ R.H.S

24. Show that $r + r_3 + r_1 - r_2 = 4R\cos B$.

A: L.H.S$=r+r_3+r_1-r_2=(r_3+r_1)+(r-r_2)$

$$=\left(4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}+4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}\right)+\left(4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}-4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}\right)$$

$$=4R\left(\cos\frac{B}{2}\left(\cos\frac{A}{2}\sin\frac{C}{2}+\sin\frac{A}{2}\cos\frac{C}{2}\right)-\sin\frac{B}{2}\left(\cos\frac{A}{2}\cos\frac{C}{2}-\sin\frac{A}{2}\sin\frac{C}{2}\right)\right)$$

$$=4R\left(\cos\frac{B}{2}\sin\left(\frac{A+C}{2}\right)-\sin\frac{B}{2}\cos\left(\frac{A+C}{2}\right)\right)=4R\left(\cos\frac{B}{2}\cos\frac{B}{2}-\sin\frac{B}{2}\sin\frac{B}{2}\right)$$

$$=4R\left[\cos^2\frac{B}{2}-\sin^2\frac{B}{2}\right]=4R\cos B=\text{R.H.S}\qquad\left[\because\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}=\cos\theta\right]$$