Previous IPE

SOLVED PAPERS

MARCH-2019 (AP)

PREVIOUS PAPERS

IPE: MARCH-2019(AP)

Time : 3 Hours **MATHS-2A** Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **$10 \times 2 = 20$**

1. If $(\sqrt{3}+i)^{100} = 2^{99}(a+ib)$, show that $a^2+b^2=4$. **2.** If $z=2-3i$, show that $z^2-4z+13=0$
3. If $1,\omega,\omega^2$ are the cube roots of unity, find $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$.
4. If $x^2-15-m(2x-8)=0$ has equal roots then find m.
5. If the product of the roots of $4x^3+16x^2-9x-a=0$ is 9, then find a.
6. Find the number of ways of arranging the letters of the word INTERMEDIATE
7. If ${}^{n}C_5 = {}^{n}C_6$, then find ${}^{13}C_n$ **8.** Find the 6^{th} term in $\left(\frac{2x}{3}+\frac{3y}{2}\right)^9$
9. Find the mean deviation about median for the data 4, 6, 9, 3, 10, 13, 2
10. The mean and variance of a bionomial distribution are 4 and 3 respectively. Find the distribution and find $P(X \geq 1)$

SECTION-B

II. Answer any FIVE of the following SAQs: **$5 \times 4 = 20$**

11. If $x+iy = \frac{1}{1+\cos\theta+i\sin\theta}$ then, show that $4x^2-1=0$
12. If x is real, prove that $\frac{x}{x^2-5x+9}$ lies between 1 and $\frac{-1}{11}$.
13. If the letters of the word 'MASTER' are permuted in all possible ways and the words thus formed are arranged in the dictionary order, then findthe rank of the word "MASTER"
14. Simplify ${}^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_4$ **15.** Resolve $\frac{x^3}{(x-1)(x+2)}$ into partial fractions.
16. A,B,C are three horses in a race. The probability of A to win the race is twice that of B, and probability of B is twice that of C. What are the probabilities of A,B, and C to win the race?
17. A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.

SECTION-C

III. Answer any FIVE of the following Long Answer Questions: **$5 \times 7 = 35$**

18. If α,β are the roots of the equation $x^2-2x+4=0$, then show that $\alpha^n+\beta^n = 2^{n+1}\cos\left(\frac{n\pi}{3}\right)$
19. Solve $18x^3+81x^2+121x+60=0$, given that a root is equal half the sum of the remaining roots.
20. If the coefficient of x^{10} in the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-10} in the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$, find the relation between a and b where a and b are real numbers.
21. If $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+....$ then prove that $9x^2+24x=11$
22. Find the variance and standard deviation of the following frequency distribution.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

23. State and Prove Baye's theorem
24. A random variable X has the following probability distribution.

$X=x_i$	1	2	3	4	5
$P(X=x_i)$	k	2k	3k	4k	5k

Find the value of k and the mean and variance of X

IPE AP MARCH-2019

SOLUTIONS

SECTION-A

1. If $(\sqrt{3}+i)^{100} = 2^{99}(a+ib)$, show that $a^2+b^2 = 4$.

Sol: Given that $(\sqrt{3}+i)^{100} = 2^{99}(a+ib)$

$\Rightarrow \left|(\sqrt{3}+i)^{100}\right| = \left|2^{99}(a+ib)\right| \Rightarrow (\sqrt{(\sqrt{3})^2+1^2})^{100} = 2^{99}\sqrt{a^2+b^2}$

$\Rightarrow \cancel{2^{100}} = \cancel{2^{99}}\sqrt{a^2+b^2} \Rightarrow 2 = \sqrt{a^2+b^2}$. Squaring on both sides, we get a $a^2+b^2 = 4$.

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2. If $z=2-3i$, show that $z^2-4z+13=0$

Sol: Given that $z=2-3i \Rightarrow z-2=-3i \Rightarrow (z-2)^2=9i^2 \Rightarrow z^2-4z+4=-9 \Rightarrow z^2-4z+13=0$

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3. If $1,\omega,\omega^2$ are the cube roots of unity, find $(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$.

Sol: G.E $=(1-\omega+\omega^2)^5+(1+\omega-\omega^2)^5$

$=(1+\omega^2-\omega)^5+(1+\omega-\omega^2)^5= (-\omega-\omega)^5+(-\omega^2-\omega^2)^5$

$=(-2\omega)^5+(-2\omega^2)^5=-2^5[\omega^2+\omega]=-32(-1)=32$

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4. Find the values of m, if the equation $x^2-15-m(2x-8)=0$ have equal roots.

Sol: Given equation is $x^2-15-m(2x-8)=0$

$\Rightarrow x^2-15-2mx+8m=0 \Rightarrow x^2-2mx+(8m-15)=0$

Comparing with $ax^2+bx+c=0$ we get $a=1, b=-2m, c=8m-15$

But roots are equal. $\therefore\ \Delta=b^2-4ac=0$

$\Rightarrow (-2m)^2-4(1)(8m-15)=0$

$\Rightarrow 4m^2-32m+60=0 \Rightarrow 4(m^2-8m+15)=0$

$\Rightarrow m^2-8m+15=0 \Rightarrow m^2-5m-3m+15=0$

$\Rightarrow m(m-5)-3(m-5)=0 \Rightarrow (m-3)(m-5)=0 \Rightarrow m=3, 5$

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5. If the product of the roots of $4x^3+16x^2-9x-a=0$ is 9, then find a.

Sol: From the given equation we get, $a_0=4, a_1=16, a_2=-9, a_3=-a$

Product of the roots is $9 \Rightarrow S_3 = -\frac{a_3}{a_0} = 9 \Rightarrow \frac{a}{4} = 9 \Rightarrow a = 4\times 9 = 36$

6. Find the number of ways of arranging the letters of the word INTERMEDIATE

A: Given word INTERMEDIATE contains 12 letters.

Here, 3 'E's, 2 'I's, 2 'T's are alike

$\therefore$ Number of arrangements $= \dfrac{n!}{p!q!r!} = \dfrac{12!}{3!2!2!}$

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7. If $^nC_5 = {}^nC_6$, then find $^{13}C_n$

Sol : **Formula:** $^nC_r = {}^nC_s \Rightarrow r+s=n$ (or) $r=s$

$\therefore {}^nC_5 = {}^nC_6 \Rightarrow n = 5+6 = 11$

$\therefore {}^{13}C_n = {}^{13}C_{11} = {}^{13}C_{13-11} = {}^{13}C_2 = \dfrac{13\times \cancel{12}}{1\times \cancel{2}} = 13\times 6 = 78$

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8. Find the 6th term in $\left(\dfrac{2x}{3}+\dfrac{3y}{2}\right)^9$

Sol: We know in $(x+y)^n$, $T_{r+1} = {}^nC_r x^{n-r} y^r$.

$$\therefore T_6 = T_{5+1} = {}^9C_5\left(\frac{2x}{3}\right)^{9-5}\left(\frac{3y}{2}\right)^5 = {}^9C_5\left(\frac{2}{3}\right)^4 x^4\left(\frac{3}{2}\right)^5 y^5$$

$$= \frac{\cancel{9}\times\cancel{8}\times 7\times 6}{1\times\cancel{2}\times\cancel{3}\times\cancel{4}}\times\frac{\cancel{2^4}}{\cancel{3^4}}\times\frac{\cancel{3^5}}{\cancel{2^5}}x^4y^5 = 27\times 7x^4y^5 = 189x^4y^5$$

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9. Find the mean deviation about median for the data 4, 6, 9, 3, 10, 13, 2

Sol: **Given data:** 4, 6, 9, 3, 10, 13, 2.

Its ascending order : 2,3,4,<u>6</u>,9,10,13.

Number of observations n = 7 is odd .

$\therefore$ Median is the middle most term $\Rightarrow M=6$

Deviations from the median:

2–6= –4; 3–6= –3; 4–6= –2; 6–6=0; 9–6=3;10–6=4; 13–6=7

Absolute values of these deviations: 4, 3, 2, 0, 3, 4, 7

$\therefore$ M.D from Median is $M.D = \dfrac{\Sigma|x_i - M|}{7} = \dfrac{4+3+2+0+3+4+7}{7} = \dfrac{23}{7} = 3.29$

10. The mean and variance of a binomial distribution are 4 and 3 respectively. Find the distribution and find P(X≥1)

Sol: Given mean np = 4 ,variance npq = 3

Now, (np)q = 3 $\Rightarrow (4)q = 3 \Rightarrow q = \frac{3}{4} \Rightarrow p = 1 - q = 1 - \frac{3}{4} = \frac{4-3}{4} = \frac{1}{4}$

Take $np = 4 \Rightarrow n\left(\frac{1}{4}\right) = 4 \Rightarrow n = 4(4) = 16$

$\therefore$ n=16 , q=3/4 and p=1/4

Binomial distribution is $P(X = r) = {}^{n}C_r q^{n-r}.p^r = {}^{16}C_r \left(\frac{3}{4}\right)^{16-r}.\left(\frac{1}{4}\right)^r$

$\therefore P(X\geq 1) = 1 - P(X=0) = 1 - q^n = 1 - \left(\frac{3}{4}\right)^n = 1 - \left(\frac{3}{4}\right)^{16}$

SECTION-B

11. If $x+iy=\frac{1}{1+\cos\theta+i\sin\theta}$ then, show that $4x^2-1=0$

Sol: Given that $x+iy=\frac{1}{(1+\cos\theta)+i\sin\theta}=\frac{1}{(2\cos^2\frac{\theta}{2})+i(2\sin\frac{\theta}{2}\cos\frac{\theta}{2})}$

$$\Rightarrow x+iy=\frac{1}{(2\cos\frac{\theta}{2})(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})}=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{(2\cos\frac{\theta}{2})(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})(\cos\frac{\theta}{2}-i\sin\frac{\theta}{2})}$$

$$=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\left(2\cos\frac{\theta}{2}\right)\left(\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}\right)}=\frac{\cos\frac{\theta}{2}-i\sin\frac{\theta}{2}}{\left(2\cos\frac{\theta}{2}\right)(1)}=\frac{\cos\frac{\theta}{2}}{2\cos\frac{\theta}{2}}-\frac{i\sin\frac{\theta}{2}}{2\cos\frac{\theta}{2}}=\frac{1}{2}-\frac{1}{2}i\tan\frac{\theta}{2}$$

Equating the real parts, we get $x=\frac{1}{2}\Rightarrow 2x=1\Rightarrow(2x)^2=1^2\Rightarrow 4x^2=1\Rightarrow 4x^2-1=0$

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12. If x is real, prove that $\frac{x}{x^2-5x+9}$ lies between 1 and $\frac{-1}{11}$.

Sol: Let $y=\frac{x}{x^2-5x+9}\Rightarrow y(x^2-5x+9)=x\Rightarrow yx^2-5yx+9y-x=0$

$\Rightarrow yx^2-(5y+1)x+9y=0$ ....(1)

(1) is a quadratic in x and its roots are reals.

$\therefore\ \Delta=b^2-4ac\geq 0$

$\Rightarrow(5y+1)^2-4(y)(9y)\geq 0$

$\Rightarrow(25y^2+10y+1)-36y^2\geq 0$

$\Rightarrow -11y^2+10y+1\geq 0$

$\Rightarrow 11y^2-10y-1\leq 0$

$\Rightarrow 11y^2-11y+y-1\leq 0\Rightarrow 11y(y-1)+(y-1)\leq 0$

$\Rightarrow(11y+1)(y-1)\leq 0$

$\Rightarrow\ y\in\left[-\frac{1}{11},1\right]\Rightarrow\frac{-1}{11}\leq y\leq 1$

$\therefore$ The given expression lies between $\frac{-1}{11}$ and 1

13. If the letters of the word MASTER are permutted in all possible ways and the words thus formed are arranged in dictionary order, then find the rank of the word MASTER.

Sol: The alphabetical order of the letters of the word MASTER is

A,E,M,R,S,T

The number of words that begin with A $-----$ = 5! = 120

The number of words that begin with E $-----$ = 5! = 120

The number of words that begin with MAE $---$ = 3! = 6

The number of words that begin with MAR $---$ = 3! = 6

The number of words that begin with MASE $--$ = 2! = 2

The number of words that begin with MASR $--$ = 2! = 2

The next word is MASTER = 1! = 1

$\therefore$ Rank of the word MASTER = 2(120) + 2(6) + 2(2) + 1

= 240 + 12 + 4 + 1 = 257

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14. Simplify $^{34}C_5 + \sum_{r=0}^{4} {}^{(38-r)}C_4$

Sol: $G.E. = \sum_{r=0}^{4} {}^{(38-r)}C_4 + {}^{34}C_5$

$= \left[{}^{38}C_4 + {}^{37}C_4 + {}^{36}C_4 + {}^{35}C_4 + {}^{34}C_4\right] + {}^{34}C_5$

$= \left[{}^{38}C_4 + {}^{37}C_4 + {}^{36}C_4 + {}^{35}C_4\right] + \left[{}^{34}C_4 + {}^{34}C_5\right]$

$= \left[{}^{38}C_4 + {}^{37}C_4 + {}^{36}C_4\right] + \overline{{}^{35}C_4 + {}^{35}C_5}$

$[\because {}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r]$ $= \left[{}^{38}C_4 + {}^{37}C_4\right] + \overline{{}^{36}C_4 + {}^{36}C_5}$

$= \left[{}^{38}C_4\right] + \overline{{}^{37}C_4 + {}^{37}C_5} = {}^{38}C_4 + {}^{38}C_5 = {}^{39}C_5$

15. Resolve $\frac{x^3}{(x-1)(x+2)}$ into partial fractions.

Sol: Here, the degree of numerator 3≥ degree of denominator 2. So, it is an improper function.

Also $(x-1)(x+2)=x^2+x-2.$

Now on dividing x^3 by x^2+x-2, we have $\frac{x^3}{(x-1)(x+2)}=(x-1)+\frac{3x-2}{x^2+x-2}$

Now $\frac{3x-2}{x^2+x-2}=\frac{3x-2}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}=\frac{A(x+2)+B(x-1)}{(x-1)(x+2)}$

$\Rightarrow A(x+2)+B(x-1)=3x-2$ ....(1)

Putting x = 1 in (1), we get $A(3)+B(0)=3-2 \Rightarrow 3A=1 \Rightarrow A=\frac{1}{3}$

Putting x = –2 in (1), we get $A(0)+B(-3)=3(-2)-2 \Rightarrow -3B=-8 \Rightarrow B=\frac{8}{3}$

$\Rightarrow \frac{3x-2}{x^2+x-2}=\frac{A}{x-1}+\frac{B}{x+2}=\frac{1}{3(x-1)}+\frac{8}{3(x+1)}$

$\therefore \frac{x^3}{(x-1)(x+2)}=x-1+\frac{1}{3(x-1)}+\frac{8}{3(x+1)}$

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16. A,B,C are three horses in a race. The probability of A to win the race is twice that of B, and probability of B is twice that of C. What are probabilities of A,B,C to win the race?

Sol : Let A, B, C be the events of winning the race by the horses A, B, C respectively.

Given that P(A) = 2P(B) and P(B) = 2P(C)

Hence, P(A) = 2P(B) = 2[2P(C)] = 4P(C)

Now, $P(A\cup B\cup C) = P(A)+P(B)+P(C)$ [$\because$ A,B,C are disjoint]

$\Rightarrow P(S) = 4P(C)+2P(C)+P(C)$ [$\because$ only A,B,C run the race $\Rightarrow A\cup B\cup C = S$]

$\Rightarrow 1=7P(C) \Rightarrow P(C) = 1/7$ [$\because P(S)=1$]

$\therefore P(A)=4P(C) = 4\times\frac{1}{7}=\frac{4}{7}$; $P(B) = 2P(C) = 2\times\frac{1}{7}=\frac{2}{7}$

$\therefore$ P(A)=4/7, P(B) =2/7, P(C) =1/7

17. **A speaks truth in 75% of the cases and B in 80% of the cases. What is the prob ability that their statements about an incident do not match.**

Sol: Let A,B denote the events of speaking truth by A,B respectively

$$P(A)=\frac{75}{100}=\frac{3}{4};\ P(B)=\frac{80}{100}=\frac{4}{5}$$

$$\therefore P(\bar{A})=1-P(A)=1-\frac{3}{4}=\frac{1}{4};\quad P(\bar{B})=1-P(B)=1-\frac{4}{5}=\frac{1}{5}$$

Let E be the event that A and B contradict to each other

$$\Rightarrow P(E)=P\left[(A\cap\bar{B})\cup(\bar{A}\cap B)\right]=P(A\cap\bar{B})+P(\bar{A}\cap B)$$

$$=P(A)P(\bar{B})+P(\bar{A})P(B)\ [\because \text{A,B are independent}]=\frac{3}{4}.\frac{1}{5}+\frac{1}{4}.\frac{4}{5}=\frac{7}{20}$$

SECTION-C

18. If α,β are roots of the equation $x^2-2x+4=0$, then show that $\alpha^n+\beta^n=2^{n+1}\cos\left(\frac{n\pi}{3}\right)$

Sol: Given $x^2-2x+4=0 \Rightarrow x=\frac{2\pm\sqrt{4-4(1)(4)}}{2(1)}=\frac{2\pm\sqrt{-12}}{2}=\frac{2\pm 2i\sqrt{3}}{2}=1\pm i\sqrt{3}$

Now, we find the mod-amp form of $1+i\sqrt{3}$

Let $x+iy=1+i\sqrt{3} \Rightarrow x=1,\ y=\sqrt{3}$

$\therefore\ r=\sqrt{x^2+y^2}=\sqrt{1^2+(\sqrt{3})^2}=\sqrt{4}=2.$

Also, $\tan\theta=\frac{y}{x}=\frac{\sqrt{3}}{1}=\tan\frac{\pi}{3}\Rightarrow\theta=\frac{\pi}{3}$

$\therefore$ mod– Amp form of $1+i\sqrt{3}$ is $r(\cos\theta+i\sin\theta)=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

$\Rightarrow\ (1+i\sqrt{3})^n=\left(2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right)^n$

$=\ (2)^n\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^n=2^n\left(\cos n\frac{\pi}{3}+i\sin n\frac{\pi}{3}\right)$ ..... (1) (by Demoivre's theorem)

Similarly, $(1-i\sqrt{3})^n=2^n\left(\cos n\frac{\pi}{3}-i\sin n\frac{\pi}{3}\right)$ ..... (2)

Adding (1) & (2), we get $\alpha^n+\beta^n=(1+i\sqrt{3})^n+(1-i\sqrt{3})^n$

$=2^n\left(\left(\cos n\frac{\pi}{3}+i\sin n\frac{\pi}{3}\right)+\left(\cos n\frac{\pi}{3}-i\sin n\frac{\pi}{3}\right)\right)=2^n\left(2\cos\frac{n\pi}{3}\right)=2^{n+1}.\cos\frac{n\pi}{3}$

19. Solve $18x^3+81x^2+121x+60=0$, given that a root is equal half the sum of the remaining roots

Sol: Easy Method: Let α,β,γ be the roots of $18x^3+81x^2+121x+60=0$

Given that one root is half the sum of remaining two roots. Let $\beta=\frac{\alpha+\gamma}{2}$

$\Rightarrow \alpha, \beta, \gamma$ are in A.P

$\therefore$ we take $\alpha=a-d$, $\beta=a$, $\gamma=a+d$

The given equation is $18x^3+81x^2+121x+60=0$

$$S_1=\alpha+\beta+\gamma=-\frac{b}{a}\Rightarrow(a-d)+a+(a+d)=-\frac{81}{18}=-\frac{9}{2}\Rightarrow 3a=-\frac{9}{2}\Rightarrow a=-\frac{3}{2}$$

$$S_3=\alpha\beta\gamma=-\frac{d}{a}\Rightarrow(a-d)(a)(a+d)=-\frac{60}{18}=-\frac{10}{3}\Rightarrow a(a^2-d^2)=-\frac{10}{3}$$

$$\Rightarrow -\frac{3}{2}\left(\left(-\frac{3}{2}\right)^2-d^2\right)=-\frac{10}{3}\Rightarrow \frac{9}{4}-d^2=\frac{10}{3}\times\frac{2}{3}=\frac{20}{9}\Rightarrow d^2=\frac{9}{4}-\frac{20}{9}=\frac{81-80}{36}=\frac{1}{36}\Rightarrow d=\frac{1}{6}$$

Now $a-d=-\frac{3}{2}-\frac{1}{6}=\frac{-9-1}{6}=-\frac{10}{6}=-\frac{5}{3}$, Also, $a+d=-\frac{3}{2}+\frac{1}{6}=\frac{-9+1}{6}=-\frac{8}{6}=-\frac{4}{3}$

$\therefore$ The roots a–d, a, a+d are $-\frac{5}{3},-\frac{3}{2},-\frac{4}{3}$

20. If the coefficient of x^{10} in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-10} in the expansion of $\left(ax - \frac{1}{bx^2}\right)^{11}$, find the relation between a and b where a and b are real numbers.

Sol: In $\left(ax^2 + \frac{1}{bx}\right)^{11}$, the general term is $T_{r+1} = {}^{11}C_r (ax^2)^{11-r}\left(\frac{1}{bx}\right)^r$

$$= {}^{11}C_r \frac{a^{11-r}}{b^r}.x^{22-3r} \quad .....(1)$$

Put $22-3r = 10 \Rightarrow 3r = 22-10 \Rightarrow 3r = 12 \Rightarrow r = 4$

From (1), the coefficient of x^{10} is ${}^{11}C_4 \frac{a^{11-4}}{b^4} = {}^{11}C_4 \frac{a^7}{b^4}$ .....(2)

In $\left(ax - \frac{1}{bx^2}\right)^{11}$ the general term is $T_{r+1} = {}^{11}C_r (ax)^{11-r}\left(-\frac{1}{bx^2}\right)^r$

$$= (-1)^r \, {}^{11}C_r \frac{a^{11-r}}{b^r} x^{11-3r} \quad ......(3)$$

Put $11 - 3r = -10 \Rightarrow 3r = 21 \Rightarrow r = 7$

From (3), the coefficient of x^{-10} is $(-1)^7 \, {}^{11}C_7 \frac{a^{11-7}}{b^7} = -{}^{11}C_7.\frac{a^4}{b^7}$ .....(4)

Given that the two coefficients are equal

$\therefore$ From (2), (4), we have

$${}^{11}C_4 \frac{a^7}{b^4} = -{}^{11}C_7 \frac{a^4}{b^7} \Rightarrow \frac{a^7}{b^4} = -\frac{a^4}{b^7} \quad (\because {}^{11}C_4 = {}^{11}C_7)$$

$$\Rightarrow a^3 = -\frac{1}{b^3} \Rightarrow a^3b^3 = -1 \Rightarrow ab = -1$$

21. If $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \ldots\ldots$ then prove that $9x^2+24x = 11$

Sol: Given that $x = \frac{1.3}{3.6} + \frac{1.3.5}{3.6.9} + \frac{1.3.5.7}{3.6.9.12} + \ldots\ldots = \frac{1.3}{2!}\left(\frac{1}{3}\right)^2 + \frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3 + \frac{1.3.5.7}{4!}\left(\frac{1}{3}\right)^4 + ..$

Adding $1+\frac{1}{3}$ on both sides, we get $1+\frac{1}{3}+x = 1+\frac{1}{1!}\left(\frac{1}{3}\right)+\frac{1.3}{2!}\left(\frac{1}{3}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3+\ldots\ldots$

Comparing the above series with $1+\frac{p}{1!}\left(\frac{y}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{y}{q}\right)^2+\ldots.. = (1-y)^{-p/q}$

we get, $p=1, p+q=3 \Rightarrow 1+q=3 \Rightarrow q=2$

Also, $\frac{y}{q}=\frac{1}{3} \Rightarrow y=\frac{q}{3}=\frac{2}{3}$

$\therefore 1+\frac{1}{3}+x=(1-y)^{-p/q}=\left(1-\frac{2}{3}\right)^{-1/2}=\left(\frac{1}{3}\right)^{-1/2}=(3)^{1/2}=\sqrt{3}$

$\Rightarrow \frac{4}{3}+x=\sqrt{3} \Rightarrow x=\sqrt{3}-\frac{4}{3}=\frac{3\sqrt{3}-4}{3}=\frac{3\sqrt{3}-4}{3} \Rightarrow 3x=3\sqrt{3}-4 \Rightarrow 3x+4=3\sqrt{3}$

$\Rightarrow (3x+4)^2=(3\sqrt{3})^2 \Rightarrow 9x^2+24x+16=27 \Rightarrow 9x^2+24x=11$

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22. Find the variance and standard deviation of the following frequency distribution.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Sol: Here $N=\Sigma f_i=3+5+9+5+4+3+1=30$

Also $\Sigma f_i x_i=4(3)+8(5)+11(9)+17(5)+20(4)+24(3)+32(1)=420$ $\quad\therefore \bar{x}=\frac{\Sigma f_i x_i}{N}=\frac{420}{30}=14$

x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^2$
4	3	12	–10	100	300
8	5	40	–6	36	180
11	9	99	–3	9	81
17	5	85	3	9	45
20	4	80	6	36	144
24	3	72	10	100	300
32	1	32	18	324	324
		$\Sigma f_i x_i=420$			$\Sigma f_i(x_i - \bar{x})^2=1374$

Variance $(\sigma^2)=\frac{1}{N}\Sigma f_i(x_i-\bar{x})^2=\frac{1}{30}(1374)=45.8$. Standard Deviation $\sigma=\sqrt{45.8}=6.77$

23. State and Prove Baye's theorem on Probability.

Sol: ***Statement*:** If $E_1, E_2 ... E_n$ are mutually exclusive and exhaustive events in a sample space S and A is any event intersecting with any E_i such that $P(A) \neq 0$ then $P(E_k | A) = \frac{P(E_k)P(A/E_k)}{\sum_{i=1}^{n} P(Ei).P(A/E_i)}$

Proof: From the definition of conditional probability: $P(E_k | A) = \frac{P(E_k \cap A)}{P(A)} = \frac{P(E_k).P(A | E_k)}{P(A)}$ ...(1)

Given that $E_1, E_2 .... E_n$ are mutually exclusive and exhaustive events in a sample space S

$\Rightarrow \bigcup_{i=1}^{n} E_i = S$ and $A \cap E_1, A \cap E_2, ... A \cap E_n$ are mutually disjoint $\Rightarrow A \cap E_i = \phi$

Now, $P(A) = P(S \cap A) = P\left(\left(\bigcup_{i=1}^{n} E_i\right) \cap A\right) = P\left(\bigcup_{i=1}^{n} (E_i \cap A)\right) = \sum_{i=1}^{n} P(E_i \cap A) = \sum_{i=1}^{n} P(E_i)P(A | E_i)$

$\therefore$ From (1), $P(E_k | A) = \frac{P(E_k)P(A | E_k)}{\sum_{i=1}^{n} P(E_i)P(A | E_i)}$

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24. A random variable X has the following probability distribution.

$X=x_i$	1	2	3	4	5
$P(X=x_i)$	k	2k	3k	4k	5k

Find k and the mean and variance of X.

Sol: We know $\sum P(X = x_i) = 1$

$\Rightarrow k + 2k + 3k + 4k + 5k = 1 \Rightarrow 15k=1 \quad \Rightarrow k = 1/15$

Mean $\mu = \sum_{i=1}^{5} x_i.P(X = x_i)$

$=1(k)+2(2k)+3(3k)+4(4k)+5(5k)=k(1+4+9+16+25)$

$= k(55) = \frac{1}{15}(55) = \frac{55}{15} = \frac{11}{3}$

Variance $\sigma^2 = \Sigma x_i^2.P(X = x_i) - \mu^2 = 1(k) + 4(2k) + 9(3k) + 16(4k) + 25(5k) - \left(\frac{11}{3}\right)^2$

$= k(1 + 8 + 27 + 64 + 125) - \left(\frac{11}{3}\right)^2 = \frac{1}{15}(225) - \left(\frac{11}{3}\right)^2 = 15 - \frac{121}{9} = \frac{14}{9}$