Previous IPE

SOLVED PAPERS

MARCH -2019 (AP)

PREVIOUS PAPERS

IPE: MARCH-2019(AP)

Time : 3 Hours

MATHS-1B

Max.Marks : 75

SECTION-A

I. Answer ALL the following VSAQ: **$10 \times 2 = 20$**

1. Find the angle which the straight line $y = \sqrt{3}x - 4$ makes with the Y- axis.
2. Find the distance between the parallel lines $3x+4y-3 = 0$ and $6x+8y-1=0$.
3. If the distance between the points $(5,-1,7)$ and $(x,5,1)$ is 9 units, find the values of x.
4. Write the equation of the plane $4x-4y+2z+5=0$ in the intercept form.
5. Find $\underset{x\to 0}{Lt} \dfrac{e^{3+x} - e^3}{x}$
6. Evaluate $\underset{x\to 3}{Lt} \dfrac{x^2+3x+2}{x^2-6x+9}$
7. Find the derivative function of $\tan^{-1}(\log x)$
8. If $y = \dfrac{2x+3}{4x+5}$ then find y"
9. Derive relative error and percentage error of the variable 'y'
10. Find the absolute extremum of $f(x) = x^2$ defined on $[-2, 2]$

SECTION-B

II. Answer any FIVE of the following SAQs: **$5 \times 4 = 20$**

11. A(5,3) & B(3,–2) are 2 fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9 sq.units.
12. When the origin is shifted to the point (3, – 4) and transformed equation is $x^2 + y^2 = 4$. Find the original equation.
13. If the straight lines $ax + by + c = 0$, $bx + cy + a = 0$ and $cx + ay + b = 0$ are concurrent, then prove that $a^3 + b^3 + c^3 = 3abc$
14. Find $\underset{x\to a}{Lt} \left(\dfrac{x\sin a - a\sin x}{x-a}\right)$
15. Find the derivative of cotx from the first principle.
16. Find the approximate value of $\sqrt[3]{999}$
17. The distance-time formula for the motion of a particle along a straight line is $s = t^3-9t^2 + 24t -18$. Find when and where the velocity is zero.

SECTION-C

III. Answer any FIVE of the following LAQs: **$5 \times 7 = 35$**

18. If Q(h, k) is the image of the point $P(x_1, y_1)$ with respect to the straight line $ax + by + c = 0$ then prove that $(h - x_1) : a = (k - y_1) : b = -2\,(ax_1 + by_1 + c) : (a^2 + b^2)$.
19. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel lines then Prove that (i) $h^2 = ab$ (ii) $af^2 = bg^2$ (iii) the distance between the parallel lines is $2\sqrt{\dfrac{g^2-ac}{a(a+b)}}$ or $2\sqrt{\dfrac{f^2-bc}{b(a+b)}}$
20. Find the value of k, if the lines joining the origin with the points of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the line $x + 2y = k$ are mutually perpendicular.
21. Find the angle between the lines whose d.c's are related by $l + m + n = 0$ & $l^2 + m^2 - n^2 = 0$
22. If $y = x\sqrt{a^2+x^2} + a^2\log\left(x + \sqrt{a^2+x^2}\right)$, then show that $\dfrac{dy}{dx} = 2\sqrt{a^2+x^2}$
23. Find the condition for the orthogonality of the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$
24. Find the points of local extrema and local extrema for the function $f(x) = \cos 4x$ defined on $\left(0, \dfrac{\pi}{2}\right)$

IPE AP MARCH-2019

SOLUTIONS

SECTION-A

1. Find the angle which the straight line $y = \sqrt{3}x - 4$ makes with the Y- axis.

A: Given line is $y=\sqrt{3}x-4$. This is in the form y = mx + c $\Rightarrow$ slope $m=\sqrt{3}$

$\Rightarrow \tan\theta = \sqrt{3} = \tan 60^\circ \Rightarrow$ Angle made by the line with the X-axis is $\theta = 60^\circ$

$\therefore$ Angle made by the line with Y-axis is $90^\circ - 60^\circ = 30^\circ$

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2. Find the distance between the parallel lines 3x+4y–3 = 0 and 6x+8y–1=0.

A: We write the first line 3x+4y–3=0 as 6x+8y–6 = 0 ....(1)

Second line is 6x+8y–1=0 .....(2)

$\therefore$ The distance between (1) & (2) is $\frac{|c_1-c_2|}{\sqrt{a^2+b^2}} = \frac{|-6+1|}{\sqrt{6^2+8^2}} = \frac{|-5|}{\sqrt{36+64}} = \frac{5}{\sqrt{100}} = \frac{5}{10} = \frac{1}{2}$

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3. If the distance between the points (5,–1,7) and (x,5,1) is 9 units, find the values of x.

A: Let A=(5,–1,7) and B=(x,5,1)

Given that AB=9 $\Rightarrow \sqrt{(5-x)^2+(-1-5)^2+(7-1)^2} = 9$

Squaring both sides we have $(5-x)^2 + 36 + 36 = 81 \Rightarrow (5-x)^2 + 72 = 81 \Rightarrow (5-x)^2 = 9$

$\Rightarrow 5 - x = \pm 3 \Rightarrow x = 5 \pm 3 \Rightarrow x = 8$ or 2

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4. Write the equation of the plane 4x–4y+2z+5=0 in the intercept form.

A: The given equation of the plane is 4x–4y+2z+5=0 $\Rightarrow$ 4x–4y+2z = –5

$\Rightarrow \frac{4x}{-5} + \frac{-4y}{-5} + \frac{2z}{-5} = 1 \Rightarrow \frac{x}{\left(\frac{-5}{4}\right)} + \frac{y}{\left(\frac{5}{4}\right)} + \frac{z}{\left(\frac{-5}{2}\right)} = 1$ which is in the intercept form $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$

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5. Find $\lim\limits_{x\to 0} \frac{e^{3+x} - e^3}{x}$

A: $\lim\limits_{x\to 0} \frac{e^{3+x}-e^3}{x} = \lim\limits_{x\to 0} \frac{e^3(e^x-1)}{x} = e^3 \lim\limits_{x\to 0} \frac{e^x-1}{x} = e^3(1) = e^3$

6. Evaluate $\underset{x\to 3}{Lt}\dfrac{x^2+3x+2}{x^2-6x+9}$

A: $\underset{x\to 3}{Lt}\dfrac{x^2+3x+2}{x^2-6x+9}=\underset{x\to 3}{Lt}\dfrac{x^2+3x+2}{(x-3)^2}=\dfrac{9+9+2}{(3-3)^2}=\infty$

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7. Find the derivative function of $Tan^{-1}(\log x)$

A : Given $y=Tan^{-1}(\log x)$, then $\dfrac{dy}{dx}=\dfrac{d}{dx}Tan^{-1}(\log x)$

$$=\frac{1}{1+(\log x)^2}\frac{d}{dx}(\log x)=\left(\frac{1}{1+(\log x)^2}\right)\frac{1}{x}$$

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8. If $y=\dfrac{2x+3}{4x+5}$, then find $\dfrac{dy}{dx}$

A: Formula: $\dfrac{d}{dx}\left(\dfrac{u}{v}\right)=\dfrac{v.\frac{d}{dx}(u)-u.\frac{d}{dx}(v)}{v^2}$

$$\frac{dy}{dx}=\frac{(4x+5)\frac{d}{dx}(2x+3)-(2x+3).\frac{d}{dx}(4x+5)}{(4x+5)^2}=\frac{(4x+5)(2)-(2x+3)(4)}{(4x+5)^2}$$

$$=\frac{\cancel{8x}+10-\cancel{8x}-12}{(4x+5)^2}=\frac{-2}{(4x+5)^2}$$

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9. Derive relative error and percentage error of the variable 'y'

A: Error: If an error Δx occurs in x of y = f(x) then error in y is $\Delta y = f'(x)\,\Delta x$

Relative Error: If an error Δx occurs in x of y = f(x) then relative error in y is $\dfrac{\Delta y}{y}$

Percentage Error: If an error Δx occurs in x of y = f(x) then percentage error in yis $\dfrac{\Delta y}{y}\times 100$

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10. Find the absolute extremum of $f(x) = x^2$ defined on [– 2, 2]

A: Given $f(x)=x^2 \Rightarrow f'(x)=2x$. Now $f'(x)=0 \Rightarrow 2x=0 \Rightarrow x=0$

Also $f(x) = x^2$ is continuous on [–2, 2]. Now $f(2)=2^2=4$; $f(-2)=(-2)^2=4$

Also $f(0)=0^2=0$

Hence the minimum value is 0. The maximum is 4.

$\therefore$ Absolute extremums = {0, 4}

SECTION-B

11. A(5,3) and B(3,–2) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9 sq.units.

A: ★ Let the locus point P=(x,y)

- Given points A=(5,3), B=(3,–2)

Given condition:

Area of Δ PAB = 9 sq.units

★ $\Delta = \frac{1}{2}\begin{vmatrix} x_1 - x_2 & x_1 - x_3 \\ y_1 - y_2 & y_1 - y_3 \end{vmatrix} = 9$

★ $\Rightarrow \frac{1}{2}\begin{vmatrix} 5-3 & 5-x \\ 3+2 & 3-y \end{vmatrix} = 9$

● $\Rightarrow \begin{vmatrix} 2 & 5-x \\ 5 & 3-y \end{vmatrix} = 2(9)$

● $\Rightarrow |2(3-y) - 5(5-x)| = 18$

● $\Rightarrow |6 - 2y - 25 + 5x| = 18$

● $\Rightarrow |5x - 2y - 19| = 18$

★ $\Rightarrow 5x - 2y - 19 = \pm 18$

● $\Rightarrow 5x - 2y - 19 = 18 \text{ (or) } 5x - 2y - 19 = -18$

● $\Rightarrow 5x - 2y - 37 = 0 \text{ (or) } 5x - 2y - 1 = 0$

★ $\Rightarrow (5x - 2y - 37)(5x - 2y - 1) = 0$

Hence, locus of P is $(5x - 2y - 37)(5x - 2y - 1) = 0$

12. When the origin is shifted to the point (3, – 4) and transformed equation is $x^2 + y^2 = 4$. Find the original equation.

A: ★ Given transformed(new) equation is taken as $X^2 + Y^2 = 4$.........(1)

- We take origin (h, k) = (3, –4), then

★ $X = x - h \Rightarrow X = x - 3$

★ $Y = y - k \Rightarrow Y = y + 4$

- From (1), original equation is $(x - 3)^2 + (y + 4)^2 = 0$
- $\Rightarrow (x^2 + 9 - 6x) + (y^2 + 16 + 8y) = 0$
- $\Rightarrow -18+12y+17x-34-7y+21-11=0$
- $\Rightarrow x^2 + y^2 - 6x + 8y + 21 = 0.$

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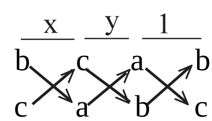
13. If the straight lines ax + by + c = 0, bx + cy + a = 0 and cx + ay + b = 0 are concurrent, then prove that $a^3 + b^3 + c^3 = 3abc$.

A: Given lines $ax + by + c = 0$ ....(1)

$bx + cy + a = 0$ ....(2)

Solving (1) and (2), we get P, $\dfrac{x}{ba-(c)(c)} = \dfrac{y}{cb-(a)(a)} = \dfrac{1}{ac-(b)(b)}$



$$\Rightarrow \frac{x}{ab-c^2} = \frac{y}{bc-a^2} = \frac{1}{ca-b^2} \Rightarrow x = \frac{ab-c^2}{ca-b^2} \text{ and } y = \frac{bc-a^2}{ca-b^2}$$

$\therefore$ Point of intersection $P = \left(\dfrac{ab-c^2}{ca-b^2}, \dfrac{bc-a^2}{ca-b^2}\right)$

But $P\left(\dfrac{ab-c^2}{ca-b^2}, \dfrac{bc-a^2}{ca-b^2}\right)$ lies on $cx + ay + b = 0$ [$\because$ Given lines are concurrent.]

$$\Rightarrow c\left(\frac{ab-c^2}{ca-b^2}\right) + a\left(\frac{bc-a^2}{ca-b^2}\right) + b = 0 \Rightarrow c(ab-c^2) + a(bc-a^2) + b(ca-b^2) = 0$$

$$\Rightarrow cab - c^3 + abc - a^3 + bca - b^3 = 0 \Rightarrow 3abc = a^3 + b^3 + c^3$$

Hence, proved that $a^3 + b^3 + c^3 = 3abc$

14. Find $\underset{x\to a}{Lt}\left(\frac{x\sin a - a\sin x}{x-a}\right)$

A: $\underset{x\to a}{Lt}\frac{x\sin a - a\sin x}{x-a} = \underset{x\to a}{Lt}\frac{(x\sin a - a\sin a)-(a\sin x - a\sin a)}{x-a}$ ($\because$ Subtract and Add asina)

$$= \underset{x\to a}{Lt}\frac{\sin a(x-a)-a(\sin x-\sin a)}{x-a} = \underset{x\to a}{Lt}\left(\frac{(x-a)\sin a}{x-a}\right) - a\underset{x\to a}{Lt}\left(\frac{\sin x - \sin a}{x-a}\right)$$

$$= \underset{x\to a}{Lt}\sin a - a\underset{x\to a}{Lt}\frac{2\cos\left(\frac{x+a}{2}\right)\sin\left(\frac{x-a}{2}\right)}{x-a}$$

$$= \sin a - 2a\underset{x\to a}{Lt}\cos\left(\frac{x+a}{2}\right).\underset{x\to a}{Lt}\frac{\sin\left(\frac{x-a}{2}\right)}{(x-a)} = \sin a - 2a\cos\left(\frac{a+a}{2}\right).\frac{1}{2} = \sin a - a\cos a$$

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15. Find the derivative of cotx from the first principle.

A: We take f(x) = cotx , then

f(x + h) = cot(x + h)

From the first principle,

$$f'(x) = \underset{h\to 0}{Lt}\frac{f(x+h)-f(x)}{h}$$

$$= \underset{h\to 0}{Lt}\frac{\cot(x+h)-\cot(x)}{h} = \underset{h\to 0}{Lt}\frac{1}{h}\left[\frac{\cos(x+h)}{\sin(x+h)} - \frac{\cos x}{\sin x}\right]$$

$$= \underset{h\to 0}{Lt}\frac{1}{h}\left[\frac{\cos(x+h).\sin x - \sin(x+h).\cos x}{\sin(x+h).\sin x}\right]$$

$$= \underset{h\to 0}{Lt}\frac{1}{h}\left[\frac{-\sin((x+h)-x)}{\sin(x+h)\sin x}\right] \qquad [\because \cos A\sin B - \sin A\cos B = -\sin(A-B)]$$

$$= \underset{h\to 0}{Lt}\frac{1}{h}\left[\frac{-\sinh}{\sin(x+h)\sin x}\right]$$

$$= \underset{h\to 0}{Lt}\left(\frac{-\sinh}{h}\right).\underset{h\to 0}{Lt}\frac{1}{\sin(x+h).\sin x}$$

$$= -1\left(\frac{1}{\sin x.\sin x}\right) = -\operatorname{cosec}^2 x$$

16. Find the approximate value of $\sqrt[3]{999}$

A: Given $\sqrt[3]{999} = \sqrt[3]{1000-1}$

$\therefore$ known value x = 1000 and $\Delta x = -1$.

Let, $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$$\Rightarrow f'(x) = \frac{1}{3}x^{\frac{1}{3}-1} = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{2/3}}$$

Formula: $f(x+\Delta x) = [f(x) + f'(x)\Delta x]_{\text{at known } x}$

$$\therefore \sqrt[3]{999} \cong \sqrt[3]{x} + \frac{1}{3x^{2/3}}\Delta x$$

$$= \sqrt[3]{1000} + \frac{1}{3(1000)^{2/3}}(-1)$$

$$= 10 + \frac{1}{3(10^3)^{2/3}}(-1) = 10 - \frac{1}{3(10^2)} = 10 - \frac{1}{3(100)} = 10 - \frac{1}{300} = 10 - 0.0033 = 9.9967$$

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17. The distance-time formula for the motion of a particle along a straight line is $s = t^3 - 9t^2 + 24t - 18$. Find when and where the velocity is zero.

A: Given that $s = t^3 - 9t^2 + 24t - 18$

$$\Rightarrow \text{velocity } v = \frac{ds}{dt} = 3t^2 - 18t + 24 = 3(t^2 - 6t + 8) = 3(t-2)(t-4)$$

If velocity is zero then $(t-2)(t-4) = 0 \Rightarrow t = 2$ or 4

$\therefore$ the velocity becomes zero after 2 seconds and 4 seconds.

If t = 2 then $s = t^3 - 9t^2 + 24t - 18 = 2^3 - 9(2^2) + 24(2) - 18 = 8 - 36 + 48 - 18 = 56 - 54 = 2$

If t = 4 then $s = t^3 - 9t^2 + 24t - 18$

$$= 4^3 - 9(4^2) + 24(4) - 18 = 64 - 144 + 96 - 18 = 160 - 162 = -2$$

$\therefore$ the particle is at a distance of 2 units on either side of the starting point.

SECTION-C

18. If Q(h, k) is the image of the point $P(x_1, y_1)$ w.r.to the straight line $ax + by + c = 0$ then prove that $(h - x_1) : a = (k - y_1) : b = -2(ax_1 + by_1 + c) : (a^2 + b^2)$.

A: • Given $P = (x_1, y_1)$, $Q = (h, k)$

- So, slope of $\overline{PQ}$ is $m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{k - y_1}{h - x_1}$
- Slope of the line $ax + by + c = 0$ is $m_2 = -\frac{a}{b}$
- Two lines are perpendicular $\Rightarrow m_1 m_2 = -1$

$\Rightarrow \left(\frac{k - y_1}{h - x_1}\right)\left(\not- \frac{a}{b}\right) = \not-1 \Rightarrow \left(\frac{k - y_1}{h - x_1}\right)\left(\frac{a}{b}\right) = 1$

$\Rightarrow \frac{k - y_1}{h - x_1} = \frac{b}{a} \Rightarrow \frac{k - y_1}{b} = \frac{h - x_1}{a}$

- We take $\frac{h - x_1}{a} = \frac{k - y_1}{b} = r$ ....(1)

$\therefore \frac{h - x_1}{a} = r \Rightarrow h - x_1 = ar \Rightarrow h = x_1 + ar$

$\frac{k - y_1}{b} = r \Rightarrow k - y_1 = br \Rightarrow k = y_1 + br$

Now, $P = (x_1, y_1)$, $Q = (h, k)$ $\Rightarrow$Mid point of $\overline{PQ}$ is $R = \left(\frac{x_1 + h}{2}, \frac{y_1 + k}{2}\right)$

- But R lies on $ax + by + c = 0$

$\Rightarrow a\left(\frac{x_1 + h}{2}\right) + b\left(\frac{y_1 + k}{2}\right) + c = 0$

$\Rightarrow \frac{a(x_1 + h) + b(y_1 + k) + 2c}{2} = 0$

$\Rightarrow a(x_1 + h) + b(y_1 + k) + 2c = 0$

$\Rightarrow a[x_1 + (x_1 + ar)] + b[y_1 + (y_1 + br)] + 2c = 0$

$\Rightarrow a[2x_1 + ar] + b[2y_1 + br] + 2c = 0$

$\Rightarrow 2ax_1 + a^2 r + 2by_1 + b^2 r + 2c = 0$

$\Rightarrow a^2 r + b^2 r + 2ax_1 + 2by_1 + 2c = 0$

$\Rightarrow r(a^2 + b^2) = -2(ax_1 + by_1 + c)$

$\Rightarrow r = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$............(2)

- From (1) & (2), $\frac{h - x_1}{a} = \frac{k - y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$

19. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel lines then Prove that (i) $h^2 = ab$ (ii) $af^2 = bg^2$ (iii) the distance between the parallel lines is $2\sqrt{\dfrac{g^2 - ac}{a(a+b)}}$ (or) $2\sqrt{\dfrac{f^2 - bc}{b(a+b)}}$

A: ★ Let $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0 \equiv (lx + my + n_1)(lx + my + n_2)$

- On equating like term coeff. , we get

★ $a = l^2,\ b = m^2,\ h = lm,\ 2g = l(n_1 + n_2),\ 2f = m(n_1 + n_2),\ c = n_1 n_2$

★ (i) $h^2 = (lm)^2 = l^2 m^2 = ab \Rightarrow h^2 = ab$

★

$$\text{(ii)}\ \ af^2 = l^2\left(\frac{m(n_1+n_2)}{2}\right)^2 = \frac{l^2 m^2 (n_1+n_2)^2}{4} = \frac{m^2 l^2 (n_1+n_2)^2}{4} = m^2\left(\frac{l(n_1+n_2)}{2}\right)^2 = bg^2$$

★ (iii) Distance between $lx + my + n_1 = 0,\ lx + my + n_2 = 0$ is $\dfrac{|n_1 - n_2|}{\sqrt{l^2 + m^2}}$

★

$$= \frac{\sqrt{(n_1+n_2)^2 - 4n_1 n_2}}{\sqrt{a+b}} = \sqrt{\frac{\left(\frac{2g}{l}\right)^2 - 4c}{a+b}} = \sqrt{\frac{\frac{4g^2}{l^2} - 4c}{a+b}} = \sqrt{\frac{\frac{4g^2}{a} - 4c}{a+b}} = \sqrt{\frac{4g^2 - 4ac}{a(a+b)}} = 2\sqrt{\frac{g^2 - ac}{a(a+b)}}$$

★ Similarly, by taking $n_1 + n_2 = \dfrac{2f}{m}$ we get, the distance between the lines $2\sqrt{\dfrac{f^2 - bc}{b(a+b)}}$

20. Find the value of k, if the lines joining the origin with the points of intersection of the curve $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$ and the line $x + 2y = k$ are mutually perpendicular.

A:
- The given line is x+2y=k $\Rightarrow \frac{x+2y}{k} = 1$ ...(1)
- Given curve is $2x^2 - 2xy + 3y^2 + 2x - y - 1 = 0$............(2)
- Homogenising (1) & (2), we get

★ $2x^2 - 2xy + 3y^2 + 2x(1) - y(1) - (1)^2 = 0$

★ $\Rightarrow 2x^2 - 2xy + 3y^2 + 2x\left(\frac{x+2y}{k}\right) - y\left(\frac{x+2y}{k}\right) - \frac{(x+2y)^2}{k^2} = 0$

★ $\Rightarrow \frac{k^2(2x^2 - 2xy + 3y^2) + k(2x^2 + 4xy) - k(xy + 2y^2) - (x^2 + 4y^2 + 4xy)}{k^2} = 0$

★ $\Rightarrow k^2(2x^2 - 2xy + 3y^2) + k(2x^2 + 4xy) - k(xy + 2y^2) - (x^2 + 4y^2 + 4xy) = 0$

★ $\Rightarrow x^2(2k^2 + 2k - 1) + y^2(3k^2 - 2k - 4) + xy(-2k^2 + 3k - 4) = 0$

- If this pair or lines are perpendicular then

★ Coeff. x^2 + Coeff. y^2 =0

- $\Rightarrow (2k^2 \cancel{+2k} - 1) + (3k^2 \cancel{-2k} - 4) = 0 \Rightarrow 5k^2 - 5 = 0$
- $\Rightarrow \cancel{5}(k^2 - 1) = 0 \Rightarrow k^2 - 1 = 0 \Rightarrow k^2 = 1 \Rightarrow k = \pm 1$

Hence, value of $k = \pm 1$

21. Find the angle between the lines whose d.c's are related by $l+m+n=0$ and $l^2+m^2-n^2=0$

A:
- Given $l + m + n = 0 \Rightarrow l = -(m + n)$ .....(1),
- $l^2 + m^2 - n^2 = 0$ ......(2)
- Solving (1) & (2) we get $[-(m + n)]^2 + m^2 - n^2 = 0$

$\Rightarrow (m^2 \cancel{+n^2} +2mn)+m^2 \cancel{-n^2} =0$

$\Rightarrow 2m^2 + 2mn=0 \Rightarrow \cancel{2}(m^2 +mn)=0$

$\Rightarrow m^2 + mn = 0 \Rightarrow m(m + n) = 0$

★ $\Rightarrow m = 0$ (or) $m + n = 0 \Rightarrow m = 0$ (or) $m = -n$

- **Case (i):** Put m = 0 in (1), then

$l = -(0 + n) = -n$

$\therefore l = -n$

Now, $l : m : n = \cancel{-n} : 0 : \cancel{n} = -1 : 0 : 1$

★So, d.r's of $L_1 = (a_1, b_1, c_1) = (-1, 0, 1)$ ......(3)

Case (ii): Put $m = -n$ in (1), then $l = -(-n + n) = 0$

$\therefore l = 0$

Now, $l : m : n = 0 : -\cancel{n} : \cancel{n} = 0 : -1 : 1$

So, d.r's of $L_2 = (a_2, b_2, c_2) = (0, -1, 1)$ ...(4)

- If θ is the angle between the lines then from (3), (4), we get

★ $\cos\theta = \dfrac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{(a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2)}}$

$= \dfrac{|(-1)(0) + (0)(-1) + 1(1)|}{\sqrt{((-1)^2 + 0^2 + 1^2)(0^2 + (-1)^2 + 1^2)}}$

★ $= \dfrac{1}{\sqrt{(2)(2)}} = \dfrac{1}{\sqrt{4}} = \dfrac{1}{2} = \cos 60^\circ \Rightarrow \theta = 60^\circ$

Hence angle between the lines is 60°.

22. If $y = x\sqrt{a^2+x^2} + a^2\log\left(x+\sqrt{a^2+x^2}\right)$, then show that $\frac{dy}{dx} = 2\sqrt{a^2+x^2}$

A:
- Given $y = x\sqrt{a^2+x^2} + a^2\log(x+\sqrt{a^2+x^2})$;
- On diff. w.r.to x, we get

★ $\frac{dy}{dx} = \left[x\frac{d}{dx}\sqrt{a^2+x^2} + \sqrt{a^2+x^2}\frac{d}{dx}(x)\right] + a^2\left[\frac{d}{dx}\log(x+\sqrt{a^2+x^2})\right]$

★ $= \left(x\frac{1}{2\sqrt{a^2+x^2}}\frac{d}{dx}(a^2+x^2) + (\sqrt{a^2+x^2})(1)\right) + a^2\left(\frac{1}{x+\sqrt{a^2+x^2}}\frac{d}{dx}(x+\sqrt{a^2+x^2})\right)$

- $= \left(\frac{x}{\cancel{2}\sqrt{a^2+x^2}}(\cancel{2}x) + \sqrt{a^2+x^2}\right) + \left(\frac{a^2}{x+\sqrt{a^2+x^2}}\right)\left(1+\frac{1}{\cancel{2}\sqrt{a^2+x^2}}(\cancel{2}x)\right)$

- $= \left(\frac{x^2}{\sqrt{a^2+x^2}} + \sqrt{a^2+x^2}\right) + \left(\frac{a^2}{x+\sqrt{a^2+x^2}}\right)\left(1+\frac{x}{\sqrt{a^2+x^2}}\right)$

★ $= \left(\frac{x^2+(a^2+x^2)}{\sqrt{a^2+x^2}}\right) + \left(\frac{a^2}{\cancel{x+\sqrt{a^2+x^2}}}\right)\left(\frac{\cancel{\sqrt{a^2+x^2}+x}}{\sqrt{a^2+x^2}}\right)$

- $= \frac{a^2+2x^2}{\sqrt{a^2+x^2}} + \frac{a^2}{\sqrt{a^2+x^2}}$

- $= \frac{a^2+2x^2+a^2}{\sqrt{a^2+x^2}}$

- $= \frac{2a^2+2x^2}{\sqrt{a^2+x^2}}$

- $= \frac{2(a^2+x^2)}{\sqrt{a^2+x^2}}$

★ $= 2\sqrt{a^2+x^2}$ $\quad [\because \frac{a}{\sqrt{a}} = \sqrt{a}]$

$\therefore \boxed{\frac{dy}{dx} = 2\sqrt{a^2+x^2}}$

23. Find the condition for the orthogonality of the curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$.

A: 1) Finding Point of intersection:

- Take $P(x_1, y_1)$ as the point of intersection
- So, $P(x_1, y_1)$ lies on both curves $ax^2 + by^2 = 1$ and $a_1x^2 + b_1y^2 = 1$

★ $\Rightarrow ax_1^2 + by_1^2 = 1$ and $a_1x_1^2 + b_1y_1^2 = 1$

★ $\Rightarrow ax_1^2 + by_1^2 = a_1x_1^2 + b_1y_1^2 \Rightarrow ax_1^2 - a_1x_1^2 = b_1y_1^2 - by_1^2 \Rightarrow x_1^2(a - a_1) = y_1^2(b_1 - b)$

★ $\Rightarrow \dfrac{x_1^2}{y_1^2} = \dfrac{b_1 - b}{a - a_1} = \dfrac{-(b - b_1)}{a - a_1}$ ....(1)

2) Finding Derivatives & Slopes:

- Given $ax^2 + by^2 = 1$, on differentiating w.r.to x, we get
- $2ax + 2by\dfrac{dy}{dx} = 0 \Rightarrow \dfrac{dy}{dx} = -\dfrac{ax}{by}$

★ So slope of the tangent at $P(x_1,y_1)$ is $m_1 = -\dfrac{ax_1}{by_1}$

★ Similarly, we get $m_2 = -\dfrac{a_1x_1}{b_1y_1}$

3) Applying orthogonal condition:

★ But $m_1m_2 = -1$ [$\because$ Curves intersect orthogonally]

★ $\Rightarrow \left(\dfrac{-ax_1}{by_1}\right)\left(\dfrac{-a_1x_1}{b_1y_1}\right) = -1 \Rightarrow \dfrac{aa_1x_1^2}{bb_1y_1^2} = -1 \Rightarrow \dfrac{x_1^2}{y_1^2} = \dfrac{-bb_1}{aa_1}$ ....(2)

★ Equating (1) & (2), $\dfrac{bb_1}{aa_1} = \dfrac{b - b_1}{a - a_1} \Rightarrow \dfrac{a - a_1}{aa_1} = \dfrac{b - b_1}{bb_1}$

- $\Rightarrow \dfrac{a}{aa_1} - \dfrac{a_1}{aa_1} = \dfrac{b}{bb_1} - \dfrac{b_1}{bb_1}$

★ $\Rightarrow \dfrac{1}{a_1} - \dfrac{1}{a} = \dfrac{1}{b_1} - \dfrac{1}{b} \Rightarrow \dfrac{1}{a_1} - \dfrac{1}{b_1} = \dfrac{1}{a} - \dfrac{1}{b}$

Hence proved.

24. Find the points of local extrema and local extrema for f(x) = cos4x defined on $(0, \pi/2)$

A: Given $f(x) = \cos 4x$ and is defined on $\left(0, \frac{\pi}{2}\right)$

$\therefore f'(x) = -4\sin 4x$ and $f''(x) = -16\cos 4x$

Stationary points are the roots of $f'(x) = 0$ lying in the domain $\left(0, \frac{\pi}{2}\right)$

$f'(x) = 0 \Rightarrow -4\sin 4x = 0 \Rightarrow \sin 4x = 0$

$\Rightarrow 4x = 0, \pi, 2\pi, 3\pi, 4\pi, \ldots\ldots\ldots\ldots \Rightarrow x = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \ldots\ldots\ldots\ldots$

The point lying in the given domain is $x = \frac{\pi}{4}$ only.

Thus $x = \frac{\pi}{4}$ is the stationary point of the given function,

Now $f''\left(\frac{\pi}{4}\right) = -16\cos 4\left(\frac{\pi}{4}\right) = -16\pi = -16(-1) = 16 > 0.$

$\therefore$ The function f has local maximum at $x = \frac{\pi}{4}$ and

its local minimum value is $f\left(\frac{\pi}{4}\right) = \cos 4\left(\frac{\pi}{4}\right) = \cos \pi = -1$