

9. HYPERBOLIC FUNCTIONS

1 x 2 = 2 Marks

 IMP FORMULAS, KEY CONCEPTS 

Hyperbolic function	Definition	Domain	Range
1. $\sinh x$	$\frac{e^x - e^{-x}}{2}$	\mathbb{R}	\mathbb{R}
2. $\cosh x$	$\frac{e^x + e^{-x}}{2}$	\mathbb{R}	$[1, \infty)$
3. $\tanh x$	$\frac{e^x - e^{-x}}{e^x + e^{-x}}$	\mathbb{R}	$(-1, 1)$
4. $\coth x$	$\frac{e^x + e^{-x}}{e^x - e^{-x}}$	$\mathbb{R} - \{0\}$	$(-\infty, -1) \cup (1, \infty)$
5. $\operatorname{sech} x$	$\frac{2}{e^x + e^{-x}}$	\mathbb{R}	$(0, 1]$
6. $\operatorname{csch} x$	$\frac{2}{e^x - e^{-x}}$	$\mathbb{R} - \{0\}$	$\mathbb{R} - \{0\}$
7. $\operatorname{Sinh}^{-1} x$	$\log(x + \sqrt{x^2 + 1})$	\mathbb{R}	\mathbb{R}
8. $\operatorname{Cosh}^{-1} x$	$\log(x + \sqrt{x^2 - 1})$	$[1, \infty)$	$[0, \infty)$
9. $\operatorname{Tanh}^{-1} x$	$\frac{1}{2} \log\left(\frac{1+x}{1-x}\right)$	$(-1, 1)$	\mathbb{R}
10. $\operatorname{Coth}^{-1} x$	$\frac{1}{2} \log\left(\frac{x+1}{x-1}\right)$	$(-\infty, -1) \cup (1, \infty)$	$\mathbb{R} - \{0\}$
11. $\operatorname{Sech}^{-1} x$	$\log\left(\frac{1 + \sqrt{1 - x^2}}{x}\right)$	$(0, 1]$	$[0, \infty)$
12. $\operatorname{Csch}^{-1} x$	$\log\left(\frac{1 + \sqrt{1 + x^2}}{x}\right)$	$(0, \infty)$	$\mathbb{R} - \{0\}$

Hyperbolic identities

Verses

Trigonometric identities

$$13. \sinh 2x = 2 \sinh x \cosh x.$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$14. \cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$15. \cosh^2 x - \sinh^2 x = 1$$

$$\cos^2 \theta + \sin^2 \theta = 1$$

$$16. \operatorname{sech}^2 x + \tanh^2 x = 1$$

$$\sec^2 \theta - \tan^2 \theta = 1$$

$$17. \operatorname{coth}^2 x - \operatorname{csch}^2 x = 1$$

$$\operatorname{csc}^2 \theta - \cot^2 \theta = 1$$