

9. DIFFERENTIATION

(2 x 2) + (1 x 4) + (1 x 7) = 15 Marks

1. Derivative of $f(x)$ using First principles $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$; $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

2. If u, v are functions of x then $\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$; $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

3. Chain Rule: $y = f(u), u = g(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ i.e., $y = f(g(x)) \Rightarrow \frac{dy}{dx} = f'(g(x)) \cdot g'(x)$

4. Derivatives of some standard & useful functions:

$$\frac{d}{dx} x^n = nx^{n-1}; \frac{d}{dx}(k) = 0; \frac{d}{dx}(kx) = k; \frac{d}{dx} kx^2 = 2kx; \frac{d}{dx} kx^3 = 3kx^2; \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$$

$$\frac{d}{dx} \frac{1}{x^n} = \frac{-n}{x^{n+1}}; \frac{d}{dx} \left(\frac{1}{x}\right) = \frac{-1}{x^2}, \frac{d}{dx} \frac{1}{\sqrt{x}} = \frac{-1}{2x\sqrt{x}}$$

$$\frac{d}{dx}(ax + b) = a; \frac{d}{dx}(ax + b)^2 = 2(ax + b)(a); \frac{d}{dx} \sqrt{ax + b} = \frac{a}{2\sqrt{ax + b}}$$

$$\frac{d}{dx} \log x = \frac{1}{x}; \frac{d}{dx} \log_a x = \frac{1}{x} \log_a e; \frac{d}{dx} \log f(x) = \frac{1}{f(x)} \cdot f'(x)$$

$$\frac{d}{dx} e^x = e^x; \frac{d}{dx} e^{f(x)} = e^{f(x)} f'(x); \frac{d}{dx} a^x = a^x \log_e a; \frac{d}{dx} x^x = x^x (1 + \log x)$$

Trigonometric

Hyperbolic

Inverse Trigonometric

Inverse Hyperbolic

$$1. \frac{d}{dx} \sin x = \cos x$$

$$\frac{d}{dx} \sinh x = \cosh x$$

$$\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \sinh^{-1} x = \frac{1}{\sqrt{1+x^2}}$$

$$2. \frac{d}{dx} \cos x = -\sin x$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx} \cosh^{-1} x = \frac{1}{\sqrt{x^2-1}}$$

$$3. \frac{d}{dx} \tan x = \sec^2 x$$

$$\frac{d}{dx} \tanh x = \operatorname{sech}^2 x$$

$$\frac{d}{dx} \tan^{-1} x = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \tanh^{-1} x = \frac{1}{1-x^2}$$

$$4. \frac{d}{dx} \cot x = -\operatorname{csc}^2 x$$

$$\frac{d}{dx} \coth x = -\operatorname{csch}^2 x$$

$$\frac{d}{dx} \cot^{-1} x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx} \coth^{-1} x = \frac{1}{1-x^2}$$

$$5. \frac{d}{dx} \sec x = \sec x \tan x;$$

$$\frac{d}{dx} \operatorname{sech} x = -\operatorname{sech} x \tanh x;$$

$$\frac{d}{dx} \sec^{-1} x = \frac{1}{|x| \sqrt{x^2-1}}$$

$$\frac{d}{dx} \operatorname{sech}^{-1} x = \frac{-1}{|x| \sqrt{1-x^2}}$$

$$6. \frac{d}{dx} \csc x = -\csc x \cot x;$$

$$\frac{d}{dx} \operatorname{csch} x = -\operatorname{csch} x \coth x;$$

$$\frac{d}{dx} \csc^{-1} x = \frac{-1}{|x| \sqrt{x^2-1}}$$

$$\frac{d}{dx} \operatorname{csch}^{-1} x = \frac{-1}{|x| \sqrt{1+x^2}}$$

5. **Logarithmic Differentiation:** If $y = f(x)^{g(x)}$ then $\log y = g(x) \log f(x) \Rightarrow \frac{1}{y} \frac{dy}{dx} = g(x) \frac{1}{f(x)} f'(x) + \log f(x) g'(x)$

6. **Derivative of a function w.r.t another function:** If $y = f(x), z = g(x)$ then $\frac{dy}{dz} = \frac{df}{dg} = \frac{f'(x)}{g'(x)}$

7. **Parametric differentiation:** If $x = f(t), y = g(t)$ then $\frac{dy}{dx} = \frac{g'(t)}{f'(t)}$ and $\frac{d^2y}{dx^2} = \left[\frac{d}{dt} \left(\frac{dy}{dx} \right) \right] \left(\frac{dt}{dx} \right)$

BULLET MASTER'S
MATH BEATS!

DIFFERENTIATION in Calculus Vs STRAIGHT LINES in Geometry

IPE Weightage for Differentiation (2 + 4 + 7 = 15 Marks)

IPE Weightage for Straight Lines (2 + 4 + 7 = 15 Marks)

Straight Lines లో Slope లాంటిదే Differentiation లో Derivative.

Derivative అంటే Slope Formula వంటిదే అనే Secret కొద్ది మందికే తెలుసు!

Ex : $y = 2x + 3$; Slope $m = 2$ and Derivative $dy/dx = 2$

So, Straight Lines లో Line Equations కి Slope కట్టడం వంటిదే...

Derivatives లో Curve Equations కి Derivative కట్టడం!

Straight Lines లో Line కి Slope ను findout చేశాక **Line Equation** $y - y_1 = m(x - x_1)$

Derivatives లో Tangent కి Slope ను findout చేశాక **Tangent equation** $y - y_1 = m(x - x_1)$

Straight Lines లో Line కి Slope ను findout చేశాక **Perpendicular Line equation** $y - y_1 = -1/m(x - x_1)$

Derivatives లో Tangent కి Slope findout చేశాక **Normal equation** $y - y_1 = -1/m(x - x_1)$