# 8. LIMITS & CONTINUITY

(2 x 2) + (1 x 4) = 8 Marks

## IMP FORMULAS, KEY CONCEPTS

**1) Standard Limits:**

**1.1)** $\underset{x\to a}{Lt}\frac{x^n-a^n}{x-a}=na^{n-1}$; $\underset{x\to a}{Lt}\frac{x^m-a^m}{x^n-a^n}=\frac{m}{n}a^{m-n}$

**1.2)** $\underset{x\to 0}{Lt}\frac{\sin x}{x}=1$; $\underset{x\to 0}{Lt}\frac{\tan x}{x}=1$; $\underset{x\to 0}{Lt}\frac{\sin(kx)}{x}=k$; $\underset{x\to 0}{Lt}\frac{\tan kx}{x}=k$

**1.3)** $\underset{x\to 0}{Lt}\frac{a^x-1}{x}=\log_e a$

**1.4)** $\underset{x\to 0}{Lt}(1+x)^{\frac{1}{x}}=e$; $\underset{x\to\infty}{Lt}\left(1+\frac{1}{x}\right)^x=e$

**2) Indeterminate forms:** $\frac{0}{0},\frac{\infty}{\infty},1^\infty, 0x\infty, \infty-\infty, 0^\infty, \infty^0, 0^0$

**3) Evaluation of $\underset{x\to a}{Lt}f(x)$:** First, check whether f(a) exists as a real number or not. If f(a) exists as a real then $\underset{x\to a}{Lt}f(x)=f(a)$. For if, f(a) takes the indeterminate form such as $\frac{0}{0},\frac{\infty}{\infty},1^\infty$ etc., then reduce the given limit into standard limit forms (or) rationalise the numerator / denominator or both accordingly (or) factorise; (or) take proper substitutions and proceed accordingly.

**4.1) Left handed Limit(L.H.L):** If x approaches a from the left i.e., through values just smaller than a then the limit of f defined in the usual sense is called the left limit of f(x) and it is written as $\underset{x\to a-}{Lt}f(x)$

☞ L.H.L : $\underset{x\to a-}{Lt}f(x)=\underset{h\to 0+}{Lt}f(a-h)=\underset{x\to 0}{Lt}f(a-x)$, $(x\to a-\Rightarrow x<a)$

**4.2) Right handed Limit(R.H.L):** If x approaches a from the right i.e., through values just larger than a then the limit of f defined in the usual sense is called the right limit of f(x) and it is written as $\underset{x\to a+}{Lt}f(x)$

☞ R.H.L : $\underset{x\to a+}{Lt}f(x)=\underset{h\to 0+}{Lt}f(a+h)=\underset{x\to 0}{Lt}f(a+x)$, $(x\to a+\Rightarrow x>a)$

**Note:** If L.H.L=R.H.L then we say the limit exists, otherwise the limit does not exist.

**5)** A function f(x) is said to be **continuous** at x=a if

(i) f(a) exists (naturally or by attribution) (ii) $f(a)=\underset{x\to a+}{Lt}f(x)=\underset{x\to a-}{Lt}f(x)$

If any of the above conditions is not satisfied then f(x) is said be discontinuous at x = a.