7. THE PLANE

1 x 2 = 2 Marks

IMP FORMULAS, KEY CONCEPTS

1.1) The general equation of a plane is $ax + by + cz + d = 0$

1.2) In the equation $ax + by + cz + d = 0$, the coefficients a,b,c represent the d.r's of the normal to the plane.

1.3) The equation of any plane parallel to $ax + by + cz + d = 0$ is of the form $ax + by + cz + k = 0$

1.4) The equation of the plane passing through (x_1, y_1, z_1) and perpendicular to the line with d.r's (a, b, c) is $a(x - x_1) + b(y - y_1) + c(z - z_1) = 0$

2) The equation of a plane which is at a distance p from the origin and with l,m,n as d.c's of the normal, is $lx + my + nz = p$ (normal form).

3) The perpendicular distance from the origin O(0, 0, 0) to the plane $ax + by + cz + d = 0$ is $\frac{|d|}{\sqrt{a^2 + b^2 + c^2}}$

4) The perpendicular distance from $A(x_1, y_1, z_1)$ to the plane $ax + by + cz + d = 0$ is $\frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$

5) The distance between the parallel planes $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$ is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$

6) The equation of the plane with x-intercept a, y-intercept b, z-intercept c is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

7) The intercepts of the plane $ax + by + cz + d = 0$ are $-\frac{d}{a}, \frac{-d}{b}, \frac{-d}{c}$

8) The equation of the plane passing through 3 non-collinear points $A(x_1, y_1, z_1)$,$B(x_2, y_2, z_2)$, $C(x_3, y_3, z_3)$ is $\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$

9) If θ is an angle between the planes $a_1x + b_1y + c_1z + d_1 = 0$, $a_2x + b_2y + c_2z + d_2 = 0$ then

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{(a_1^2 + b_1^2 + c_1^2)(a_2^2 + b_2^2 + c_2^2)}}$$

10) The planes $\Pi_1 = a_1x + b_1y + c_1z + d_1 = 0$, $\Pi_2 = a_2x + b_2y + c_2z + d_2 = 0$ are

(i) parallel $\Leftrightarrow a_1 : b_1 : c_1 = a_2 : b_2 : c_2$ (ii) perpendicular $\Leftrightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$