5.PRODUCT OF VECTORS

(1 x 2) + (1 x 4) + (1 x 7) = 13 Marks

1. Scalar(dot) Product Verses Vector (Cross) Product

	Scalar Product	Vector Product
1.	$\bar{a}.\bar{b} = \lvert\bar{a}\rvert\lvert\bar{b}\rvert\cos\theta$	$\bar{a}\times\bar{b} = \lvert\bar{a}\rvert\lvert\bar{b}\rvert\sin\theta\ \hat{n}$
2.	$\cos\theta = \dfrac{\bar{a}.\bar{b}}{\lvert\bar{a}\rvert\lvert\bar{b}\rvert}$	$\sin\theta = \dfrac{\lvert\bar{a}\times\bar{b}\rvert}{\lvert\bar{a}\rvert\lvert\bar{b}\rvert}$; $\hat{n} = \dfrac{\bar{a}\times\bar{b}}{\lvert\bar{a}\times\bar{b}\rvert}$ ☞ a vector of magnitude k, ⊥ to $\bar{a}$ & $\bar{b} = k\left(\dfrac{\bar{a}\times\bar{b}}{\lvert\bar{a}\times\bar{b}\rvert}\right)$
3.	$\bar{a}.\bar{b} = \bar{b}.\bar{a}$ (commutative law)	$\bar{a}\times\bar{b} \neq \bar{b}\times\bar{a}$ but $\bar{a}\times\bar{b} = -\bar{b}\times\bar{a}$
4.	$\bar{a}.\bar{a} = \lvert\bar{a}\rvert^2 = a^2$	$\bar{a}\times\bar{a} = \bar{0}$
5.	$\bar{a},\bar{b}$ are perpendicular $\Rightarrow \bar{a}.\bar{b} = 0$	$\bar{a},\bar{b}$ are parallel $\Rightarrow \bar{a}\times\bar{b} = \bar{0}$
6.	$\bar{i}.\bar{j} = 0, \bar{j}.\bar{k} = 0, \bar{k}.\bar{i} = 0$ $\bar{i}.\bar{i} = 1, \bar{j}.\bar{j} = 1, \bar{k}.\bar{k} = 1$	$\bar{i}\times\bar{i} = \bar{0}, \bar{j}\times\bar{j} = \bar{0}, \bar{k}\times\bar{k} = \bar{0}$ $\bar{i}\times\bar{j} = \bar{k}, \bar{j}\times\bar{k} = \bar{i}, \bar{k}\times\bar{i} = \bar{j}$
7.	If $\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}; \bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k}$ then $\bar{a}.\bar{b} = a_1b_1 + a_2b_2 + a_3b_3$	$\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}; \bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k} \Rightarrow \bar{a}\times\bar{b} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$
8.	**Projections**: (i) Projection Vector of $\bar{b}$ on $\bar{a} = \dfrac{(\bar{a}.\bar{b})\bar{a}}{\lvert\bar{a}\rvert^2}$ (ii) Length of projection of $\bar{b}$ on $\bar{a} = \dfrac{\lvert\bar{a}.\bar{b}\rvert}{\lvert\bar{a}\rvert}$ (iii) Component of $\bar{b}$ ⊥^{r} to $\bar{a} = \bar{b} - \dfrac{(\bar{a}.\bar{b})\bar{a}}{\lvert\bar{a}\rvert^2}$	**(Vector) Areas**: (i) Area of the parallelogram with adjacent sides $\bar{a},\bar{b} = \lvert\bar{a}\times\bar{b}\rvert$ (ii) Area of the parallelogram with diagonals $\bar{d}_1,\bar{d}_2 = \frac{1}{2}\lvert\bar{d}_1\times\bar{d}_2\rvert$ (iii) Area of the quadrilateral with diagonals $\overline{AC},\overline{BD} = \frac{1}{2}\lvert\overline{AC}\times\overline{BD}\rvert$ (iv) Area of $\Delta ABC = \frac{1}{2}\lvert\overline{AB}\times\overline{AC}\rvert$

2. The vector equation of the plane in the normal form is $\bar{r}.\hat{n} = p$, where $\hat{n}$ is the unit normal from the origin to the plane and p is the perpendicular distance from the origin to the plane.

3. The vector equation of the plane passing through the point $A(\bar{a})$ and perpendicular to $\bar{n}$ is $(\bar{r}-\bar{a}).\bar{n} = 0$ or $\bar{r}.\bar{n} = \bar{a}.\bar{n}$

4. The angle θ between the planes $\bar{r}.\bar{n}_1 = p_1$ and $\bar{r}.\bar{n}_2 = p_2$ is $\theta = \cos^{-1}\dfrac{\bar{n}_1.\bar{n}_2}{\lvert\bar{n}_1\rvert\lvert\bar{n}_2\rvert}$

BABY BULLET-Q

TEACHING POINTS

TRIPLE PRODUCT OF VECTORS

I) SCALAR TRIPLE PRODUCT[STP]:

1) If $\bar{a}, \bar{b}, \bar{c}$ are any three vectors then their Scalar Triple Product is $(\bar{a} \times \bar{b}).\bar{c}$ (or) $\bar{a}.(\bar{b} \times \bar{c})$

Scalar Triple product of three vectors $\bar{a}, \bar{b}, \bar{c}$ is denoted by $[\bar{a}\ \bar{b}\ \bar{c}]$ read as box $[\bar{a}\ \bar{b}\ \bar{c}]$.

2) $[\bar{i}\ \bar{j}\ \bar{k}] = (\bar{i} \times \bar{j}).\bar{k} = \bar{k}.\bar{k} = 1$ Also $[\bar{i}\ \bar{k}\ \bar{j}] = (\bar{i} \times \bar{k}).\bar{j} = -\bar{j}.\bar{j} = -1$

Hence $[\bar{i}\ \bar{j}\ \bar{k}] = 1 = [\bar{j}\ \bar{k}\ \bar{i}] = [\bar{k}\ \bar{i}\ \bar{j}] = -[\bar{i}\ \bar{k}\ \bar{j}] = -[\bar{j}\ \bar{i}\ \bar{k}] = -[\bar{k}\ \bar{j}\ \bar{i}]$.

3) $[\bar{a}\ \bar{b}\ \bar{c}] = [\bar{b}\ \bar{c}\ \bar{a}] = [\bar{c}\ \bar{a}\ \bar{b}]$ (STP is same, when $\bar{a}, \bar{b}, \bar{c}$ are in the same cyclic order).

$[\bar{a}\ \bar{b}\ \bar{c}] = -[\bar{b}\ \bar{a}\ \bar{c}] = -[\bar{a}\ \bar{c}\ \bar{b}] = -[\bar{c}\ \bar{b}\ \bar{a}]$ (STP changes its sign if the cyclic order of $\bar{a}, \bar{b}, \bar{c}$

4.1) If $\bar{a} = a_1\bar{i} + a_2\bar{j} + a_3\bar{k}, \bar{b} = b_1\bar{i} + b_2\bar{j} + b_3\bar{k}, \bar{c} = c_1\bar{i} + c_2\bar{j} + c_3\bar{k}$ then $[\bar{a}\ \bar{b}\ \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

4.2) If $\bar{\alpha} = x_1\bar{a} + y_1\bar{b} + z_1\bar{c}, \bar{\beta} = x_2\bar{a} + y_2\bar{b} + z_2\bar{c}, \bar{\gamma} = x_3\bar{a} + y_3\bar{b} + z_3\bar{c}$ then $[\bar{\alpha}\ \bar{\beta}\ \bar{\gamma}] = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} [\bar{a}\ \bar{b}\ \bar{c}]$

5) The volume of the Parallelopiped with $\bar{a}, \bar{b}, \bar{c}$ as its coterminous edges is $V = |[\bar{a}\ \bar{b}\ \bar{c}]|$

6) The volume of the tetrahedron with $\bar{a}, \bar{b}, \bar{c}$ as its coterminous edges is $V = \frac{1}{6}|[\bar{a}\ \bar{b}\ \bar{c}]|$

7) The volume of the tetrahedron ABCD is $V = \frac{1}{6}|[\overline{AB}\ \overline{AC}\ \overline{AD}]|$

8) Three vectors $\bar{a}, \bar{b}, \bar{c}$ are coplanar $\Leftrightarrow$ $[\bar{a}, \bar{b}, \bar{c}] = 0$

9) Three vectors $\bar{a}, \bar{b}, \bar{c}$ are non-coplanar $\Leftrightarrow$ $[\bar{a}, \bar{b}, \bar{c}] \neq 0$

10) If $[\bar{a}, \bar{b}, \bar{c}] = 0$ then (i) $\bar{a}, \bar{b}, \bar{c}$ are coplanar when $\bar{a}, \bar{b}, \bar{c}$ are non-zero vectors.
(ii) any two of $\bar{a}, \bar{b}, \bar{c}$ are parallel (iii) at least one of $\bar{a}, \bar{b}, \bar{c}$ is a zero vector.

II) PRODUCT OF FOUR VECTORS:

11) The scalar product of 4 vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ is

$$(\bar{a} \times \bar{b}).(\bar{c} \times \bar{d}) = \begin{vmatrix} \bar{a}.\bar{c} & \bar{a}.\bar{d} \\ \bar{b}.\bar{c} & \bar{b}.\bar{d} \end{vmatrix} = (\bar{a}.\bar{c})(\bar{b}.\bar{d}) - (\bar{b}.\bar{c})(\bar{a}.\bar{d})$$

12) The vector product of 4 vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ is

$$(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = [\bar{a}\ \bar{c}\ \bar{d}]\bar{b} - [\bar{b}\ \bar{c}\ \bar{d}]\bar{a} = [\bar{a}\ \bar{b}\ \bar{d}]\bar{c} - [\bar{a}\ \bar{b}\ \bar{c}]\bar{d}$$

III) VECTOR PRODUCT OF THREE VECTORS:

13) $(\bar{a} \times \bar{b}) \times \bar{c} = (\bar{a}.\bar{c})\bar{b} - (\bar{b}.\bar{c})\bar{a}$ (This vector lies in the plane of $\bar{a}, \bar{b}$)

14) $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a}.\bar{c})\bar{b} - (\bar{a}.\bar{b})\bar{c}$ (This vector lies in the plane of $\bar{b}, \bar{c}$)

BABY BULLET-Q

BULLET MASTER'S

MATH BEATS!

PRODUCT OF VECTORS ⇒ రెండు తలల రారాజు!

In any Branch of Maths, if we find the Product of two entities, we get a Unique Answer.

But, in case of Vectors it is not like that.

Product of **Two Reals** is Unique: $2 \times 3 = 6$.

Product of **Two Functions(fg)** is Unique: $(2x)(3x) = 6x^2$.

Product of **Two Matrices** (AB) is Unique

Product (Cartesian) of **Two Sets** $A \times B$ is Unique.

Product of **Two Trigonometric Functions** is Unique: $(\tan\theta)(\cot\theta) = 1$

Derivative of Product of **Two functions** (uv) is Unique: (uv)'

Integral of Product of **Two functions** is Unique : $\int uv$

But, **Product of Two Vectors** $\bar{a}, \bar{b}$ is **not Unique (??).**

That too, Product of two Vectors is amazingly **a Scalar (or) a Vector.**

That is, Dot (Scalar) Product of Two Vectors gives a **Scalar Quantity.**

Ofcourse, Cross (Vector) Product of Two Vectors is a **Vector Quantity.**

A Special Case: $|\bar{a} \times \bar{b}| = |\bar{a}.\bar{b}| \Rightarrow$ Angle between $\bar{a}, \bar{b}$ is 45^o

BABY BULLET-Q