4. PAIR OF LINES

2 x 7 = 14 Marks

IMP FORMULAS, KEY CONCEPTS

1) If $h^2 \geq ab$ then $ax^2 + 2hxy + by^2 = 0$ represents the equation of a pair of lines passing through the origin.

2.1) If $ax^2 + 2hxy + by^2 \equiv (y - m_1x)(y - m_2x)$ then $m_1 + m_2 = -\frac{2h}{b}, m_1m_2 = \frac{a}{b}$

2.2) If $ax^2 + 2hxy + by^2 \equiv (l_1x + m_1y)(l_2x + m_2y)$ then $l_1l_2 = a, m_1m_2 = b, l_1m_2 + l_2m_1 = 2h$

3.1) If θ is the angle between the pair of lines $ax^2 + 2hxy + by^2 = 0$ then

(i) $\cos\theta = \frac{a+b}{\sqrt{(a-b)^2 + 4h^2}}$　　(ii) $\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$

3.2) The pair of lines $ax^2 + 2hxy + by^2 = 0$ are at right angle $\Leftrightarrow a + b = 0$
i.e., coefficient of x^2 + coefficient of $y^2 = 0$

4) The equation of pair of lines passing through (x_1, y_1) and
(i) parallel to $ax^2 + 2hxy + by^2 = 0$ is $a(x - x_1)^2 + 2h(x - x_1)(y - y_1) + b(y - y_1)^2 = 0$
(ii) perpendicular to $ax^2 + 2hxy + by^2 = 0$ is $b(x - x_1)^2 - 2h(x - x_1)(y - y_1) + a(y - y_1)^2 = 0$

5) The equation of the bisectors of the angle between $ax^2 + 2hxy + by^2 = 0$ is
$h(x^2 - y^2) = (a - b)xy$

6) The area of the triangle formed by $ax^2 + 2hxy + by^2 = 0$ & the line $lx + my + n = 0$ is $\frac{n^2\sqrt{h^2 - ab}}{|am^2 - 2hlm + bl^2|}$

7) The product of perpendiculars drawn from a point $P(\alpha, \beta)$ to the pair of lines $ax2 + 2hxy + by2 = 0$ is $\frac{|a\alpha^2 + 2h\alpha\beta + b\beta^2|}{\sqrt{(a-b)^2 + 4h^2}}$

8) If $S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents the equation of a pair of lines then
(i) $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$　　(ii) $h^2 \geq ab,\ g^2 \geq ac,\ f^2 \geq bc.$

9) The point of intersection of the pair of lines $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ is $\left(\frac{hf - bg}{ab - h^2}, \frac{gh - af}{ab - h^2}\right)$

10) If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel lines then (i) $h^2 = ab$ (ii) $af^2 = bg^2$

(iii) the distance between the parallel lines is $2\sqrt{\frac{g^2 - ac}{a(a+b)}}$ or $2\sqrt{\frac{f^2 - bc}{b(a+b)}}$

11) Homogenisation: The equation to the pair of lines joining the origin to the points of intersection of the curve $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ and the line $lx + my + n = 0$ is

$$ax^2 + 2hxy + by^2 + 2gx\left(\frac{lx + my}{-n}\right) + 2fy\left(\frac{lx + my}{-y}\right) + c\left(\frac{lx + my}{-n}\right)^2 = 0$$