BULLET

MODEL PAPER

A 'MULTI QUESTION PAPER' WITH 'BULLET ANSWERS'

SAQ & LAQ SECTIONS

SAQ SECTION-B

Q11: COMPLEX NUMBERS:

- **Show that the four points in the Argand plane represented by the complex numbers 2+i, 4+3i, 2+5i, 3i are the vertices of a square.**

A: Given complex numbers are taken as A(2,1), B(4,3), C(2,5), D(0,3)

$AB=\sqrt{(2-4)^2+(1-3)^2}=\sqrt{8}$;

$BC=\sqrt{(4-2)^2+(3-5)^2}=\sqrt{8}$

$CD=\sqrt{(2-0)^2+(5-3)^2}=\sqrt{8}$;

$DA=\sqrt{(2-0)^2+(1-3)^2}=\sqrt{8}$

$AC=\sqrt{(2-2)^2+(1-5)^2}=\sqrt{16}=4$;

$BD=\sqrt{(4-0)^2+(3-3)^2}=\sqrt{16}=4$

Hence, the four sides AB, BC, CD, DA are equal. The two diagonals AC, BD are equal.
$\therefore$ A,B,C,D form a square.

- **If $(x-iy)^{1/3}=a-ib$, then show that $\frac{x}{a}+\frac{y}{b}=4(a^2-b^2)$**

A: Given that $(x-iy)^{1/3}=a-ib \Rightarrow x-iy=(a-ib)^3$

$\Rightarrow x-iy=a^3-3a^2bi+3ai^2b^2-i^3b^3$

$=a^3-3a^2bi-3ab^2+ib^3$

$=(a^3-3ab^2)-i(3a^2b-b^3)$

Equating real parts on both sides, we get

$x=a^3-3ab^2=a(a^2-3b^2) \Rightarrow \frac{x}{a}=a^2-3b^2$

Equating imaginary parts on both sides, we get

$y=3a^2b-b^3=b(3a^2-b^2) \Rightarrow \frac{y}{b}=3a^2-b^2$

$\therefore \frac{x}{a}+\frac{y}{b}=(a^2-3b^2)+(3a^2-b^2)$

$=4a^2-4b^2=4(a^2-b^2)$

- **If the real part of $\frac{z+1}{z+i}$ is 1, find the locus of z.**

A: $\frac{z+1}{z+i}=\frac{(x+iy)+1}{(x+iy)+i}=\frac{[(x+1)+iy]}{[x+i(y+1)]}\cdot\frac{[x-i(y+1)]}{[x-i(y+1)]}$

$=\frac{[x(x+1)+y(y+1)]+i[xy-(x+1)(y+1)]}{x^2+(y+1)^2}$

$=\frac{(x^2+y^2+x+y)}{x^2+(y+1)^2}+i\frac{[xy-(x+1)(y+1)]}{x^2+(y+1)^2}$

But the real part is $1 \Rightarrow \frac{x^2+y^2+x+y}{x^2+(y+1)^2}=1$

$\Rightarrow x^2+y^2+x+y=x^2+(y+1)^2$

$\Rightarrow x^2+y^2+x+y=x^2+(y^2+2y+1) \Rightarrow x-y=1$

$\therefore$ The locus of z is $x-y=1$

Q12: QUADRATIC EXPRESSIONS:

- **Find the range of $\frac{x^2+x+1}{x^2-x+1}$ for $x\in R$.**

A: Let $y=\frac{x^2+x+1}{x^2-x+1}$

$\Rightarrow y(x^2-x+1)=x^2+x+1$

$\Rightarrow yx^2-yx+y=x^2+x+1$

$\Rightarrow yx^2-x^2-yx-x+y-1=0$

$\Rightarrow x^2(y-1)-x(y+1)+(y-1)=0$

$\Rightarrow (y-1)x^2-(y+1)x+(y-1)=0$ ....(1)

(1) is a quadratic in x and its roots are reals.

$\therefore \Delta=b^2-4ac\geq 0$

$\Rightarrow (y+1)^2-4(y-1)^2\geq 0 \Rightarrow (y+1)^2-(2y-2)^2\geq 0$

$\Rightarrow (y+1+2y-2)(y+1)-(2y-2)\geq 0$

$[\because a^2-b^2=(a+b)(a-b)]$

$\Rightarrow (3y-1)(3-y)\geq 0 \Rightarrow (3y-1)(y-3)\leq 0$

$\Rightarrow y\in\left[\frac{1}{3},3\right]$ $\therefore$ Range$=\left[\frac{1}{3},3\right]$

- **If x is real, P.T $\frac{x}{x^2-5x+9}$ lies between 1 and $\frac{-1}{11}$.**

A: Let $y=\frac{x}{x^2-5x+9}$

$\Rightarrow y(x^2-5x+9)=x$

$\Rightarrow yx^2-5yx+9y-x=0$

$\Rightarrow yx^2-(5y+1)x+9y=0$ .... (1)

(1) is a quadratic in x and its roots are reals.

$\therefore \Delta=b^2-4ac\geq 0$

$\Rightarrow (5y+1)^2-4(y)(9y)\geq 0$

$\Rightarrow (25y^2+10y+1)-36y^2\geq 0$

$\Rightarrow -11y^2+10y+1\geq 0$

$\Rightarrow 11y^2-10y-1\leq 0$

$\Rightarrow 11y^2-11y+y-1\leq 0 \Rightarrow 11y(y-1)+(y-1)\leq 0$

$\Rightarrow (11y+1)(y-1)\leq 0$

$\Rightarrow y\in\left[-\frac{1}{11},1\right] \Rightarrow \frac{-1}{11}\leq y\leq 1$

$\therefore$ The given expression lies between $\frac{-1}{11}$ and 1

- **Show that $\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$ does not lie between 1 and 4, if x is real.**

A: $GE=\frac{1}{3x+1}+\frac{1}{x+1}-\frac{1}{(3x+1)(x+1)}$

$=\frac{x+1+3x+1-1}{(3x+1)(x+1)}=\frac{4x+1}{3x^2+4x+1}$

Let $y=\frac{4x+1}{3x^2+4x+1}$

$\Rightarrow y(3x^2+4x+1)=4x+1$

$\Rightarrow 3yx^2+4yx+y=4x+1$

$\Rightarrow 3yx^2+(4y-4)x+(y-1)=0$............(1)

(1) is a quadratic in x and its roots are reals.

$\therefore \Delta=b^2-4ac\geq 0$

$(4y-4)^2-4(3y)(y-1)\geq 0$

$\Rightarrow 16y^2+16-32y-12y+12y\geq 0 \Rightarrow 4y^2-20y+16\geq 0$

$\Rightarrow 4y^2-20y+16\geq 0 \Rightarrow 4(y^2-5y+4)\geq 0$

$\Rightarrow y^2-5y+4\geq 0 \Rightarrow (y-1)(y-4)\geq 0 \Rightarrow y\leq 1$ or $y\geq 4$

$\Rightarrow$ y does not lie between 1 and 4

- If the expression $\frac{x-p}{x^2-3x+2}$ takes all real values for $x\in R$ then find the bounds for p

A: Let $\mathbf{y}=\frac{x-p}{x^2-3x+2}$

$\Rightarrow y(x^2-3x+2)=x-p$

$\Rightarrow yx^2-3yx+2y=x-p$

$\Rightarrow yx^2+(-3y-1)x+(2y+p)=0$ ...... (1)

(1) is a quadratic in x and its roots are reals.

$\therefore \Delta=b^2-4ac\geq 0$

$\Rightarrow (-3y-1)^2-4y(2y+p)\geq 0$

$\Rightarrow 9y^2+6y+1-8y^2-4py\geq 0$

$\Rightarrow y^2+(6-4p)y+1\geq 0$ ..... (2)

But y is real. Also coefficient of y^2 is positive.

$\therefore$ (2) holds true only when the roots of $y^2+(6-4p)y+1=0$ are imaginary or real & equal.

$\Rightarrow \Delta=b^2-4ac\leq 0 \Rightarrow (6-4p)^2-4\leq 0$

$\Rightarrow 36+16p^2-48p-4\leq 0$

$\Rightarrow 16p^2-48p+32\leq 0 \Rightarrow 16(p^2-3p+2)\leq 0$

$\Rightarrow p^2-3p+2\leq 0 \Rightarrow (p-1)(p-2)\leq 0 \Rightarrow 1\leq p\leq 2$

But p=1 or 2 is not possible. $\therefore 1<p<2$

Q13&Q14:PERMUTATIONS & COMBINATIONS:

- **Simplify $^{34}C_5+\sum_{r=0}^{4}{}^{(38-r)}C_4$**

A: $GE=\sum_{r=0}^{4}{}^{(38-r)}C_4+{}^{34}C_5$

$=\left[{}^{38}C_4+{}^{37}C_4+{}^{36}C_4+{}^{35}C_4+{}^{34}C_4\right]+{}^{34}C_5$

$=\left[{}^{38}C_4+{}^{37}C_4+{}^{36}C_4+{}^{35}C_4\right]+\left[{}^{34}C_4+{}^{34}C_5\right]$

$=\left[{}^{38}C_4+{}^{37}C_4+{}^{36}C_4\right]+\overline{{}^{35}C_4+{}^{35}C_5}$

$(\because {}^nC_{r-1}+{}^nC_r={}^{n+1}C_r)$

$=\left[{}^{38}C_4+{}^{37}C_4\right]+\overline{{}^{36}C_4+{}^{36}C_5}$

$=\left[{}^{38}C_4\right]+\overline{{}^{37}C_4+{}^{37}C_5}={}^{38}C_4+{}^{38}C_5={}^{39}C_5$

- **Find the number of ways of selecting a cricket team of 11 players from batsmen and 6 bowlers such that there will be atleast 5 bowlers in the team.**

A: A Team of 11 players with atleast 5 bowlers can be selected in the following compositions:

Bowlers (6)	Batsmen (7)	No. of selections
5	6	$^6C_5\times{}^7C_6=6\times=42$
6	5	$^6C_6\times{}^7C_5=1\times 21=21$

$\therefore$ the total number of selections=42+21= 63

- **Find the number of ways of forming a committee of 5 members out of 6 Indians and 5 Americans so that always the Indians will be in majority in the committee.**

A: A 5 men committee out of 6 Indians, 5 Americans with majority indians can be selected in the following compositions:

Indians (6)	Americans (5)	No. of selections
5	0	$^6C_5\times{}^5C_0=6\times 1=6$
4	1	$^6C_4\times{}^5C_1=15\times 5=75$
3	2	$^6C_3\times{}^5C_2=20\times 10=200$

$\therefore$ the total number of selections =6+75+200=281

- **If the letters of the word MASTER are permutted in all possible ways and the words thus formed are arranged in dictionary order, then find the rank of the word MASTER.**

A: The alphabetrical order of the letters of the word MASTER is

A,E,M,R,S,T

The number of words that begin with A

$$\_\ \_\ \_\ \_\ \_ = 5! = 120$$

The number of words that begin with E

$$\_\ \_\ \_\ \_\ \_ = 5! = 120$$

The number of words that begin with MAE

$$\_\ \_\ \_ = 3! = 6$$

The number of words that begin with MAR

$$\_\ \_\ \_ = 3! = 6$$

The number of words that begin with MASE

$$\_\ \_ = 2! = 2$$

The number of words that begin with MASR

$$\_\ \_ = 2! = 2$$

The next word is MASTER $= 1! = 1$

$\therefore$ Rank of the word MASTER

$= 2(120) + 2(6) + 2(2) + 1$

$= 240 + 12 + 4 + 1 = 257$

- **If the letters of the word thus formed are arranged in the dictionary order, find the rank of the word EAMCET**

A: The alphabetical order of the letters of the word EAMCET is as follows:

A,C,E, E,M,T

The number of words that begin with A

$$\_\ \_\ \_\ \_\ \_ \rightarrow 5!/2! = 60$$

The number of words that begin with C

$$\_\ \_\ \_\ \_\ \_ \rightarrow 5!/2! = 60$$

The number of words that begin with E A C

$$\_\ \_\ \_ \rightarrow 3! = 6$$

The number of words that begin with E A E

$$\_\ \_\ \_ \rightarrow 3! = 6$$

The next word is E A M C E T $\rightarrow 1$

Hence the rank of the word EAMCET is

$60+60+6+6+1=133$

Q15: PARTIAL FRACTIONS:

- **Resolve $\dfrac{x+4}{(x^2-4)(x+1)}$ into partial fractions.**

A: $\text{G.E} = \dfrac{x+4}{(x^2-4)(x+1)} = \dfrac{x+4}{(x+2)(x-2)(x+1)}$

Let $\dfrac{x+4}{(x+2)(x-2)(x+1)} = \dfrac{A}{x+2} + \dfrac{B}{x-2} + \dfrac{C}{x+1}$

$= \dfrac{A(x-2)(x+1)+B(x+2)(x+1)+C(x+2)(x-2)}{(x+2)(x-2)(x+1)}$

$\therefore A(x-2)(x+1)+B(x+2)(x+1)+C(x^2-4)=x+4 \ldots\ldots(1)$

Putting $x=-2$ in (1) we get

$A(-2-2)(-2+1)+B(0)+C(0)=-2+4$

$\Rightarrow 4A=2 \Rightarrow A=1/2$

Putting $x=2$ in (1) we get

$A(0)+B(2+2)(2+1)+C(0)=2+4 \Rightarrow 12B=6 \Rightarrow B=1/2$

Putting $x=-1$ in (1) we get

$A(0)+B(0)+C(1-4)=-1+4 \Rightarrow -3C=3 \Rightarrow C=-1$

$\therefore \dfrac{x+4}{(x^2-4)(x+1)} = \dfrac{A}{(x+2)} + \dfrac{B}{(x-2)} + \dfrac{C}{(x+1)}$

$= \dfrac{1}{2(x+2)} + \dfrac{1}{2(x-2)} - \dfrac{1}{(x+1)}$

- **Resolve $\dfrac{2x^2+3x+4}{(x-1)(x^2+2)}$ into partial fractions**

A: Let $\dfrac{2x^2+3x+4}{(x-1)(x^2+2)} = \dfrac{A}{x-1} + \dfrac{Bx+C}{x^2+2}$

$= \dfrac{A(x^2+2)+(Bx+1)(x-1)}{(x-1)(x^2+2)}$

$\therefore A(x^2+2)+(Bx+C)(x-1) = 2x^2+3x+4 \ldots\ldots.(1)$

Putting $x=1$ in (1) we get

$A(1^2+2)+(Bx+C)(0) = 2(1^2)+3(1)+4$

$\Rightarrow 3A = 9 \Rightarrow A = 3$

Putting $x=0$ in (1) we get

$A(0+2)+(0+C)(0-1) = 4 \Rightarrow 2A - C = 4$

$\Rightarrow C = 2A - 4 = 2(3) - 4 = 2$

Comparing the coeff. of x^2 in (1), we get

$A + B = 2 \Rightarrow 3 + B = 2 \Rightarrow B = -1$

$\therefore \dfrac{2x^2+3x+4}{(x-1)(x^2+2)} = \dfrac{3}{x-1} + \dfrac{(-1)x+2}{x^2+2} = \dfrac{3}{x-1} + \dfrac{2-x}{x^2+2}$

Q16&17: PROBABILITY:

- **Suppose A and B are independent events with $P(A)=0.6$, $P(B)=0.7$ compute (i) $P(A\cap B)$ (ii) $P(A\cup B)$ (iii) $P(B/A)$ (iv) $P(A^c\cap B^c)$**

A: Given that A,B are independent, hence

(i) $P(A\cap B)=P(A)P(B)=0.6 \times 0.7=0.42$

(ii) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=0.6+0.7-0.42=1.3-0.42=0.88$

(iii) $P(B/A)=P(B)=0.7$

(iv) $P(A^c\cap B^c)=P(A^c).P(B^c)=[1-P(A)][1-P(B)]$

$=0.4\times 0.3=0.12$

- **If A, B are two events with $P(A\cup B)=0.65$ and $P(A\cap B)=0.15$, then find $P(A^c)+P(B^c)$.**

A: We know $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ [From Addition theorem]

$\Rightarrow P(A)+P(B)=P(A\cup B)+P(A\cap B)$

$=0.65+0.15=0.8$

$\therefore P(A^c)+P(B^c)=[1-P(A)]+[1-P(B)]$

$=2-[P(A)+P(B)]=2-0.8=1.2$

- **A problem in calculus is given to two students A and B whose chances of solving it are 1/3, 1/4 respectively. Find the probability of the problem being solved if both of them try independently.**

A: Let A,B denote the events of solving the problem by A, B respectively $\Rightarrow P(A)=\frac{1}{3}, P(B)=\frac{1}{4}$

$\therefore P(\bar{A})=1-P(A)=1-\frac{1}{3}=\frac{2}{3};$

$P(\bar{B})=1-P(B)=1-\frac{1}{4}=\frac{3}{4}$

$\therefore P(A\cup B)=1-P(\bar{A}\cap\bar{B})=1-P(\bar{A}).P(\bar{B})$

$=1-\left(\frac{2}{\not{3}}\right)\left(\frac{\not{3}}{4}\right)=1-\frac{1}{2}=\frac{1}{2}$

- **A speaks truth in 75% of the cases and B in 80% of the cases. What is the probability that their statements about an incident do not match.**

A: Let A,B denote the events of speaking truth by A,B respectively

$P(A)=\frac{75}{100}=\frac{3}{4};\ P(B)=\frac{80}{100}=\frac{4}{5}$

$\therefore P(\bar{A})=1-P(A)=1-\frac{3}{4}=\frac{1}{4};$

$P(\bar{B})=1-P(B)=1-\frac{4}{5}=\frac{1}{5}$

Let E be the event that A and B contradict to each other

$\Rightarrow P(E)=P\left[(A\cap\bar{B})\cup(\bar{A}\cap B)\right]$

$=P(A\cap\bar{B})+P(\bar{A}\cap B)$

$=P(A)P(\bar{B})+P(\bar{A})P(B)$ [$\because$ A,B are independent]

$=\frac{3}{4}.\frac{1}{5}+\frac{1}{4}.\frac{4}{5}=\frac{7}{20}$

- **A,B,C are three news papers published from a city. 20% of the population read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C and 2% all the three. Find the percentage of the population who read atleast one news paper.**

A: Given that $P(A)=\frac{20}{100}=0.2$,

$P(B)=\frac{16}{100}=0.16,\ P(C)=\frac{14}{100}=0.14$

$P(A\cap B)=\frac{8}{100}=0.08,\ P(B\cap C)=\frac{4}{100}=0.04,$

$P(A\cap C)=\frac{5}{100}=0.05,$

$P(A\cap B\cap C)=\frac{2}{100}=0.02$

$\therefore P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)$
$-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$

$=0.2+0.16+0.14-0.08-0.04-0.05+0.02$

$=0.52-0.17=0.35$

Percentage of population who read atleast one newspaper $=0.35\times 100\%=35\%$.

LAQ SECTION-C

Q 18 : DEMOIVRE'S THEOREM:

- **If n is a positive integer then show that $(1+i)^{2n}+(1-i)^{2n}=2^{(n+1)}\cos(n\pi/2)$**

A: First we find the mod-amp form of 1+i

Let $x+iy=1+i \Rightarrow x=1, y=1$

$\therefore\ r=\sqrt{x^2+y^2}=\sqrt{1^2+1^2}=\sqrt{2};$

$\tan\theta=\frac{y}{x}=\frac{1}{1}=1=\tan\frac{\pi}{4}\Rightarrow\theta=\frac{\pi}{4}$

$\therefore 1+i=r(\cos\theta+i\sin\theta)=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$\Rightarrow\ (1+i)^{2n}=\left(\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right)^{2n}$

$=(\sqrt{2})^{2n}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)^{2n}$

$=2^n\left(\cos(2n)\frac{\pi}{4}+i\sin(2n)\frac{\pi}{4}\right)$

[By Demoivre's theorem]

$=2^n\left(\cos\frac{n\pi}{2}+i\sin\frac{n\pi}{2}\right)$ ..... (1)

Similarly,

$(1-i)^{2n}=2^n\left(\cos\frac{n\pi}{2}-i\sin\frac{n\pi}{2}\right)$ ..... (2)

Adding (1) & (2), we get $(1+i)^{2n}+(1-i)^{2n}$

$=2^n\left(\left(\cos\frac{n\pi}{2}+i\sin\frac{n\pi}{2}\right)+\left(\cos\frac{n\pi}{2}-i\sin\frac{n\pi}{2}\right)\right)$

$=2^n\left(2\cos\frac{n\pi}{2}\right)=2^{n+1}.\cos\frac{n\pi}{2}$

- **Prove that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n$ $=2^{n+1}\cos^n(\theta/2)\cos(n\theta/2)$**

A: First we find the mod-amp form of $1+\cos\theta+i\sin\theta$

$1+\cos\theta+i\sin\theta=2\cos^2\frac{\theta}{2}+i2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$=2\cos\frac{\theta}{2}(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2})$

$\therefore(1+\cos\theta+i\sin\theta)^n=\left(2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2}+i\sin\frac{\theta}{2}\right)\right)^n$

$=2^n\cos^n\frac{\theta}{2}\left(\cos n\frac{\theta}{2}+i\sin n\frac{\theta}{2}\right)$ ....(1)

Similarly, $(1+\cos\theta-i\sin\theta)^n$

$=2^n\cos^n\frac{\theta}{2}\left(\cos n\frac{\theta}{2}-i\sin n\frac{\theta}{2}\right)$ ....(2)

Adding (1) & (2),

$(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n$

$=2^n\cos^n\frac{\theta}{2}\left(\left(\cos n\frac{\theta}{2}+i\sin n\frac{\theta}{2}\right)+\left(\cos n\frac{\theta}{2}-i\sin n\frac{\theta}{2}\right)\right)$

$=2^n\cos^n\frac{\theta}{2}\left(2\cos\frac{n\theta}{2}\right)=2^{n+1}\cos^n\frac{\theta}{2}.\cos\frac{n\theta}{2}$

- **If α,β are roots of the equation $x^2-2x+4=0$, then show that $\alpha^n+\beta^n=2^{n+1}\cos\left(\frac{n\pi}{3}\right)$**

Sol: Given $x^2-2x+4=0\Rightarrow x=\frac{2\pm\sqrt{4-4(1)(4)}}{2(1)}$

$=\frac{2\pm\sqrt{-12}}{2}=\frac{2\pm 2i\sqrt{3}}{2}=1\pm i\sqrt{3}$

Now, we find the mod-amp form of $1+i\sqrt{3}$

Let $x+iy=1+i\sqrt{3}\Rightarrow x=1, y=\sqrt{3}$

$\therefore\ r=\sqrt{x^2+y^2}=\sqrt{1^2+(\sqrt{3})^2}=\sqrt{4}=2.$

Also, $\tan\theta=\frac{y}{x}=\frac{\sqrt{3}}{1}=\tan\frac{\pi}{3}\Rightarrow\theta=\frac{\pi}{3}$

$\therefore 1+i\sqrt{3}=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

$\Rightarrow\ (1+i\sqrt{3})^n=\left(2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right)^n$

$=(2)^n\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)^n$

$=2^n\left(\cos n\frac{\pi}{3}+i\sin n\frac{\pi}{3}\right)$ ..... (1)

[By Demoivre's theorem]

Similarly, $(1-i\sqrt{3})^n=2^n\left(\cos n\frac{\pi}{3}-i\sin n\frac{\pi}{3}\right)$ ..... (2)

Adding (1) & (2), we get

$\alpha^n+\beta^n=(1+i\sqrt{3})^n+(1-i\sqrt{3})^n$

$=2^n\left(\left(\cos n\frac{\pi}{3}+i\sin n\frac{\pi}{3}\right)+\left(\cos n\frac{\pi}{3}-i\sin n\frac{\pi}{3}\right)\right)$

$=2^n\left(2\cos\frac{n\pi}{3}\right)=2^{n+1}.\cos\frac{n\pi}{3}$

Q 19 : THEORY OF EQUATIONS:

- **Solve $x^4-10x^3+26x^2-10x+1=0$.**

A: The degree of the given equation is n=4, which is even. Also $a_k=a_{n-k}\ \forall\ k=0,1,2,3,4$
So, the given equation is a Standard Reciprocal Equation.
Dividing the equation by x^2, we get

$$x^2-10x+26-\frac{10}{x}+\frac{1}{x^2}=0$$

$$\Rightarrow\left(x^2+\frac{1}{x^2}\right)-10\left(x+\frac{1}{x}\right)+26=0 \text{ ....(1)}$$

Put $x+\frac{1}{x}=y\Rightarrow x^2+\frac{1}{x^2}=y^2-2$

$\therefore$ (1) $\Rightarrow (y^2-2)-10y+26=0\Rightarrow y^2-10y+24=0$

$\Rightarrow y^2-6y-4y+24=0\Rightarrow y(y-6)-4(y-6)=0$

$\Rightarrow (y-4)(y-6)=0$

$\Rightarrow y-4=0$ (or) $y-6=0\Rightarrow y=4$ (or) $y=6$

If y=4 then

$$x+\frac{1}{x}=4\Rightarrow x^2+1=4x\Rightarrow x^2-4x+1=0$$

$$\Rightarrow x=\frac{4\pm\sqrt{(4)^2-4.1.1}}{2.1}=\frac{4\pm\sqrt{12}}{2}$$

$$=\frac{4\pm2\sqrt{3}}{2}=2\pm\sqrt{3}$$

If y=6 then $x+\frac{1}{x}=6\Rightarrow\frac{x^2+1}{x}=6$

$$\Rightarrow x^2+1=6x\Rightarrow x^2-6x+1=0$$

$$\Rightarrow x=\frac{6\pm\sqrt{36-4}}{2}=\frac{6\pm\sqrt{32}}{2}$$

$$=\frac{6\pm4\sqrt{2}}{2}=3\pm2\sqrt{2}$$

$\therefore$ the roots of the equation are $2\pm\sqrt{3}, 3\pm2\sqrt{2}$

- **Solve $2x^5+x^4-12x^3-12x^2+x+2=0$**

A: The degree of the given equation is n=5, which is odd. Also $a_k=a_{n-k}\ \forall\ k=0,1,2,3,4,5$
Hence the given equation is a reciprocal equation of class I of odd degree
$\therefore$ –1 is a root of the given equation.
Now dividing the expression by (x+1), we have

–1	2	1	–12	–12	1	2
	0	–2	1	11	1	2
	2	–1	–11	–1	2	0

Now, we solve the SRE $2x^4-x^3-11x^2-x+2=0$
On dividing the equation by x^2, we get

$$2x^2-x-11-\frac{1}{x}+\frac{2}{x^2}=0$$

$$\Rightarrow 2\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)-11=0 \text{ ....(1)}$$

Put $x+\frac{1}{x}=y\Rightarrow x^2+\frac{1}{x^2}=y^2-2$

$\therefore(1)\Rightarrow 2(y^2-2)-y-11=0\Rightarrow 2y^2-4-y-11=0$

$\Rightarrow 2y^2-y-15=0\Rightarrow 2y^2-6y+5y-15=0$

$\Rightarrow 2y(y-3)+5(y-3)=0\Rightarrow(y-3)(2y+5)=0$

$\Rightarrow y-3=0$ (or) $2y+5=0\Rightarrow y=3$ (or) $y=\frac{-5}{2}$

If y=3 then

$$x+\frac{1}{x}=3\Rightarrow x^2+1=3x\Rightarrow x^2-3x+1=0$$

$$\Rightarrow x=\frac{3\pm\sqrt{(-3)^2-4.1.1}}{2(1)}=\frac{3\pm\sqrt{9-4}}{2}=\frac{3\pm\sqrt{5}}{2}$$

If $y=-\frac{5}{2}$ then $x+\frac{1}{x}=-\frac{5}{2}\Rightarrow\frac{x^2+1}{x}=-\frac{5}{2}=-2\frac{1}{2}=-2-\frac{1}{2}$

$$\therefore x=-2 \text{ or } x=-\frac{1}{2}$$

$\therefore$ the roots of the equation are $-1,-2,-\frac{1}{2},\frac{3\pm\sqrt{5}}{2}$

- **Solve the equation $8x^3-36x^2-18x+81=0$ the roots being in A.P.**

A: Let the roots of $8x^3-36x^2-18x+81=0$ in A.P be taken as a–d, a, a+d

$$\text{Now, } S_1=(a-d)+a+(a+d)=\frac{36}{8}=\frac{9}{2}$$

$$\Rightarrow 3a=\frac{9}{2}\Rightarrow a=\frac{3}{2}$$

$$S_3=(a-d)a(a+d)=\frac{-81}{8}\Rightarrow a(a^2-d^2)=\frac{-81}{8}$$

$$\Rightarrow\frac{3}{2}\left(\frac{9}{4}-d^2\right)=\frac{-81}{8}\Rightarrow\left(\frac{9}{4}-d^2\right)=\frac{-81}{8}\times\frac{2}{3}=\frac{-27}{4}$$

$$\Rightarrow\frac{9}{4}-d^2=-\frac{27}{4}\Rightarrow d^2=\frac{9}{4}+\frac{27}{4}=\frac{36}{4}=9$$

$\Rightarrow d=\pm3$

$\therefore$ the roots of the given equation are a–d, a, a+d

$$\Rightarrow\frac{3}{2}-3,\frac{3}{2},\frac{3}{2}+3\Rightarrow-\frac{3}{2},\frac{3}{2},\frac{9}{2}$$

Q 20 & 21: BINOMIAL THEOREM:

- **If the coefficients of 4 consecutive terms in the expansion of $(1+x)^n$ are a_1, a_2, a_3, a_4 respectively, then show that $\dfrac{a_1}{a_1+a_2}+\dfrac{a_3}{a_3+a_4}=\dfrac{2a_2}{a_2+a_3}$**

A: We take the coefficients of 4 consecutive terms of $(1+x)^n$ as follows:

$a_1={}^nC_r,\ a_2={}^nC_{r+1},\quad a_3={}^nC_{r+2},\quad a_4={}^nC_{r+3}.$

$$L.H.S=\frac{a_1}{a_1+a_2}+\frac{a_3}{a_3+a_4}$$

$$=\frac{{}^nC_r}{{}^nC_r+{}^nC_{r+1}}+\frac{{}^nC_{r+2}}{{}^nC_{r+2}+{}^nC_{r+3}}$$

$$=\frac{{}^nC_r}{{}^{(n+1)}C_{r+1}}+\frac{{}^nC_{r+2}}{{}^{n+1}C_{r+3}}\left(\because {}^nC_r+{}^nC_{r+1}={}^{(n+1)}C_{r+1}\right)$$

$$=\frac{{}^nC_r}{\left(\frac{n+1}{r+1}\right).{}^nC_r}+\frac{{}^nC_{r+2}}{\left(\frac{n+1}{r+3}\right).{}^nC_{r+2}}$$

$$\left(\because {}^nC_r=\left(\frac{n}{r}\right){}^{n-1}C_{r-1}\right)$$

$$=\frac{r+1}{n+1}+\frac{r+3}{n+1}=\frac{r+1+r+3}{n+1}=\frac{2r+4}{n+1}=\frac{2(r+2)}{n+1}\ ......(1)$$

$$R.H.S=\frac{2a_2}{a_2+a_3}=\frac{2({}^nC_{r+1})}{{}^nC_{r+1}+{}^nC_{r+2}}=\frac{2({}^nC_{r+1})}{{}^{(n+1)}C_{r+2}}$$

$$=\frac{2({}^nC_{r+1})}{\left(\frac{n+1}{r+2}\right)({}^nC_{r+1})}=\frac{2}{\frac{n+1}{r+2}}=\frac{2(r+2)}{n+1}\ ...(2)$$

From (1) & (2), L.H.S=R.H.S

22. Prove that $C_0.C_r+C_1.C_{r+1}+C_2.C_{r+2}+....+C_{n-r}.C_n={}^{2n}C_{(n+r)}$ for $0\le r\le n$

Hence deduce that

(i) $C_0^2+C_1^2+C_2^2+......+C_n^2={}^{2n}C_n$

(ii) $C_0C_1+C_1C_2+C_2C_3+.....+C_{n-1}.C_n={}^{2n}C_{n+1}$

A: We have $(1+x)^n=C_0+C_1x+C_2x^2+.......$

$+C_rx^r+C_{r+1}x^{r+1}+C_{r+2}x^{r+2}+.......+C_nx^n\ .....(1)$

$\Rightarrow (x+1)^n=C_0x^n+C_1x^{n-1}+C_2x^{n-2}+.......$

$+C_{n-r}x^{n-r}+......+C_n\quad .....(2)$

Multiplying (2) and (1), we get

$[C_0x^n+C_1x^{n-1}+C_2x^{n-2}+....$

$+C_{n-r}x^{n-r}+C_n][C_0+C_1x+C_2x^2+......$

$+C_rx^r+C_{r+1}x^{r+1}+C_{r+2}x^{r+2}+....+C_nx^n]$

$=(x+1)^n(1+x)^n=(1+x)^{2n}$

Comparing coefficients of x^{n+r} both sides, we get

$C_0C_r+C_1C_{r+1}+C_2C_{r+2}+....+C_{n-r}C_n={}^{2n}C_{n+r}$

(i) On substituting r=0 , we get

$C_0^2+C_1^2+C_2^2+......+C_n^2={}^{2n}C_n$

(ii) On substituting r=1, we get

$C_0C_1+C_1C_2+C_2C_3+.....+C_{n-1}.C_n={}^{2n}C_{n+1}$

- **If n is a positive integer and x is any nonzero real number, then prove that**
$C_0+C_1\dfrac{x}{2}+C_2\dfrac{x^2}{3}+C_3\dfrac{x^3}{4}+......+C_n\dfrac{x^n}{n+1}=\dfrac{(1+x)^{n+1}-1}{(n+1)x}$

A: Let $S=C_0+C_1.\dfrac{x}{2}+C_2.\dfrac{x^2}{3}+....+C_n.\dfrac{x^n}{n+1}$

$$={}^nC_0+{}^nC_1\frac{x}{2}+{}^nC_2\frac{x^2}{3}+......+{}^nC_n.\frac{x^n}{n+1}$$

$$\Rightarrow xS={}^nC_0.x+{}^nC_1\frac{x^2}{2}+{}^nC_2.\frac{x^3}{3}+....+{}^nC_n.\frac{x^{n+1}}{n+1}$$

$$\Rightarrow (n+1)xS=\frac{n+1}{1}.{}^nC_0.x+\frac{n+1}{2}.{}^nC_1.x^2$$

$$+\frac{n+1}{3}.{}^nC_2x^3+....+\frac{n+1}{n+1}{}^nC_n.x^{n+1}$$

$$={}^{n+1}C_1.x+{}^{n+1}C_2x^2+{}^{n+1}C_3x^3+....+{}^{n+1}C_{n+1}.x^{n+1}$$

$$\left(\because \left(\frac{n+1}{r+1}\right).{}^nC_r={}^{(n+1)}C_{r+1}\right)$$

$$\Rightarrow (n+1)xS=(1+x)^{n+1}-1$$

$$(\because {}^nC_1x+{}^nC_2x^2+.....+{}^nC_nx^n=(1+x)^n-1)$$

$$\therefore S=\frac{(1+x)^{n+1}-1}{(n+1).x}$$

- **If $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+...\infty$, then find $3x^2+6x$**

A: Given $x=\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+...\infty$

Adding 1 on both sides, we get

$$1+x=1+\frac{1}{5}+\frac{1.3}{5.10}+\frac{1.3.5}{5.10.15}+...\infty$$

$$=1+\frac{1}{1!}\left(\frac{1}{5}\right)+\frac{1.3}{2!}\left(\frac{1}{5}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{5}\right)^3+...\infty$$

Comparing the above series with

$$1+\frac{p}{1!}\left(\frac{y}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{y}{q}\right)^2+.......=(1-y)^{-p/q}$$

we get $p=1$, $p+q=3 \Rightarrow q=2$ and

$$\frac{y}{q}=\frac{1}{5}\Rightarrow y=\frac{q}{5}=\frac{2}{5}$$

$$\therefore 1+x=(1-y)^{\frac{-p}{q}}=\left(1-\frac{2}{5}\right)^{\frac{-1}{2}}=\left(\frac{3}{5}\right)^{\frac{-1}{2}}=\left(\frac{5}{3}\right)^{\frac{1}{2}}=\sqrt{\frac{5}{3}}$$

$$\Rightarrow (1+x)^2=\frac{5}{3}\Rightarrow 1+2x+x^2=\frac{5}{3}\Rightarrow 3+6x+3x^2=5$$

$$\Rightarrow 3x^2+6x=2.$$

- **If $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+......$ then prove that $9x^2+24x=11$**

A: Given $x=\frac{1.3}{3.6}+\frac{1.3.5}{3.6.9}+\frac{1.3.5.7}{3.6.9.12}+.....$

$$=\frac{1.3}{2!}\left(\frac{1}{3}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3+\frac{1.3.5.7}{4!}\left(\frac{1}{3}\right)^4+..$$

Adding $1+\frac{1}{3}$ on both sides, we get

$$1+\frac{1}{3}+x=1+\frac{1}{1!}\left(\frac{1}{3}\right)+\frac{1.3}{2!}\left(\frac{1}{3}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{3}\right)^3+......$$

Comparing the above series with

$$1+\frac{p}{1!}\left(\frac{y}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{y}{q}\right)^2+.....=(1-y)^{-p/q}$$

we get, $p=1, p+q=3\Rightarrow 1+q=3\Rightarrow q=2$

Also, $\frac{y}{q}=\frac{1}{3}\Rightarrow y=\frac{q}{3}=\frac{2}{3}$

$$\therefore 1+\frac{1}{3}+x=(1-y)^{-p/q}=\left(1-\frac{2}{3}\right)^{-1/2}$$

$$=\left(\frac{1}{3}\right)^{-1/2}=(3)^{1/2}=\sqrt{3}$$

$$\Rightarrow\frac{4}{3}+x=\sqrt{3}\Rightarrow x=\sqrt{3}-\frac{4}{3}=\frac{3\sqrt{3}-4}{3}=\frac{3\sqrt{3}-4}{3}$$

$$\Rightarrow 3x=3\sqrt{3}-4\Rightarrow 3x+4=3\sqrt{3}$$

$$\Rightarrow (3x+4)^2=(3\sqrt{3})^2\Rightarrow 9x^2+24x+16=27$$

$$\Rightarrow 9x^2+24x=11$$

29. Find the sum of the infinite series

$$\frac{7}{5}\left(1+\frac{1}{10^2}+\frac{1.3}{1.2}\cdot\frac{1}{10^4}+\frac{1.3.5}{1.2.3}\cdot\frac{1}{10^6}+....\right)$$

Sol: Let $S=1+\frac{1}{10^2}+\frac{1.3}{1.2}\cdot\frac{1}{10^4}+\frac{1.3.5}{1.2.3}\cdot\frac{1}{10^6}+...$

$$=1+\frac{1}{1!}\frac{1}{100}+\frac{1.3}{2!}\left(\frac{1}{100}\right)^2+\frac{1.3.5}{3!}\left(\frac{1}{100}\right)^3+......$$

Comparing the above series with

$$1+\frac{p}{1!}\left(\frac{x}{q}\right)+\frac{p(p+q)}{2!}\left(\frac{x}{q}\right)^2+....=(1-x)^{-p/q}$$

we get, $p=1, p+q=3\Rightarrow 1+q=3\Rightarrow q=2$

Also $\frac{x}{q}=\frac{1}{100}\Rightarrow x=\frac{q}{100}=\frac{2}{100}=\frac{1}{50}$

$$\therefore S=(1-x)^{-p/q}=\left(1-\frac{1}{50}\right)^{-1/2}=\left(\frac{49}{50}\right)^{-1/2}$$

$$=\left(\frac{50}{49}\right)^{1/2}=\sqrt{\frac{50}{49}}=\frac{5\sqrt{2}}{7}$$

$\therefore$ the given series is $\frac{7}{5}(S)=\frac{\not{7}}{\not{5}}\left(\frac{\not{5}\sqrt{2}}{\not{7}}\right)=\sqrt{2}$

Q 22: MEASURES OF DISPERSION:

Find the mean deviation about the mean for the given data using 'step deviation method':

Marks	0-10	10-20	20-30	30-40	40-50
Students	5	8	15	16	6

A: We take the assumed mean A =25. Here, C=10. Hence, we form the following table.

Class	f_i	x_i	d_i	f_id_i	$\lvert x_i-\bar{x}\rvert$	$f_i\lvert x_i-\bar{x}\rvert$
0-10	5	5	–2	–10	22	110
10-20	8	15	–1	–8	12	96
20-30	15	25A	0	0	2	30
30-40	16	35	1	16	8	128
40-50	6	45	2	12	18	108
Total	50			10		472

Here, N=50, $\Sigma f_id_i=10$, $\Sigma f_i\lvert x_i-\bar{x}\rvert=472$

So, Mean $\bar{x}=A+C\left(\frac{\Sigma f_id_i}{N}\right)=25+10\left(\frac{10}{50}\right)=25+2=27$.

$\therefore$ M.D= $\frac{1}{N}\sum_{i=1}^{5}f_i\lvert x_i-\bar{x}\rvert=\frac{1}{50}(472)=9.44$

- **Find the mean deviation about median for the following data:**

x_i	6	9	3	12	15	13	21	22
f_i	4	5	3	2	5	4	4	3

A: The statistical table is as follows:

x_i	f_i	c.f	$\lvert x_i-M\rvert$	$f_i\lvert x_i-M\rvert$
3	3	3	10	30
6	4	7	7	28
9	5	12	4	20
12	2	14	1	2
13	4	18	0	0
15	5	23	2	10
21	4	27	8	32
22	3	30	9	27
	N=30			149

$\frac{N}{2}=\left(\frac{30}{2}\right)=15$. So, Median $M=13$

$\therefore$ $M.D=\frac{\Sigma f_i\lvert x_i-M\rvert}{N}=\frac{149}{30}=4.97$

Q 23: PROBABILITY:

- **State and prove addition theorem on Probability.**

A: Statement: If E_1,E_2 are the 2events of a sample space S then $P(E_1\cup E_2)=P(E_1)+P(E_2)-P(E_1\cap E_2)$

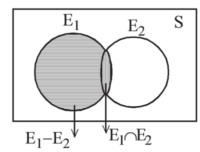
Proof: Case (i): When $E_1\cap E_2=\phi$

$E_1\cap E_2=\phi\Rightarrow P(E_1\cap E_2)=0$

$\therefore P(E_1\cup E_2)=P(E_1)+P(E_2)$ [From the union axiom]

$=P(E_1)+P(E_2)-0=P(E_1)+P(E_2)-P(E_1\cap E_2)$

Case (ii) : When $E_1\cap E_2\neq\phi$



$E_1\cup E_2$ is the union of disjoint sets (E_1-E_2), E_2

$\therefore P(E_1\cup E_2)=P[(E_1-E_2)\cup E_2]$

$=P(E_1-E_2)+P(E_2)$ ....(1)

E_1 is the union of disjoint sets (E_1-E_2), $(E_1\cap E_2)$.

$\therefore P(E_1)=P[(E_1-E_2)\cup(E_1\cap E_2)]$

$=P(E_1-E_2)+P(E_1\cap E_2)$

$\Rightarrow P(E_1-E_2)=P(E_1)-P(E_1\cap E_2)$

$\therefore$ from (1), $P(E_1\cup E_2)=[P(E_1)-P(E_1\cap E_2)]+P(E_2)$

$=P(E_1)+P(E_2)-P(E_1\cap E_2)$.

Hence proved.

- **State and Prove Baye's theorem on Probability.**

A: Statement: If $E_1, E_2...E_n$ are mutually exclusive and exhaustive events in a sample space S and A is any event intersecting with any E_i such that

$P(A)\neq0$ then $P(E_k\mid A)=\frac{P(E_k)P(A/E_k)}{\sum_{i=1}^{n}P(Ei).P(A/E_i)}$

Proof: From the definition of conditional probability:

$P(E_k\mid A)=\frac{P(E_k\cap A)}{P(A)}=\frac{P(E_k).P(A\mid E_k)}{P(A)}$ ...(1)

Given that $E_1,E_2....E_n$ are mutually exclusive and exhaustive events in a sample space S

$$\Rightarrow \bigcup_{i=1}^{n} E_i = S \text{ and } A\cap E_1, A\cap E_2, \ldots, A\cap E_n$$

are mutually disjoint $\Rightarrow A\cap E_i = \phi$

Now,

$$P(A) = P(S\cap A) = P\left(\left(\bigcup_{i=1}^{n} E_i\right)\cap A\right) = P\left(\bigcup_{i=1}^{n}(E_i\cap A)\right)$$

$$= \sum_{i=1}^{n} P(E_i\cap A) = \sum_{i=1}^{n} P(E_i)P(A\,|\,E_i)$$

$$\therefore \text{From (1), } P(E_k\,|\,A) = \frac{P(E_k)P(A\,|\,E_k)}{\sum_{i=1}^{n} P(E_i)P(A\,|\,E_i)}$$

- **Suppose that an urn B_1 contains 2 white and 3 black balls and another urn B_2 contains 3 white and 4 black balls. One urn is selected at random and a ball is drawn from it. If the ball drawn is found black, find the probability that the urn chosen was B_1.**

A: Let E_1, E_2 denote the events of selecting urn B_1 and urn B_2 respectively and

B be the event of drawing a black ball.

Then $P(E_1) = P(E_2) = \frac{1}{2}$ and

$$P(B\,|\,E_1) = \frac{3}{5};\ P(B\,|\,E_2) = \frac{4}{7}$$

$\therefore$ by Baye's theorem, the required probability is

$$P(E_1\,|\,B) = \frac{P(E_1)P(B\,|\,E_1)}{P(E_1).P(B\,|\,E_1) + P(E_2)P(B\,|\,E_2)}$$

$$= \frac{\frac{1}{2}\times\frac{3}{5}}{\left(\frac{1}{2}\times\frac{3}{5}\right)+\left(\frac{1}{2}\times\frac{4}{7}\right)} = \frac{\frac{3}{10}}{\frac{3}{10}+\frac{2}{7}} = \frac{\frac{3}{10}}{\frac{21+20}{70}}$$

$$= \frac{3}{10}\times\frac{70}{41} = \frac{21}{41}$$

- **Three boxes numbered I, II, III contains the balls as follows:**

	White	Black	Red
I	1	2	3
II	2	1	1
III	4	5	3

One box is randomly selected and a ball is drawn from it. If the ball is red, then find the probability that it is from box II.

A: Let B_1, B_2, B_3 be the events of selecting boxes B_1, B_2, B_3 and R be the event of getting drawing a red ball

$$\therefore P(B_1) = \frac{1}{3}, P(B_2) = \frac{1}{3}, P(B_3) = \frac{1}{3} \text{ and}$$

$$P\left(\frac{R}{B_1}\right) = \frac{3}{6} = \frac{1}{2},\ P\left(\frac{R}{B_2}\right) = \frac{1}{4},\ P\left(\frac{R}{B_3}\right) = \frac{3}{12} = \frac{1}{4}$$

$\therefore$ by Baye's theorem, the required probability is

$$P\left(\frac{B_2}{R}\right) = \frac{P(B_2)P\left(\frac{R}{B_2}\right)}{P(B_1)P\left(\frac{R}{B_1}\right)+P(B_2)P\left(\frac{R}{B_2}\right)+P(B_3)P\left(\frac{R}{B_3}\right)}$$

$$= \frac{\frac{1}{3}\times\frac{1}{4}}{\left(\frac{1}{3}\times\frac{1}{2}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{4}\right)} = \frac{\frac{1}{3}\times\frac{1}{4}}{\frac{1}{3}\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right)}$$

$$= \frac{\frac{1}{4}}{\frac{1}{2}+\frac{1}{4}+\frac{1}{4}} = \frac{\left(\frac{1}{4}\right)}{\left(\frac{4}{4}\right)} = \frac{1}{4}$$

Q 24: RANDOM VARIABLES:

- **A random variable X has the following probability distribution.**

$X=x_i$	1	2	3	4	5
$P(X=x_i)$	k	2k	3k	4k	5k

Find the k and the mean and variance of X.

A: We know $\sum P(X=x_i)=1$

$\Rightarrow k+2k+3k+4k+5k=1 \Rightarrow 15k=1$

$\Rightarrow k=1/15$

Mean $\mu = \sum_{i=1}^{5} x_i.P(X=x_i)$

$=1(k)+2(2k)+3(3k)+4(4k)+5(5k)$

$=k(1+4+9+16+25)=k(55)=\frac{1}{15}(55)=\frac{55}{15}=\frac{11}{3}$

Variance $\sigma^2 = \Sigma x_i^2.P(X=x_i)-\mu^2$

$=1(k)+4(2k)+9(3k)+16(4k)+25(5k)-\left(\frac{11}{3}\right)^2$

$=k(1+8+27+64+125)-\left(\frac{11}{3}\right)^2$

$=\frac{1}{15}(225)-\left(\frac{11}{3}\right)^2=15-\frac{121}{9}=\frac{14}{9}$

- **A random variable X has the range {1,2,3,....}. If $P(X=k)=\frac{c^k}{k!}$ for k=1,2,3,.... then find c and $P(0<X<3)$.**

A: Given that $P(X=k)=\frac{c^k}{k!}, (k=1,2,3,......)$

We know that the sum of probabilities

$\frac{c^1}{1!}+\frac{c^2}{2!}+\frac{c^3}{3!}+....=1$

$\Rightarrow 1+\frac{c}{1!}+\frac{c^2}{2!}+\frac{c^3}{3!}+....=1+1$

$\Rightarrow e^c=2 \Rightarrow c=\log_e 2$

Also, $P(0<X<3)=P(X=1)+P(X=2)$

$=\frac{c^1}{1!}+\frac{c^2}{2!}=c+\frac{c^2}{2}=\log_e 2+\frac{(\log_e 2)^2}{2}$

- **A cubical die is thrown. Find the mean and variance of X, giving the number on the face that shows up.**

A: Let S be the sample space of throwing a die and X be the random variable.

Then P(X) is given by the following table.

$X=x_i$	1	2	3	4	5	6
$P(X=x_i)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Mean of X is $\mu = \sum_{i=1}^{6} X_i.P(X=x_i)$

$=1.\frac{1}{6}+2.\frac{1}{6}+3.\frac{1}{6}+4.\frac{1}{6}+5.\frac{1}{6}+6.\frac{1}{6}$

$=\frac{1}{6}(1+2+3+4+5+6)=\frac{21}{6}=\frac{7}{2}$

Variance of X is $\sigma^2=\sum_{i=1}^{6} x_i^2.P(X=x_i)-\mu^2$

$=1^2.\frac{1}{6}+2^2.\frac{1}{6}+3^2.\frac{1}{6}+4^2.\frac{1}{6}+5^2.\frac{1}{6}+6^2.\frac{1}{6}-\left(\frac{7}{2}\right)^2$

$=\frac{1}{6}(1+4+9+16+25+36)-\frac{49}{4}$

$=\frac{91}{6}-\frac{49}{4}=\frac{182-147}{12}=\frac{35}{12}$