BULLET

MODEL PAPER

A 'MULTI QUESTION PAPER' WITH 'BULLET ANSWERS'

SAQ & LAQ SECTIONS

SAQ SECTION-B

Q11 : LOCUS:

- **Find the equation of locus of P, if the line segment joining (4,0) and (0,4)subtends a right angle at P.**

A: We take A=(4,0), B=(0,4) and P=(x,y) is a point on the locus.

Given condition: $\angle APB=90^\circ \Rightarrow PA^2+PB^2=AB^2$

$\Rightarrow[(x-4)^2+(y-0)^2]+[(x-0)^2+(y-4)^2]=(4-0)^2+(0-4)^2$

$\Rightarrow (x^2-8x+16)+y^2+x^2+(y^2-8y+16)=16+16$

$\Rightarrow 2x^2+2y^2-8x-8y=0 \Rightarrow 2(x^2+y^2-4x-4y)=0$

$\Rightarrow x^2+y^2-4x-4y=0$

- **A(5,3) and B(3,–2) are two fixed points. Find the equation of locus of P, so that the area of triangle PAB is 9 sq.units.**

A: Given that A=(5,3), B=(3,–2) and P=(x, y) be a point on the locus.

Given condition: Area of Δ PAB = 9 sq.units

$$\Delta=\frac{1}{2}\begin{vmatrix} x_1-x_2 & x_1-x_3 \\ y_1-y_2 & y_1-y_3\end{vmatrix}=9 \Rightarrow \frac{1}{2}\begin{vmatrix} 5-3 & 5-x \\ 3+2 & 3-y\end{vmatrix}=9$$

$$\Rightarrow\begin{vmatrix} 2 & 5-x \\ 5 & 3-y\end{vmatrix}=2(9) \Rightarrow |2(3-y)-5(5-x)|=18$$

$\Rightarrow|6-2y-25+5x|=18 \Rightarrow|5x-2y-19|=18$

$\Rightarrow 5x-2y-19=\pm18$

$\Rightarrow 5x-2y-19=18$ (or) $5x-2y-19=-18$

$\Rightarrow 5x-2y-37=0$ (or) $5x-2y-1=0$

$\Rightarrow (5x-2y-37)(5x-2y-1)=0$

Hence, locus of P is (5x–2y–37)(5x–2y–1)=0

- **If the distance from 'P' to the points (2,3), (2,–3) are in the ratio 2:3, then find the equation of locus of P.**

A: We take A=(2,3), B=(2,–3), P= (x,y) is a point on locus.

Condition: $\frac{PA}{PB}=\frac{2}{3} \Rightarrow 3PA=2PB \Rightarrow 9PA^2=4PB^2$

$\Rightarrow 9[(x-2)^2+(y-3)^2] = 4[(x-2)^2+(y+3)^2]$

$\Rightarrow 9[(x^2+4-4x)+(y^2+9-6y)]$

$=4[(x^2+4-4x)+(y^2+9+6y)]$

$\Rightarrow 9x^2+36-36x+9y^2+81-54y$

$=4x^2+16-16x+4y^2+36+24y$

$\Rightarrow 9x^2-4x^2+9y^2-4y^2-36x+16x-54y-24y+81-16=0$

$\Rightarrow 5x^2+5y^2-20x-78y+65=0$

Hence, locus of P is $5x^2+5y^2-20x-78y+65=0$.

- **Find the equation of locus of a point P, if A=(2,3), B=(2,-3) and PA+PB=8.**

A: Given A=(2,3), B=(2,–3),P=(x,y) is a point on the locus.

Given condition: PA+PB=8

$\Rightarrow PA=8-PB \Rightarrow PA^2=(8-PB)^2$

$\Rightarrow PA^2=64+PB^2-16PB \Rightarrow 16PB=64+PB^2-PA^2$

$\Rightarrow 16PB=64+[(x-2)^2+(y+3)^2]-[(x-2)^2+(y-3)^2]$

$\Rightarrow 16PB=64+(y+3)^2-(y-3)^2$

$\Rightarrow 16PB=64+4(3)y$ [$\because (a+b)^2-(a-b)^2=4ab$]

$\Rightarrow 16PB=4(16+3y)$

$\Rightarrow 4PB=(16+3y)$, Squaring on both sides

$\Rightarrow 16PB^2=(16+3y)^2$

$\Rightarrow 16[(x-2)^2+(y+3)^2]=256+9y^2+96y$

$\Rightarrow 16[(x^2+4-4x)+(y^2+9+6y)]=256+9y^2+96y$

$\Rightarrow 16x^2+64-64x+16y^2+144+96y$

$-256-9y^2-96y=0$

$\Rightarrow 16x^2+7y^2-64x-48=0.$

Hence, locus of P is $16x^2+7y^2-64x-48=0$

- **Find the equation of locus of a point the difference of whose distances from (–5,0) and (5,0) is 8 units.**

A: We take A=(–5,0), B=(5,0) and P= (x,y) is a point on the locus.

Given condition: |PA–PB|=8 ⇒PA–PB=±8

$\Rightarrow PA=\pm8+PB \Rightarrow PA^2=(\pm8+PB)^2$

$\Rightarrow PA^2=64+PB^2\pm16PB \Rightarrow \pm16PB=64+PB^2-PA^2$

$\Rightarrow\pm16PB=64+[(x-5)^2+(y-0)^2]-[(x+5)^2+(y-0)^2)]$

$\Rightarrow\pm16PB=64+(x-5)^2-(x+5)^2$

$\Rightarrow\pm16PB=64-4(x)(5)$ [$\because (a-b)^2-(a+b)^2=-4ab$]

$\Rightarrow\pm16PB=64-20x \Rightarrow \pm16PB=4(16-5x)$

$\Rightarrow\pm4PB=16-5x$, Squaring on both sides

$\Rightarrow 16PB^2=(16-5x)^2$

$\Rightarrow 16[(x-5)^2+(y-0)^2]=256+25x^2-160x$

$\Rightarrow 16(x^2+25-10x+y^2)=256-160x+25x^2$

$\Rightarrow 16x^2+400-160x+16y^2=256-160x+25x^2$

$\Rightarrow (25x^2-16x^2)-16y^2=400-256 \Rightarrow 9x^2-16y^2=144$

Hence locus of P is $9x^2-16y^2=144$.

Q12 : TRANSFORMATIONS:

- **When the origin is shifted to (–1,2), find the transformed equation of $x^2+y^2+2x-4y+1=0$**

A: Given equation (original) is $x^2+y^2+2x-4y+1=0$....(1)
We take new origin (h,k)=(–1,2), then
$x=X+h \Rightarrow x=X-1$; $y=Y+k \Rightarrow y=Y+2$
From (1), transformed equation is
$(X-1)^2+(Y+2)^2+2(X-1)-4(Y+2)+1=0$
$\Rightarrow (X^2+1-2X)+(Y^2+4+4Y)+2X-2-4Y-8+1=0$
$\Rightarrow X^2+Y^2-4=0$

- **Find the transformed equation of $3x^2+10xy+3y^2=9$ when the axes are rotated through an angle $\pi/4$.**

A: Given equation (original) is $3x^2+10xy+3y^2=9$ ......(1)
Angle of rotation $\theta=\pi/4=45^o$, then
$x=X\cos\theta-Y\sin\theta \Rightarrow x=X\cos 45^o-Y\sin 45^o$
$=X\left(\frac{1}{\sqrt{2}}\right)-Y\left(\frac{1}{\sqrt{2}}\right) \Rightarrow x=\frac{X-Y}{\sqrt{2}}$
$y=Y\cos\theta+X\sin\theta \Rightarrow y=Y\cos 45^o+X\sin 45^o$
$=Y\left(\frac{1}{\sqrt{2}}\right)+X\left(\frac{1}{\sqrt{2}}\right) \Rightarrow y=\frac{X+Y}{\sqrt{2}}$
From (1), transformed equation is
$3\left(\frac{X-Y}{\sqrt{2}}\right)^2+10\left(\frac{X-Y}{\sqrt{2}}\right)\left(\frac{X+Y}{\sqrt{2}}\right)+3\left(\frac{X+Y}{\sqrt{2}}\right)^2=9$
$\Rightarrow 3\left(\frac{X^2+Y^2-2XY}{2}\right)+10\left(\frac{X^2-Y^2}{2}\right)+3\left(\frac{X^2+Y^2+2XY}{2}\right)=9$
$\Rightarrow \frac{3X^2+3y^2-6XY+10X^2-10Y^2+3X^2+3Y^2+6XY}{2}=9$
$\Rightarrow 16X^2-4Y^2=2(9) \Rightarrow 2(8X^2-2Y^2)=2(9)$
$\Rightarrow 8X^2-2Y^2=9$

- **When the axes are rotated through an angle 45°, the transformed equation of a curve is $17x^2-16xy+17y^2=225$. Find the original equation.**

A: Given transformed(new) equation is taken as
$17X^2-16XY+17Y^2=225$ ....(1)
Angle of rotation $\theta=45^o$, then
$X=x\cos\theta+y\sin\theta=x\cos45^o+y\sin45^o$
$=x\left(\frac{1}{\sqrt{2}}\right)+y\left(\frac{1}{\sqrt{2}}\right) \Rightarrow X=\frac{x+y}{\sqrt{2}}$
$Y=y\cos\theta-x\sin\theta=y\cos45^o-x\sin45^o$
$=y\left(\frac{1}{\sqrt{2}}\right)-x\left(\frac{1}{\sqrt{2}}\right) \Rightarrow Y=\frac{y-x}{\sqrt{2}}$
From (1),original equation is
$17\left(\frac{x+y}{\sqrt{2}}\right)^2-16\left(\frac{x+y}{\sqrt{2}}\right)\left(\frac{y-x}{\sqrt{2}}\right)+17\left(\frac{y-x}{\sqrt{2}}\right)^2=225$
$\Rightarrow 17\left(\frac{x^2+y^2+2xy}{2}\right)-16\left(\frac{y^2-x^2}{2}\right)+17\left(\frac{y^2+x^2-2xy}{2}\right)=225$
$\Rightarrow \frac{17x^2+17y^2+34xy-16y^2+16x^2+17x^2+17y^2-34xy}{2}=225$
$\Rightarrow 50x^2+18y^2=2(225) \Rightarrow 2(25x^2+9y^2)=2(225)$
$\Rightarrow 25x^2+9y^2=225$

Q13 : STRAIGHT LINES:

- **Find the value of k, if the lines $2x-3y+k=0$, $3x-4y-13=0$, $8x-11y-33=0$ are concurrent.**

A: Given lines $3x-4y-13=0$ ....(1); $8x-11y-33=0$ ....(2)
Solving (1) and (2), we get P $\frac{x}{(-4)(-33)-(-11)(-13)}$
$=\frac{y}{-13(8)-(-33)(3)}=\frac{1}{3(-11)-8(-4)}$
$\Rightarrow \frac{x}{132-143}=\frac{y}{-104+99}=\frac{1}{-33+32}$
$\Rightarrow \frac{x}{-11}=\frac{y}{-5}=\frac{1}{-1} \Rightarrow x=\frac{-11}{-1}=11; y=\frac{-5}{-1}=5$
$\therefore$ Point of intersection is P(11, 5)
Given lines are concurrent.
So, P(11,5) lies on $2x-3y+k=0$
$\Rightarrow 2(11)-3(5)+k=0 \Rightarrow 22-15+k=0 \Rightarrow 7+k=0 \Rightarrow k=-7$
Hence, value of k = –7

- **Find the value of k if the angle between the straight lines $4x-y+7=0$, $kx-5y-9=0$ is 45°**

A: Given line is $4x-y+7=0$. It's slope $m_1=\frac{-a}{b}=\frac{-4}{-1}=4$
Another line is $kx-5y-9=0$.
It's slope is $m_2=\frac{-a}{b}=\frac{-k}{-5}=\frac{k}{5}$
Angle between the lines is 45°, then $\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$
$\Rightarrow \tan 45^o=\left|\frac{4-(k/5)}{1+4(k/5)}\right|$
$\Rightarrow 1=\left|\frac{20-k}{5+4k}\right| \Rightarrow |5+4k|=|20-k|$
$\Rightarrow 5+4k=\pm(20-k) \Rightarrow 5+4k=20-k \Rightarrow 5k=15 \Rightarrow k=3$
(or) $5+4k=-(20-k)=k-20 \Rightarrow 3k=-25 \Rightarrow k=-25/3$
$\therefore$ k=3 or –25/3

- **Find the points on the line $3x-4y-1=0$ which are at a distance of 5 units from the point (3, 2).**

A: Given line is $3x-4y-1=0$. It's slope $m=\frac{-a}{b}=\frac{-3}{-4}=\frac{3}{4}$
$m=\tan\theta=\frac{3}{4} \Rightarrow \cos\theta=\frac{4}{5}$, $\sin\theta=\frac{3}{5}$
Given distance |r|=5; Given point $(x_1,y_1)=(3, 2)$
Required points $(x,y)=(x_1\pm r\cos\theta, y_1\pm r\sin\theta)$
$=\left(3\pm 5\left(\frac{4}{5}\right), 2\pm 5\left(\frac{3}{5}\right)\right)=(3\pm 4, 2\pm 3)$
= (3+4,2+3)=(7,5) (or) (3–4,2–3)=(–1,–1)
The required points are (7,5) and (–1,–1)

Q14 : CONTINUITY:

- **Is f defined by $f(x)=\begin{cases}\frac{\sin 2x}{x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$ continuous at 0?**

A: (a) Given f(0)=1 ....(1)

(b) $\underset{x\to 0}{Lt} f(x)=\underset{x\to 0}{Lt}\frac{\sin 2x}{x}=2$ ......(2) $\left(\because \underset{x\to 0}{Lt}\frac{\sin kx}{x}=k\right)$

$\therefore$ From (1) & (2), $\underset{x\to 0}{Lt} f(x)\neq f(0)$

Hence, proved that f is not continuous at x=0

- **Is f given by $f(x)=\begin{cases}\frac{x^2-9}{x^2-2x-3} & \text{if } 0<x<5, x\neq 3\\ 1.5 & \text{if } x=3\end{cases}$, continuous at the point 3.**

A: (a) Given f(3)=1.5.........(1)

(b) $\underset{x\to 3}{Lt} f(x)=\underset{x\to 3}{Lt}\frac{x^2-9}{x^2-2x-3}=\underset{x\to 3}{Lt}\frac{(x-3)(x+3)}{(x-3)(x+1)}$

$=\underset{x\to 3}{Lt}\frac{x+3}{x+1}=\frac{3+3}{3+1}=\frac{6}{4}=\frac{3}{2}=1.5$ ....(2)

From (1) & (2), $\underset{x\to 0}{Lt} f(x)=f(0)$

Hence proved that f(x) is continuous at x=3

- **If f is given by $f(x)=\begin{cases}k^2x-k & \text{if } x\geq 1\\ 2 & \text{if } x<1\end{cases}$ is a continuous function on R, then find k.**

A: (a) When x<1, L.H.L $=\underset{x\to 1-}{Lt} f(x)=\underset{x\to 1-}{Lt} 2=2$ .....(1)

(b) When x>1,

R.H.L $=\underset{x\to 1+}{Lt} f(x)=\underset{x\to 1+}{Lt} k^2x-k=k^2(1)-k=k^2-k$ ....(2)

From (1) & (2), L.H.L= R.H.L

[$\because$ f(x) is continuous at x=1as it is continuous on R]

So, $k^2-k=2\Rightarrow k^2-k-2=0\Rightarrow(k-2)(k+1)=0\Rightarrow k=2$ (or) -1

- **Check the continuity of $f(x)=\begin{cases}\frac{1}{2}(x^2-4) & \text{if } 0<x<2\\ 0 & \text{if } x=2\\ 2-8x^{-3} & \text{if } x>2\end{cases}$ at 2.**

A: (a) When x<2,

L.H.L= $\underset{x\to 2-}{Lt} f(x)=\underset{x\to 2-}{Lt}\frac{1}{2}(x^2-4)=\frac{1}{2}(4-4)=0$ .....(1)

(b) When x>2, R.H.L= $\underset{x\to 2+}{Lt} f(x)$

$=\underset{x\to 2+}{Lt}(2-8x^{-3})=\underset{x\to 2+}{Lt}\left(2-\frac{8}{x^3}\right)=2-\frac{8}{8}=2-1=1$.......(2)

From (1) & (2), L.H.L≠R.H.L

Hence proved that f(x) is not continuous at 2.

Q15 : DIFFERENTIATION:

- **Find the derivative of sin2x from the first principle.**

A: We take f(x)=sin2x, then f(x+h)=sin2(x+h)=sin (2x+2h)

From the first principle, $f'(x)=\underset{h\to 0}{Lt}\frac{f(x+h)-f(x)}{h}$

$=\underset{h\to 0}{Lt}\frac{\sin(2x+2h)-\sin 2x}{h}$

$=\underset{h\to 0}{Lt}\frac{1}{h}\left(2\cos\left(\frac{(2x+2h)+2x}{2}\right)\sin\left(\frac{(2x+2h)-2x}{2}\right)\right)$

$=2\underset{h\to 0}{Lt}\frac{1}{h}\cos\left(\frac{4x+2h}{2}\right)\sin\left(\frac{2h}{2}\right)$

$=2\underset{h\to 0}{Lt}\frac{1}{h}\cos\left(\frac{2(2x+h)}{2}\right)\sin(h)$

$=2\underset{h\to 0}{Lt}\frac{1}{h}\cos(2x+h)\sin(h)=2\underset{h\to 0}{Lt}\cos(2x+h).\underset{h\to 0}{Lt}\frac{\sinh}{h}$

$=2\cos(2x+0)(1)=2\cos 2x$

- **Find the derivative of cosax from the first Principle.**

A: We take f(x)=cosax,then f(x+h)=cosa(x+h)=cos(ax+ah)

From the first principle, $f'(x)=\underset{h\to 0}{Lt}\frac{f(x+h)-f(x)}{h}$

$=\underset{h\to 0}{Lt}\frac{\cos(ax+ah)-\cos(ax)}{h}$

$=\underset{h\to 0}{Lt}\frac{1}{h}\left[-2\sin\left(\frac{(ax+ah)+ax}{2}\right)\sin\left(\frac{(ax+ah)-ax}{2}\right)\right]$

$=-2\underset{h\to 0}{Lt}\frac{1}{h}\left[\sin\left(\frac{2ax+ah}{2}\right)\sin\left(\frac{ah}{2}\right)\right]$

$=-2\underset{h\to 0}{Lt}\frac{1}{h}\left[\sin\left(ax+\frac{ah}{2}\right)\sin\frac{ah}{2}\right]$

$=-2\underset{h\to 0}{Lt}\sin\left(ax+\frac{ah}{2}\right)\underset{h\to 0}{Lt}\frac{\sin(ah/2)}{h}$

$=-2\sin(ax+0)\left(\frac{a}{2}\right)=-2\sin ax\left(\frac{a}{2}\right)=-a\sin ax$

- **Find the derivative of tan2x from the first principle.**

A: We take f(x)=tan(2x), then f(x+h)=tan2(x+h)=tan(2x+2h)

From the first principle, $f'(x)=\underset{h\to 0}{Lt}\frac{f(x+h)-f(x)}{h}$

$=\underset{h\to 0}{Lt}\frac{\tan(2x+2h)-\tan(2x)}{h}$

$=\underset{h\to 0}{Lt}\frac{1}{h}\left[\frac{\sin(2x+2h)}{\cos(2x+2h)}-\frac{\sin(2x)}{\cos(2x)}\right]$

$=\underset{h\to 0}{Lt}\frac{1}{h}\left[\frac{\sin(2x+2h)\cos(2x)-\cos(2x+2h)\sin(2x)}{\cos(2x+2h)\cos(2x)}\right]$

$=\underset{h\to 0}{Lt}\frac{1}{h}\frac{\sin[(2x+2h)-2x]}{\cos(2x+2h)\cos(2x)}$

$[\because \sin A\cos B-\cos A\sin B=\sin(A-B)]$

$=\underset{h\to 0}{Lt}\frac{\sin 2h}{h}\underset{h\to 0}{Lt}\frac{1}{\cos(2x+2h)\cos(2x)}$

$=2.\frac{1}{\cos^2(2x)}=2\sec^2(2x)$

Q16 : TANGENT & NORMAL:

- **Find the equations of the tangent and the normal to the curve xy=10 at (2, 5)**

A: Given curve is xy=10 $\Rightarrow y=\frac{10}{x} \Rightarrow \frac{dy}{dx}=-\frac{10}{x^2}$

Slope of the tangent at (2, 5) is

$$m=\left(\frac{dy}{dx}\right)_{(2,5)}=\frac{-10}{2^2}=\frac{-10}{4}=\frac{-5}{2}$$

(i) Equation of the tangent at (2,5) with slope $\frac{-5}{2}$ is $y-y_1=m(x-x_1)$

$$\Rightarrow y-5=\frac{-5}{2}(x-2) \Rightarrow 2y-10=-5x+10$$

$$\Rightarrow 5x+2y-20=0$$

(ii) Slope of the normal is $\frac{-1}{m}=\frac{2}{5}$

Equation of the normal at (2,5) with slope $\frac{2}{5}$ is

$$y-y_1=-\frac{1}{m}(x-x_1)$$

$$\Rightarrow y-5=\frac{2}{5}(x-2) \Rightarrow 5y-25=2x-4$$

$$\Rightarrow 2x-5y+21=0$$

- **Show that the curves $x^2+y^2=2$, $3x^2+y^2=4x$ have a common tangent at the point (1,1)**

A: Given first curve is $x^2+y^2=2 \Rightarrow 2x+2y\frac{dy}{dx}=0$

$$\Rightarrow \not{2}y\frac{dy}{dx}=-\not{2}x \Rightarrow \frac{dy}{dx}=-\frac{x}{y}$$

So, slope of the tangent at P(1,1) is $m_1=\frac{-1}{1}=-1$.......(1)

Given second curve is $3x^2+y^2=4x$

$$\Rightarrow 6x+2y.\frac{dy}{dx}=4 \Rightarrow \not{2}(3x+y.\frac{dy}{dx})=\not{2}(2)$$

$$\Rightarrow 3x+y.\frac{dy}{dx}=2 \Rightarrow y.\frac{dy}{dx}=2-3x \Rightarrow \frac{dy}{dx}=\frac{2-3x}{y}$$

So, slope of the tangent at P(1,1) is

$$m_2=\frac{2-3}{1}=-\frac{1}{1}=-1 \text{.......(2)}$$

From (1) & (2), $m_1=m_2$. So slopes are equal.
Hence proved.

- **Show that at any point (x,y) on the curve $y=be^{x/a}$, the length of subtangent is a constant and the length of the subnormal is y^2/a.**

A: Let P(x,y) be point on the curve $y=be^{x/a}$

On diff. w.r.to x, we get $\frac{dy}{dx}=be^{\frac{x}{a}}.\left(\frac{1}{a}\right)=\frac{b}{a}e^{\frac{x}{a}}$

$$\therefore \text{ Slope } m=\left(\frac{dy}{dx}\right)_P=\frac{b}{a}e^{\frac{x}{a}}$$

(i)Length of subtangent

$$=\left|\frac{y}{m}\right|=\left|\frac{\not{b}\,\not{e}^{\frac{x}{a}}}{\frac{\not{b}}{a}\not{e}^{\frac{x}{a}}}\right|=a=\text{constant}$$

(i) Length of subnormal

$$=|y.m|=\left|be^{\frac{x}{a}}.\frac{b}{a}e^{\frac{x}{a}}\right|=\left|\frac{\left(be^{\frac{x}{a}}\right)^2}{a}\right|=\left|\frac{y^2}{a}\right|$$

Q17 : RATE MEASURE:

- **A particle is moving along a line according $s=f(t)=4t^3-3t^2+5t-1$ where s is measured in meters and t is measured in seconds. Find the velocity and acceleration at time t. At what time the acceleration is zero.**

A: Given that, $s=f(t)=4t^3-3t^2+5t-1$

$\therefore$ Velocity $v=\frac{ds}{dt}=12t^2-6t+5$

Acceleration $a=\frac{dV}{dt}=24t-6$

If the acceleration is 0 then $24t-6=0 \Rightarrow t=\frac{1}{4}$

The acceleration of the particle is zero at $t=\frac{1}{4}$ sec.

- **The volume of a cube is increasing at a rate of 9 cubic centimeters per second. How fast is the surface area increasing when the length of the edge is 10 centimeters?**

A: For the cube, we take
length of the edge =x , Volume=V and
Surface area=S

Given $\frac{dV}{dt}=9$ and x=10 cm

Volume of the cube $V=x^3$

On diff. w.r.t 't', we get $\frac{dV}{dt}=3x^2\frac{dx}{dt}$

$$\Rightarrow 9=3x^2\frac{dx}{dt} \Rightarrow \frac{dx}{dt}=\frac{\not{9}}{\not{3}x^2}=\frac{3}{x^2}$$

Surface area $S=6x^2$

On diff. w.r.t 't', we get $\frac{dS}{dt}=12x\frac{dx}{dt}$

$$=12\not{x}\left(\frac{3}{\not{x^2}}\right)=\frac{36}{x}=\frac{36}{10}=3.6\ cm^2/sec$$

LAQ SECTION-C

Q18 : STRAIGHT LINES:

- **Find the circumcentre of the triangle whose vertices are (1,3), (–3,5),(5,–1).**

A: We take S(x,y)as circumcentre and vertices A=(1,3), B=(–3,5), C=(5,–1).
Then SA=SB=SC
First we take, SA= SB

$$\Rightarrow\sqrt{(x-1)^2+(y-3)^2}=\sqrt{(x+3)^2+(y-5)^2}$$

Squaring on both sides, we get
$(x-1)^2+(y-3)^2=(x+3)^2+(y-5)^2$

$$\Rightarrow(\cancel{x^2}+1-2x)+(\cancel{y^2}+9-6y)=(\cancel{x^2}+9+6x)+(\cancel{y^2}+25-10y)$$

$$\Rightarrow 6x+2x-10y+6y+25\cancel{+9}\cancel{-9}-1=0$$

$$\Rightarrow 8x-4y+24=0\Rightarrow\cancel{4}(2x-y+6)=0$$

$\Rightarrow$ 2x–y+6=0 ....(1)

Now, we take SB=SC

$$\Rightarrow\sqrt{(x+3)^2+(y-5)^2}=\sqrt{(x-5)^2+(y+1)^2}$$

Squaring on both sides, we get
$(x+3)^2+(y-5)^2=(x-5)^2+(y+1)^2$

$$\Rightarrow(\cancel{x^2}+9+6x)+(\cancel{y^2}+25-10y)=(\cancel{x^2}+25-10x)+(\cancel{y^2}+1+2y)$$

$$\Rightarrow 6x+10x-10y-2y+9\cancel{+25}\cancel{-25}-1=0$$

$$\Rightarrow 16x-12y+8=0\Rightarrow\cancel{4}(4x-3y+2)=0$$

$\Rightarrow$ 4x–3y+2=0 ....(2)

Solving (1) and (2), we get S;
2x–y+6=0;
4x–3y+2=0

$$\Rightarrow\frac{x}{(-1)2-(-3)6}=\frac{y}{6(4)-2(2)}=\frac{1}{2(-3)-4(-1)}$$

$$\Rightarrow\frac{x}{-2+18}=\frac{y}{24-4}=\frac{1}{-6+4}$$

$$\Rightarrow\frac{x}{16}=\frac{y}{20}=\frac{1}{-2}$$

$$\Rightarrow x=\frac{-16}{2}=-8,\ y=\frac{-20}{2}=-10$$

∴ Circumcentre S (x,y) = (–8, –10)

- **Find the orthocentre of the triangle whose vertices are(–2,–1),(6, –1),(2, 5)**

A: We take O as orthocentre and vertices A=(–2,–1), B=(6,–1), C= (2,5)

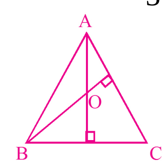
First we find altitude through A(–2,–1):

Slope of $\overline{BC}$ is $m=\frac{y_2-y_1}{x_2-x_1}=\frac{5+1}{2-6}=\frac{6}{-4}=-\frac{3}{2}$

So, slope of its perpendicular is $\frac{-1}{m}=\frac{-1}{-\frac{3}{2}}=\frac{2}{3}$

Perpendicular through A(–2,–1) with slope $\frac{2}{3}$

is $y-y_1=\frac{-1}{m}(x-x_1)$



$$\Rightarrow y+1=\frac{2}{3}(x+2)\Rightarrow 3y+3=2x+4$$

$\Rightarrow 2x-3y+1=0$.........(1)

Now, we find altitude through B(6,–1):

Slope of $\overline{AC}$ is

$$m=\frac{y_2-y_1}{x_2-x_1}=\frac{5+1}{2+2}=\frac{6}{4}=\frac{3}{2}$$

So, slope of its perpendicular is $\frac{-1}{m}=\frac{-1}{\frac{3}{2}}=-\frac{2}{3}$

Perpendicular through B(6,–1)with slope $-\frac{2}{3}$

is $y-y_1=\frac{-1}{m}(x-x_1)$

$$\Rightarrow y+1=-\frac{2}{3}(x-6)\Rightarrow 3y+3=-2x+12$$

$\Rightarrow 2x+3y-9=0$.........(2)

Solving (1), (2), we get 'O'; 2x–3y+1=0
2x+3y–9=0
(1)+(2)⇒ 4x–8=0⇒ 4x=8⇒ x=2
(1) ⇒2(2)–3(y)+1=0⇒3y =5 ⇒y = 5/3
⇒ x=2 , y=5/3
∴ Orthocentre O (x,y) =(2 ,5/3).

Q19: HOMOGENISATION:

- **Show that the lines joining the origin with the points of intersection of the curve $7x^2-4xy+8y^2+2x-4y-8=0$ with the line $3x-y=2$ are mutually perpendicular.**

A: Given line is $3x-y=2 \Rightarrow \frac{3x-y}{2}=1$ ....(1)

Given curve is

$7x^2-4xy+8y^2+2x-4y-8=0$ ......(2)

From (1)&(2), the homogenised equation is

$$7x^2-4xy+8y^2+2x(1)-4y(1)-8(1^2)=0$$

$$\Rightarrow 7x^2-4xy+8y^2+2x\left(\frac{3x-y}{2}\right)-4y\left(\frac{3x-y}{2}\right)-8\frac{(3x-y)^2}{4}=0$$

$$\Rightarrow 7x^2-4xy+8y^2+x(3x-y)-2y(3x-y)-2(9x^2+y^2-6xy)=0$$

$$\Rightarrow 7x^2-4xy+8y^2+3x^2-xy-6xy+2y^2-18x^2-2y^2+12xy=0$$

$$\Rightarrow -8x^2+8y^2+xy=0$$

Here, coeff. of x^2 + coeff. of y^2 is $-8+8=0$

$\therefore$ The pair of lines are perpendicular

- **Find the value of k, if the lines joining the origin with the points of intersection of the curve $2x^2-2xy+3y^2+2x-y-1=0$ and the line $x+2y=k$ are mutually perpendicular.**

A: The given line is $x+2y=k \Rightarrow \frac{x+2y}{k}=1$ .....(1)

Given curve is

$2x^2-2xy+3y^2+2x-y-1=0$ ....(2)

From (1)&(2), the homogenised equation is

$$2x^2-2xy+3y^2+2x(1)-y(1)-(1^2)=0$$

$$\Rightarrow 2x^2-2xy+3y^2+2x\left(\frac{x+2y}{k}\right)-y\left(\frac{x+2y}{k}\right)-\frac{(x+2y)^2}{k^2}=0$$

$$\Rightarrow \frac{k^2(2x^2-2xy+3y^2)+k(2x^2+4xy)-k(xy+2y^2)-(x^2+4y^2+4xy)}{k^2}=0$$

$$\Rightarrow k^2(2x^2-2xy+3y^2)+k(2x^2+4xy)-k(xy+2y^2)-(x^2+4y^2+4xy)=0$$

$$\Rightarrow x^2(2k^2+2k-1)+y^2(3k^2-2k-4)+xy(-2k^2+3k-4)=0$$

On applying perpendicular pair of lines condition, we get Coeff. x^2 + Coeff. y^2 =0

$$\Rightarrow (2k^2+2k-1)+(3k^2-2k-4)=0$$

$$\Rightarrow 5k^2-5=0$$

$$\Rightarrow 5(k^2-1)=0 \Rightarrow k^2-1=0$$

$$\Rightarrow k^2=1 \Rightarrow k=\pm 1$$

Hence, value of $k=\pm 1$

- **Find the condition for the chord $lx+my=1$ of the circle $x^2+y^2=a^2$ (whose centre is the origin) to subtend a right angle at the origin.**

A: Given chord is $lx+my=1$ ...(1)

Given circle is $x^2+y^2=a^2$

$\Rightarrow x^2+y^2-a^2=0$ ....(2)

From (1) & (2), homogenised equation is

$$\Rightarrow x^2+y^2-a^2(1^2)=0$$

$$\Rightarrow x^2+y^2-a^2(lx+my)^2=0$$

$$\Rightarrow x^2+y^2-a^2(l^2x^2+m^2y^2+2lmxy)=0$$

$$\Rightarrow x^2+y^2-a^2l^2x^2-a^2m^2y^2-2a^2lmxy=0$$

$$\Rightarrow x^2+y^2-a^2l^2x^2-a^2m^2y^2-2a^2lmxy=0$$

$$\Rightarrow x^2(1-a^2l^2)+y^2(1-a^2m^2)-2a^2lmxy=0$$

On applying perpendicular pair of lines condition,

we get Coeff. x^2 + Coeff. y^2 =0

$$\Rightarrow (1-a^2l^2)+(1-a^2m^2)=0$$

$$\Rightarrow 2-a^2l^2-a^2m^2=0$$

$$\Rightarrow a^2l^2+a^2m^2=2 \Rightarrow a^2(l^2+m^2)=2$$

Q20: PAIR OF LINES:

- **P.T the product of the perpendiculars from (α,β) to $ax^2+2hxy+by^2=0$ is $\dfrac{|a\alpha^2+2h\alpha\beta+b\beta^2|}{\sqrt{(a-b)^2+4h^2}}$**

A: We take

$ax^2+2hxy+by^2 \equiv (l_1x+m_1y)(l_2x+m_2y)$

On equating like term coeff. , we get $a=l_1l_2$, $b=m_1m_2$, $2h=l_1m_2+l_2m_1$.

Perpendicular distance from (α,β) to

$$l_1x+m_1y=0 \text{ is } p_1=\frac{|l_1\alpha+m_1\beta|}{\sqrt{l_1^2+m_1^2}}$$

Perpendicular distance from (α,β) to

$$l_2x+m_2y=0 \text{ is } p_2=\frac{|l_2\alpha+m_2\beta|}{\sqrt{l_2^2+m_2^2}}$$

$\therefore$ Product of perpendiculars is

$$p_1.p_2=\frac{|l_1\alpha+m_1\beta|}{\sqrt{l_1^2+m_1^2}}\frac{|l_2\alpha+m_2\beta|}{\sqrt{l_2^2+m_2^2}}$$

$$=\frac{|(l_1\alpha+m_1\beta)(l_2\alpha+m_2\beta)|}{\sqrt{(l_1^2+m_1^2)(l_2^2+m_2^2)}}$$

$$=\frac{|l_1l_2\alpha^2+l_1m_2\alpha\beta+l_2m_1\alpha\beta+m_1m_2\beta^2|}{\sqrt{l_1^2l_2^2+m_1^2m_2^2+l_1^2m_2^2+l_2^2m_1^2}}$$

$$=\frac{|l_1l_2\alpha^2+(l_1m_2+l_2m_1)\alpha\beta+m_1m_2\beta^2|}{\sqrt{(l_1l_2-m_1m_2)^2+\cancel{2l_1l_2m_1m_2}+(l_1m_2+l_2m_1)^2-\cancel{2l_1m_2l_2m_1}}}$$

$$=\frac{|a\alpha^2+2h\alpha\beta+b\beta^2|}{\sqrt{(a-b)^2+4h^2}}$$

- **If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents two parallel lines then Prove that (i) $h^2=ab$ (ii) $af^2=bg^2$ (iii) the distance between the parallel lines is $2\sqrt{\dfrac{g^2-ac}{a(a+b)}}$ (or) $2\sqrt{\dfrac{f^2-bc}{b(a+b)}}$**

A: We take $ax^2+2hxy+by^2+2gx+2fy+c=0$

$\equiv (lx+my+n_1)(lx+my+n_2)$

On equating like term coeff. , we get

$a=l^2$, $b=m^2$, $h=lm$, $2g=l(n_1+n_2)$,

$2f=m(n_1+n_2)$, $c=n_1n_2$

(i) $h^2=(lm)^2=l^2m^2=ab \Rightarrow h^2=ab$

$$\text{(ii) } af^2=l^2\left(\frac{m(n_1+n_2)}{2}\right)^2=\frac{l^2m^2(n_1+n_2)^2}{4}$$

$$=\frac{m^2l^2(n_1+n_2)^2}{4}=m^2\left(\frac{l(n_1+n_2)}{2}\right)^2=bg^2$$

(iii) Distance between $lx+my+n_1=0$,

$$lx+my+n_2=0 \text{ is } \frac{|n_1-n_2|}{\sqrt{l^2+m^2}}$$

$$=\frac{\sqrt{(n_1+n_2)^2-4n_1n_2}}{\sqrt{a+b}}=\sqrt{\frac{\left(\frac{2g}{l}\right)^2-4c}{a+b}}$$

$$=\sqrt{\frac{\frac{4g^2}{l^2}-4c}{a+b}}=\sqrt{\frac{\frac{4g^2}{a}-4c}{a+b}}$$

$$=\sqrt{\frac{4g^2-4ac}{a(a+b)}}=2\sqrt{\frac{g^2-ac}{a(a+b)}}$$

Similarly, by taking $n_1+n_2=\dfrac{2f}{m}$ we get, the distance between the lines $2\sqrt{\dfrac{f^2-bc}{b(a+b)}}$

Q21: DC'S & DR'S:

- **Find the angle between the lines whose d.c's are related by $l+m+n=0$ and $l^2+m^2-n^2=0$**

A: Given $l+m+n=0 \Rightarrow l=-(m+n)$.....(1), $l^2+m^2-n^2=0$ ......(2)

Solving (1) & (2) we get

$[-(m+n)]^2+m^2-n^2=0$

$\Rightarrow (m^2+n^2+2mn)+m^2-n^2=0$

$\Rightarrow 2m^2+2mn=0 \Rightarrow 2(m^2+mn)=0$

$\Rightarrow m^2+mn=0 \Rightarrow m(m+n)=0$

$\Rightarrow m=0$ (or) $m+n=0 \Rightarrow m=0$ (or) $m=-n$

Case (i): Put m=0 in(1), then $l=-(0+n)=-n$

$\therefore l=-n$

Now, $l:m:n=-n:0:n=-1:0:1$

So, d.r's of $L_1=(a_1,b_1,c_1)=(-1,0,1)$ ......(3)

Case (ii):

Put $m=-n$ in (1), then $l=-(-n+n)=0$

$\therefore l=0$

Now, $l:m:n=0:-n:n=0:-1:1$

So, d.r's of $L_2=(a_2,b_2,c_2)=(0,-1,1)$......(4)

If θ is the angle between the lines then from (3), (4), we get

$$\cos\theta=\frac{|a_1a_2+b_1b_2+c_1c_2|}{\sqrt{(a_1^2+b_1^2+c_1^2)(a_2^2+b_2^2+c_2^2)}}$$

$$=\frac{|(-1)(0)+(0)(-1)+1(1)|}{\sqrt{((-1)^2+0^2+1^2)(0^2+(-1)^2+1^2)}}$$

$$=\frac{1}{\sqrt{(2)(2)}}=\frac{1}{\sqrt{4}}=\frac{1}{2}=\cos 60^\circ \Rightarrow \theta=60^\circ$$

Hence angle between the lines is 60°.

- **Find the direction cosines of two lines which are connected by the relations $l+m+n=0$ and $mn-2nl-2lm=0$**

A: Given $l+m+n=0 \Rightarrow l=-m-n$ .....(1), $mn-2nl-2lm=0$ .....(2)

Solving (1) & (2), we get

$mn-2n(-m-n)-2m(-m-n)=0$

$\Rightarrow mn+2mn+2n^2+2m^2+2mn=0$

$\Rightarrow 2m^2+5mn+2n^2=0$

$\Rightarrow (2m+n)(m+2n)=0$

$\Rightarrow 2m+n=0$ (or) $m+2n=0$

So, $2m+n=0 \Rightarrow 2m=-n$

$\Rightarrow m=-n/2$ (or)

$m+2n=0 \Rightarrow m=-2n$

Case (i): Put $m=-\frac{n}{2}$ in (1), then

$$l=-\left(-\frac{n}{2}\right)-n=\frac{n}{2}-n=-\frac{n}{2} \Rightarrow l=\frac{-n}{2}$$

$$\therefore l:m:n=-\frac{n}{2}:-\frac{n}{2}:n$$

$$=-\frac{1}{2}:-\frac{1}{2}:1=\frac{1}{2}:\frac{1}{2}:-1=1:1:-2$$

So, d.r's of $L_1=(a_1,b_1,c_1)=(1,1,-2)$

On dividing by

$\sqrt{1^2+1^2+(-2)^2}=\sqrt{1+1+4}=\sqrt{6}$, we get

$$\text{d.c's of } L_1=\left(\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{-2}{\sqrt{6}}\right)$$

Case (ii): Put $m=-2n$ in (1), then

$l=-(-2n)-n=2n-n=n \Rightarrow l=n$

$\therefore l:m:n=n:-2n:n=1:-2:1$

So, d.r's of $L_2=(a_2,b_2,c_2)=(1,-2,1)$

On dividing by

$\sqrt{1^2+(-2)^2+1^2}=\sqrt{1+4+1}=\sqrt{6}$, we get

$$\text{d.c's of of } L_2=\left(\frac{1}{\sqrt{6}},\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}}\right)$$

Q22: DIFFERENTIATION:

- **If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$ then prove that $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$**

A: Given $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

We take x=sinα, y=sinβ, then

$$\sqrt{1-\sin^2\alpha}+\sqrt{1-\sin^2\beta}=a(\sin\alpha-\sin\beta)$$

$$\Rightarrow \cos\alpha+\cos\beta=a(\sin\alpha-\sin\beta)$$

$$\Rightarrow \frac{\cos\alpha+\cos\beta}{\sin\alpha-\sin\beta}=a$$

$$\Rightarrow \frac{2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)}{2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)}=a$$

$$\left[\begin{array}{l}\because \cos C+\cos D=2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)\\ \sin C-\sin D=2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)\end{array}\right]$$

$$\Rightarrow \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{\sin\left(\frac{\alpha-\beta}{2}\right)}=a\Rightarrow \cot\left(\frac{\alpha-\beta}{2}\right)=a$$

$$\Rightarrow \frac{\alpha-\beta}{2}=\mathrm{Cot}^{-1}(a)$$

$$\Rightarrow \alpha-\beta=2\mathrm{Cot}^{-1}(a)$$

But sinα=x ⇒ α=$\mathrm{Sin}^{-1}x$ and y=sinβ

⇒ β=$\mathrm{Sin}^{-1}y$

$$\therefore \mathrm{Sin}^{-1}x-\mathrm{Sin}^{-1}y=2\mathrm{Cot}^{-1}(a)$$

On diff. w.r.to x, we get $\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=0$

$$\Rightarrow \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}\Rightarrow \frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

Hence proved.

22. If $y=\mathrm{Tan}^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$ then find $\frac{dy}{dx}$

A: Given $y=\mathrm{Tan}^{-1}\left(\frac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1-x^2}}\right)$

We take $x^2=\cos2\theta$, then

$$y=\mathrm{Tan}^{-1}\left(\frac{\sqrt{1+\cos2\theta}+\sqrt{1-\cos2\theta}}{\sqrt{1+\cos2\theta}-\sqrt{1-\cos2\theta}}\right)$$

$$=\mathrm{Tan}^{-1}\left(\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right)$$

$$=\mathrm{Tan}^{-1}\left(\frac{\sqrt{2}[\cos\theta+\sin\theta]}{\sqrt{2}[\cos\theta-\sin\theta]}\right)$$

$$=\mathrm{Tan}^{-1}\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$$

$$=\mathrm{Tan}^{-1}\left(\frac{\frac{\cos\theta}{\cos\theta}+\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta}-\frac{\sin\theta}{\cos\theta}}\right)=\mathrm{Tan}^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$$

$$=\mathrm{Tan}^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]=\frac{\pi}{4}+\theta$$

$$\therefore y=\frac{\pi}{4}+\theta$$

But $\cos2\theta=x^2\Rightarrow 2\theta=\mathrm{Cos}^{-1}(x^2)$

$$\Rightarrow \theta=\frac{1}{2}\mathrm{Cos}^{-1}(x^2)$$

So, $y=\frac{\pi}{4}+\frac{1}{2}\mathrm{Cos}^{-1}(x^2)$

On diff. w.r.t x, we get

$$\frac{dy}{dx}=0+\frac{1}{2}\left(\frac{-1}{\sqrt{1-(x^2)^2}}(2x)\right),$$

$$\left[\because \frac{d}{dx}\mathrm{Cos}^{-1}f(x)=\frac{-1}{\sqrt{1-(f(x))^2}}\cdot\frac{d}{dx}f(x)\right]$$

$$=\frac{-x}{\sqrt{1-x^4}}$$

Q23: TANGENT & NORMAL:

- **If the tangent at a point on the curve $x^{2/3}+y^{2/3}=a^{2/3}$ intersects the coordinate axes in A,B then show that the length AB is a constant.**

A: We know that the parametric point on the given curve is $P(a\cos^3\theta, a\sin^3\theta)$, then $x=a\cos^3\theta$ and $y=a\sin^3\theta$

$$\therefore \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{\frac{d}{d\theta}(a\sin^3\theta)}{\frac{d}{d\theta}(a\cos^3\theta)}$$

$$=\frac{a.3\sin^2\theta(\cos\theta)}{a.3\cos^2\theta(-\sin\theta)}=-\frac{\sin\theta}{\cos\theta}$$

So, slope of the tangent at $P(a\cos^3\theta, a\sin^3\theta)$ is $m=-\frac{\sin\theta}{\cos\theta}$

$\therefore$ Equation of the tangent at $P(a\cos^3\theta, a\sin^3\theta)$ having slope $-\frac{\sin\theta}{\cos\theta}$ is $y-y_1=m(x-x_1)$

$$y-a\sin^3\theta=-\frac{\sin\theta}{\cos\theta}(x-a\cos^3\theta)$$

$$\Rightarrow \cos\theta(y-a\sin^3\theta)=-\sin\theta(x-a\cos^3\theta)$$

$$\Rightarrow y\cos\theta-a\sin^3\theta\cos\theta=-x\sin\theta+a\cos^3\theta\sin\theta$$

$$\Rightarrow x\sin\theta+y\cos\theta=a\sin^3\theta\cos\theta+a\cos^3\theta\sin\theta$$

$$\Rightarrow x\sin\theta+y\cos\theta=a\sin\theta\cos\theta(\sin^2\theta+\cos^2\theta)$$

[Taking sinθcosθ common]

$$\Rightarrow x\sin\theta+y\cos\theta=a\sin\theta\cos\theta\ (1)$$

$$[\because \sin^2\theta+\cos^2\theta=1]$$

$$\Rightarrow \frac{x\sin\theta}{a\sin\theta\cos\theta}+\frac{y\cos\theta}{a\sin\theta\cos\theta}=1$$

$$\Rightarrow \frac{x}{a\cos\theta}+\frac{y}{a\sin\theta}=1$$

$\therefore$ $A=(a\cos\theta, 0)$, $B=(0, a\sin\theta)$

$$\therefore AB=\sqrt{(a\cos\theta-0)^2+(0-a\sin\theta)^2}$$

$$=\sqrt{a^2\cos^2\theta+a^2\sin^2\theta}$$

$$=\sqrt{a^2(\cos^2\theta+\sin^2\theta)}=\sqrt{a^2(1)}=a$$

$\therefore$ Hence proved that AB is a constant.

- **Find the angle between the curves $y^2=4x$ and $x^2+y^2=5$.**

A: **1) Finding points of intersection:**

Given $y^2=4x$ ....(1), $x^2+y^2=5$ ....(2)

From (1) & (2),

$$x^2+4x=5\Rightarrow x^2+4x-5=0$$

$$\Rightarrow (x-1)(x+5)=0 \Rightarrow x=1 \text{ or } -5$$

If x=1 then $y^2=4(1)=4=2^2\Rightarrow y=\pm 2$

$\therefore$ Points of intersection P=(1,2), Q=(1,–2)

2) Finding derivatives:

$$y^2=4x\Rightarrow 2y\frac{dy}{dx}=4\Rightarrow \frac{dy}{dx}=\frac{4}{2y}=\frac{2}{y}$$

$$x^2+y^2=5\Rightarrow 2x+2y\frac{dy}{dx}=0$$

$$\Rightarrow 2y\frac{dy}{dx}=-2x\Rightarrow \frac{dy}{dx}=\frac{-x}{y}$$

3) Finding Slopes at P(1,2):

$$m_1=\left(\frac{dy}{dx}\right)_{(1,2)}=\frac{2}{y}=\frac{2}{2}=1;$$

$$m_2=\left(\frac{dy}{dx}\right)_{(1,2)}=\frac{-x}{y}=\frac{-1}{2}$$

4) Finding angle at P: If θ is the angle between the curves at P then

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{1-\left(-\frac{1}{2}\right)}{1+\left(1\times\left(-\frac{1}{2}\right)\right)}\right|=\left|\frac{1+\frac{1}{2}}{1-\frac{1}{2}}\right|=\frac{\frac{3}{2}}{\frac{1}{2}}=3$$

$$\therefore \theta=\text{Tan}^{-1}3$$

5) Finding slopes at Q(1,–2):

$$m_1=\left(\frac{dy}{dx}\right)_{(1,-2)}=\frac{2}{y}=\frac{2}{-2}=-1;$$

$$m_2=\left(\frac{dy}{dx}\right)_{(1,-2)}=\frac{-x}{y}=\frac{-1}{-2}=\frac{1}{2}$$

6) Finding angle at Q: If θ is the angle between the curves at Q then

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|=\left|\frac{-1-\frac{1}{2}}{1+\left(-1\times\frac{1}{2}\right)}\right|=\left|\frac{\frac{-3}{2}}{\frac{1}{2}}\right|=|-3|=3$$

$$\therefore \theta=\text{Tan}^{-1}3$$

Q24: MAXIMA & MINIMA:

- **Find two positive integers whose sum is 16 and the sum of whose squares is minimum.**

A: Let the two positive numbers be x,y

Given that $x + y = 16$

$\Rightarrow y = 16 - x$..........(1)

Let $f(x) = x^2 + y^2 = x^2 + (16-x)^2$

$\therefore f(x) = x^2 + (16-x)^2$ ......(2)

Diff. (2) w.r.t x, we get

$f'(x) = 2x + 2(16-x)(-1) = 2x - 32 + 2x$

$= 4x - 32 = 4(x-8)$ ....(3)

At max. or min. we have $f'(x) = 0$

$\Rightarrow 4(x-8) = 0 \Rightarrow x = 8$

Diff. (3) w.r.t x, we get $f''(x) = 4$ ......(4)

At x=8, from (4), $f''(8) = 4 > 0$

$\therefore$ f(x) is minimum when x= 8 and

$y = 16 - 8 = 8$

$\therefore$ Required numbers are x=8, y= 8

- **Find the maximum area of the rectangle that can be formed with fixed perimeter 20.**

A: For the rectangle, we take length=x,breadth=y

Given perimeter is 20 $\Rightarrow 2(x+y) = 20$

$\Rightarrow x + y = 10 \Rightarrow y = 10 - x$ .....(1)

Area of the rectangle is $A = xy$

From (1), $A(x) = xy = x(10-x)$

$\therefore A(x) = 10x - x^2$ .....(2)

On diff. (2) w.r.to x, we get

$A'(x) = 10 - 2x$.........(3)

At max. or min.,we have $A'(x) = 0$

$\Rightarrow 10 - 2x = 0 \Rightarrow x = 5$

Also, from (1), $y = 10 - x = 10 - 5 = 5$

Now, on diff. (3) w.r.to x, we get

$A''(x) = -2$ ......(4)

At x=5, from (4), we get $A''(5) < 0$

$\therefore$ Area is maximum at x=5 and y=5

Hence maximum area of the rectangle is

$A = xy = 5(5) = 25$ sq.units

- **From a rectangular sheet of dimensions 30cm x 80cm, four equal squares of sides x cm are removed at the corners, and the sides are then turned up so as to form an open rectangular box. What is the value of x, so that the volume of the box is the greatest?**

A: For the open box, we take

$$\left.\begin{array}{l}\text{height } h = x \\ \text{length } l = 80 - 2x \\ \text{breadth } b = 30 - 2x\end{array}\right\} \text{.....(1)}$$

Volume $V = lbh = (80-2x)(30-2x)(x)$

$= 2(40-x)\,2(15-x)(x)$

$= 4(40-x)(15-x)(x)$

$= 4(600 - 40x - 15x + x^2)x$

$= 4(600 - 55x + x^2)x$

$= 4(x^3 - 55x^2 + 600x)$

$V(x) = 4(x^3 - 55x^2 + 600x)$ .....(2)

On diff. (2) w.r.to x, we get,

$V'(x) = 4(3x^2 - 110x + 600)$ .....(3)

At max. or min., we have $V'(x) = 0$

$\Rightarrow \not{4}(3x^2 - 110x + 600) = 0$

$\Rightarrow 3x^2 - 90x - 20x + 600 = 0$

$\Rightarrow 3x(x-30) - 20(x-30) = 0 \Rightarrow (3x-20)(x-30) = 0$

$\Rightarrow x = 20/3$ (or) $x = 30$

Now, on diff. (3), w.r.to x, we get

$V''(x) = 4(6x - 110)$ ....(4)

At $x = \frac{20}{3}$, from (4), we get

$$V''\left(\frac{20}{3}\right) = 4\left(\not{6}\left(\frac{20}{\not{3}}\right) - 110\right)$$

$= 4(40 - 110) = 4(-70) = -280$

Thus, $V''\left(\frac{20}{3}\right) < 0$

$\therefore$ V(x) has maximum value at $x = \frac{20}{3}$ cm