10. PROPERTIES OF TRIANGLES

(1 x 4) + (1 x 7) = 11 Marks

IMP FORMULAS, KEY CONCEPTS

1) The semiperimeter of ΔABC is $s=\dfrac{a+b+c}{2} \Rightarrow 2s=a+b+c$

2.1) $\Delta=\sqrt{s(s-a)(s-b)(s-c)}$ **2.2)** $\Delta=\dfrac{abc}{4R}$ hence $R=\dfrac{abc}{4\Delta}$

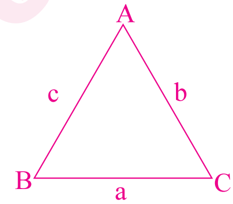
2.3) $\Delta=\dfrac{1}{2}bc\sin A=\dfrac{1}{2}ca\sin B=\dfrac{1}{2}ab\sin C$ 2.4. $\Delta=2R^2\sin A\sin B\sin C$

3) Sine rule: $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}=2R \Rightarrow a=2R\sin A,\ b=2R\sin B,\ c=2R\sin C$

4) Cosine rule: $\cos A=\dfrac{b^2+c^2-a^2}{2bc} \Rightarrow a^2=b^2+c^2-2bc\cos A$

$$\cos B=\frac{c^2+a^2-b^2}{2ac} \Rightarrow b^2=c^2+a^2-2ac\cos B$$

$$\cos C=\frac{a^2+b^2-c^2}{2ab} \Rightarrow c^2=a^2+b^2-2ab\cos C$$



5) Projection rule: $a = b\cos C + c\cos B$; $b = c\cos A + a\cos C$; $c = a\cos B + b\cos A$

6) Tangent rule: $\tan\left(\dfrac{A-B}{2}\right)=\left(\dfrac{a-b}{a+b}\right)\cot\dfrac{C}{2}$; $\tan\left(\dfrac{B-C}{2}\right)=\left(\dfrac{b-c}{b+c}\right)\cot\dfrac{A}{2}$; $\tan\left(\dfrac{C-A}{2}\right)=\left(\dfrac{c-a}{c+a}\right)\cot\dfrac{B}{2}$

7) $\sin\dfrac{A}{2}=\sqrt{\dfrac{(s-b)(s-c)}{bc}}$; $\sin\dfrac{B}{2}=\sqrt{\dfrac{(s-c)(s-a)}{ca}}$; $\sin\dfrac{C}{2}=\sqrt{\dfrac{(s-a)(s-b)}{ab}}$

$\cos\dfrac{A}{2}=\sqrt{\dfrac{s(s-a)}{bc}}$; $\cos\dfrac{B}{2}=\sqrt{\dfrac{s(s-b)}{ca}}$; $\cos\dfrac{C}{2}=\sqrt{\dfrac{s(s-c)}{ab}}$

$\tan\dfrac{A}{2}=\sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$; $\tan\dfrac{B}{2}=\sqrt{\dfrac{(s-c)(s-a)}{s(s-b)}}$; $\tan\dfrac{C}{2}=\sqrt{\dfrac{(s-a)(s-b)}{s(s-c)}}$

☞ $\tan\dfrac{A}{2}=\dfrac{\Delta}{s(s-a)}=\dfrac{(s-b)(s-c)}{\Delta}$; $\cot\dfrac{A}{2}=\dfrac{s(s-a)}{\Delta}=\dfrac{\Delta}{(s-b)(s-c)}$

8) $r=\dfrac{\Delta}{s}=4R\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}=(s-a)\tan\dfrac{A}{2}=(s-b)\tan\dfrac{B}{2}=(s-c)\tan\dfrac{C}{2}$

$r_1=\dfrac{\Delta}{s-a}=4R\sin\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}=s\tan\dfrac{A}{2}=(s-b)\cot\dfrac{C}{2}=(s-c)\cot\dfrac{B}{2}$

$r_2=\dfrac{\Delta}{s-b}=4R\cos\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}=s\tan\dfrac{B}{2}=(s-c)\cot\dfrac{A}{2}=(s-a)\cot\dfrac{C}{2}$

$r_3=\dfrac{\Delta}{s-c}=4R\cos\dfrac{A}{2}\cos\dfrac{B}{2}\sin\dfrac{C}{2}=s\tan\dfrac{C}{2}=(s-a)\cot\dfrac{B}{2}=(s-b)\cot\dfrac{A}{2}$

9) $\Delta=rs=\sqrt{rr_1r_2r_3}$ **10)** $\dfrac{1}{r}=\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}$

BULLET MASTER'S

MATH BEATS!

You know Pretty well about solving a Quadratic Equation $ax^2+bx+c=0$, using Quadratic formula.

You know well about solving two linear equations, using rule of Cross Multiplication.

You know about solving three linear equations using Cramer's Rule, Matrix Inversion Method,...

Do you ever heard of solving a Triangle?

A triangle consists of 3 angles A, B, C and 3 opposite sides a,b,c.

'**SOLVING A TRIANGLE**' is the way of finding the other 3 elements from the given 3.

'SOLUTION' OF A 'TRIANGLE' IN FEW CASES

S.No.	Given elements	Remaining elements
I.	a, b, c $\Rightarrow$ [A, B, C]	$\sin A=\frac{2\Delta}{bc};\sin B=\frac{2\Delta}{ca};\sin C=\frac{2\Delta}{ab};\ \Delta=\sqrt{s(s-a)(s-b)(s-c)}$
II.	a, b, A $\Rightarrow$ [B, C, c]	$\sin B=\frac{b}{a}\sin A;\ C=180^\circ-(A+B);\ c=\frac{a\sin C}{\sin A}$
III.	a, A, B $\Rightarrow$ [C, b, c]	$C=180^\circ-(A+B);\ b=\frac{a\sin B}{\sin A};\ c=\frac{a\sin C}{\sin A}$
IV.	a, b, C = 90° $\Rightarrow$ [A, B, c]	$\tan A=\frac{a}{b};\ B=90^\circ-A;\ c=\frac{a}{\sin A}$

BULLET MASTER'S

MATH BEATS!

ABC : Always Be Cheerful!

ABCD : AnyBody Can Dance!!

ABCDE : A Bachelor Can Dominate Everyone!!!

ABCDEF : All Boys Can Do Everything Fine!!!!

Wow! What a beautiful Pattern of 'Letter Sentences'.

Can we have such a **Beautiful Patterns** with the Elements of a Triangle?

Can we have a Triangle with 1,2,3,4,5,6 as its elements?

Ofcourse, Yes! See Q.18/241 Page.

For that Triangle, we have **$r = 1$, $r_1 = 2$, $a = 3$, $b = 4$, $c = 5$, $r_3 = 6$ (What a Beauty!)**

Certain Right Triangles have **Passion on Progressions.**

Their elements maintain A.P, G.P, H.P relations.

Sides a = 6, b = 8, c = 10 of a Triangle are in A.P & their in-radii $r_1=\frac{1}{6}, r_2=\frac{1}{8}, r_3=\frac{1}{10}$ are in H.P