10. APPLICATIONS OF DERIVATIVES

(2 x 2) + (2 x 4) + (2 x 7) = 26 Marks

IMP FORMULAS, KEY CONCEPTS

1) If $y = f(x)$ is a differentiable function of x and Δx is a small change in x then (i) the actual **change** in y is $\Delta y = f(x + \Delta x) - f(x)$ (ii) the **differential** of y is $dy = f'(x)\Delta x$.

2) **Approximations:** The approximate value of f(x) in Δx neighbourhood of known x, is $f(x + \Delta x) \cong f(x) + f'(x)\,\Delta x$. Here, Δx = given value of x – known value of x.

3) If an error Δx occurs in x of $y = f(x)$ then (i) Δy is called **error** in y
(ii) $\frac{\Delta y}{y}$ is called **relative error** in y (iii) $\frac{\Delta y}{y}\text{x}100$ is called **percentage error** in y.

4) If $P(x_1, y_1)$ is a point on $y = f(x)$ then the slope of the tangent at P is $m = \left(\frac{dy}{dx}\right)_{P(x_1,y_1)}$

5) If $P(x_1, y_1)$ is a point of intersection of the curves f(x), g(x) and θ is the angle between the 2 curves then $\tan\theta = \frac{m_1 - m_2}{1 + m_1 m_2}$ where $m_1 = (f'(x))$ at (x_1, y_1), $m_2 = (g'(x))$ at (x_1, y_1)

Note: $m_1 = m_2 \Rightarrow$ the two curves touch each other at (x_1, y_1) and
$m_1 m_2 = -1 \Rightarrow$ the 2 curves cut orthogonally.

6) If $P(x_1,y_1)$ is a point on the curve $y=f(x)$ and $m = \left(\frac{dy}{dx}\right)_{P(x_1,y_1)}$, then

(i) the **length of the tangent** to the curve at P is $\left|\frac{y_1\sqrt{1+m^2}}{m}\right|$

(ii) the **length of the normal** to the curve at P is $|\, y_1\sqrt{1+m^2}\,|$

(iii) the **length of subtangent** to the curve at P is $\left|\frac{y_1}{m}\right|$

(iv) the **length of subnormal** to the curve at P is $|y_1 m|$

7.1) The rate of change of $y=f(x)$ w.r.to t is given by $\frac{dy}{dt} = f'(x)\frac{dx}{dt}$

7.2) If $s=f(t)$ denotes the distance travelled by a body in time t then

(i) the velocity of the body at time t is $v = \frac{ds}{dt}$

(ii) the acceleration of the body at time t is $a = \frac{d^2s}{dt^2} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{dv}{dt}$

8.1) A differentiable function f is increasing at a $\Leftrightarrow f'(a)>0$

8.2) A differentiable function f is decreasing at a $\Leftrightarrow f'(a)<0$

8.3) A differentiable function f is said to be stationary at x = a if $f'(a) = 0$

Here f(a) is the stationary value & (a, f(a)) is the stationary point of f(x) at x = a

9) Let f(x) be a differentiable function in a given interval I, $a \in I$, $f'(x)$, $f''(x)$ exist at a and if
(i) $f'(a)=0$, $f''(a)<0$ then f(a) is a local maxima (ii) $f'(a) = 0$, $f''(a) > 0$ then f(a) is a local minima

BULLET MASTER'S

MATH BEATS!

APPLICATIONS OF DERIVATIVES

పార్టులు పార్టులుగా Practice చేస్తే Applications of Derivatives అన్నీ Easy గానే ఉంటాయి!

IPE Weightage for Tangents & Normals: 2M + 4 M + 7 M = 13 Marks

Tangents & Normals లో Very Easy VSAQ (Page 152)

Tangents & Normals లో Super Easy SAQ (Page 154, Page155)

Tangents & Normals లో Lucky LAQ 13/157 and Page 166 to Page 172