8. DIFFERENTIAL EQUATIONS

(1 x 2) + (1 x 4) + (1 x 7) = 13 Marks

IMP FORMULAS, KEY CONCEPTS

1) The **order** of differential equation is the **order of the highest order derivative** in it.

2) The **degree** of a differential equation is the degree of the highest order derivative, when it is expressed as a polynomial equation in derivatives after eliminating the fractional powers, if any.

3) **Forming a differential equation from a given relation:** Differentiate the given relation as many times as the number of arbitrary constants in the given relation and eliminate the arbitrary constant (s) from the given relation by replacing the arbitrary constants in the original equation with the derivatives.

4) **Solving a differential equation in the "Variables separable" form:** Separate both the variables x and y along with dx and dy and integrate accordingly. Here, the constant of integration can be modified accordingly.

5) **Procedure of solving homogeneous differential equation:**

1) Write the given differential equation in form $\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}$ ....(1)

2) Take y=vx hence we get $\frac{dy}{dx}=v+x\frac{dv}{dx}$ .....(2)

3) Find $x\frac{dv}{dx}$ by substituting the above in (1)

4) Separate the variables in x and v, and resolve into partial fractions, if required.

5) Integrate and simplify

6) Replace v by y/x in the final step to get the solution interms of x and y

6) **Non-homogeneous** differential equations are of the form $\frac{dy}{dx}=\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}$

Here, various possible cases are according as (1) $b_1+a_2=0$ (2) $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$ (3) $\frac{a_1}{a_2}=\frac{b_1}{b_2}$

7.1) The solution of the **linear differential equation** $\frac{dy}{dx}+yP(x)=Q(x)$ in y is $y(I.F)=\int(I.F)Qdx+c$

where I.F(Integrating factor)=$e^{\int Pdx}$. Thus the solution of $\frac{dy}{dx}+yP(x)=Q(x)$ is $y.e^{\int Pdx}=\int e^{\int Pdx}Qdx+c$

7.2) The solution of the **linear differential equation** $\frac{dx}{dy}+xP(y)=Q(y)$ in x is $x.e^{\int Pdy}=\int e^{\int Pdy}Qdy+c$