7. DEFINITE INTEGRALS

(2 x 2) + (1 x 4) + (1 x 7) = 15 Marks

IMP FORMULAS, KEY CONCEPTS

1) DEFINITE INTEGRALS

1) If F(x) is an integral of f(x) defined on [a,b] then $\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$

2) PROPERTIES OF DEFINITE INTEGRALS

2) $\int_a^b f(x)dx = \int_a^b f(t)dt = \int_a^b f(\theta)d\theta$

3) $\int_a^b f(x)dx = -\int_b^a f(x)dx$

4) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \quad \text{for } a < c < b$

5) $$\int_{-a}^{a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \textbf{if f(x) is an even function i.e., f(-x)=f(x)} \\ \textbf{0}, & \textbf{if f(x) is an odd function i.e., f(-x) =-f(x)} \end{cases}$$

6.1) $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

6.2) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

7.1) $$\int_0^a f(x)dx = \begin{cases} 2\int_0^{a/2} f(x)dx, & \text{if } f(a-x) = f(x) \\ 0, & \text{if } f(a-x) = -f(x) \end{cases}$$

7.2. $$\int_0^{2a} f(x)dx = \begin{cases} 2\int_0^a f(x)dx & \text{if } f(2a-x) = f(x) \\ 0 & \text{if } f(2a-x) = -f(x) \end{cases}$$

8) If f(x) is a periodic function with period a then $\int_0^{na} f(x)dx = = n\int_0^a f(x)dx$

3) REDUCTION FORMULAE

9.1) For even 'n' $\int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right).....\frac{1}{2}.\frac{\pi}{2}$

9.2) For odd 'n' $\int_0^{\pi/2} \sin^n x dx = \int_0^{\pi/2} \cos^n x dx = \left(\frac{n-1}{n}\right)\left(\frac{n-3}{n-2}\right)\left(\frac{n-5}{n-4}\right).....\frac{2}{3}.1$

10) $$\int_0^{\pi/2} \sin^m x \cos^n x dx = \begin{cases} \dfrac{[(m-1)(m-3)....(3)(1)][(n-1)(n-3)....(3)(1)]}{(m+n)(m+n-2)....(4)(2)}.\dfrac{\pi}{2}, & \text{if m is even \& n is even (i.e., both m, n are even)} \\ \dfrac{[(m-1)(m-3)....(3)(1)][(n-1)(n-3)....(4)(2)]}{(m+n)(m+n-2)....(3)(1)}, & \text{if m is even \& n is odd} \\ \dfrac{[(m-1)(m-3)....(4)(2)][(n-1)(n-3)....(\textbf{2 or 1})]}{(m+n)(m+n-2)....(\textbf{2 or 1})}, & \text{if m is odd \& n is even or odd} \end{cases}$$