# 6. INTEGRATION

**(2 x 2) + (2 x 7) = 18 Marks**

1) $\int x^n dx = \frac{x^{n+1}}{n+1} + c,\ n \neq -1$

2) $\int k dx = kx + c,$ k is a constant

3) $\int x dx = \frac{x^2}{2} + c$

4) $\int \sqrt{x} dx = \frac{2}{3} x\sqrt{x} + c$

5) $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$

6) $\int \frac{1}{x^2} dx = -\frac{1}{x} + c$

7) $\int \frac{1}{x} dx = \log_e |x| + c$

8) $\int \log_e x dx = x \log x - x + c$

9) $\int a^x dx = a^x \log_a e + c$

10) $\int e^x dx = e^x + c$

11) $\int \sin x dx = -\cos x + c$

12) $\int \cos x dx = \sin x + c$

13) $\int \tan x dx = \log|\sec x| + c$

14) $\int \cot x dx = \log|\sin x| + c$

15) $\int \sec x dx = \log|\sec x + \tan x| + c$

16) $\int \csc x dx = \log|\csc x - \cot x| + c$

17) $\int \csc^2 x\, dx = -\cot x + c$

18) $\int \sec^2 x\, dx = \tan x + c$

19) $\int \sec x \tan x\, dx = \sec x + c$

20) $\int \csc x \cot x\, dx = -\csc x + c$

21.1) $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\left(\text{Tan}^{-1}\frac{x}{a}\right) + c$

21.2) $\int \frac{1}{1 + x^2} dx = \text{Tan}^{-1} x + c$

22.1) $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \text{Sin}^{-1}\frac{x}{a} + c$

22.2) $\int \frac{1}{\sqrt{1 - x^2}} dx = \text{Sin}^{-1} x + c$

23) $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a}\left(\text{Sec}^{-1}\frac{x}{a}\right) + c$

24.1) $\int \frac{1}{\sqrt{a^2 + x^2}} dx = \text{Sinh}^{-1}\frac{x}{a} + c$ or $\log\left(x + \sqrt{a^2 + x^2}\right) + c$

24.2) $\int \frac{1}{\sqrt{1 + x^2}} dx = \text{Sinh}^{-1} x + c = \log\left(x + \sqrt{1 + x^2}\right) + c$

25.1) $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \text{Cosh}^{-1}\frac{x}{a} + c$ or $\log\left(x + \sqrt{x^2 - a^2}\right) + c$

25.2) $\int \frac{1}{\sqrt{x^2 - 1}} dx = \text{Cosh}^{-1} x + c = \log\left(x + \sqrt{x^2 - 1}\right) + c$

26) $\int \sqrt{a^2 - x^2} dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\text{Sin}^{-1}\frac{x}{a} + c$

27) $\int \sqrt{a^2 + x^2} dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\text{Sinh}^{-1}\frac{x}{a} + c$

28) $\int \sqrt{x^2 - a^2} dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\text{Cosh}^{-1}\frac{x}{a} + c$

29) $\int \frac{1}{a^2 - x^2} dx = \frac{1}{2a}\log\left|\frac{a + x}{a - x}\right| + c$

30) $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\log\left|\frac{x - a}{x + a}\right| + c$

31) $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + c$ 32) $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$

33) **By parts Rule:** $\int uv dx = u\int v dx - \int\left(\frac{du}{dx}\int v dx\right) dx$

34) $\int e^x (f(x) + f'(x))\, dx = e^x f(x) + c$

35) To find (a) $\int \frac{px + q}{ax^2 + bx + c} dx$ (b) $\int \frac{px + q}{\sqrt{ax^2 + bx + c}} dx$

(c) $\int (px + q)\sqrt{ax^2 + bx + c}\, dx$ when factorisation for $ax^2+bx+c$ is not possible then write $px+q$ as $px + q = k\frac{d}{dx}(ax^2 + bx + c) + l$ hence find k and $l$

36) To find $\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x} dx$ write Nr=A(Dr)+B $\frac{d}{dx}$ Dr and find Aand B by comparision.

37) To find $\int \frac{dx}{a + b\sin x}, \int \frac{dx}{a + b\cos x}$ put $\tan\frac{x}{2} = t,$

then $\sin x = \frac{2t}{1 + t^2}, \cos x = \frac{1 - t^2}{1 + t^2}, dx = \frac{2dt}{1 + t^2}$