6. BINOMIAL THEOREM

(1 x 2) + (2 x 7) = 16Marks

IMP FORMULAS, KEY CONCEPTS

1.1) Binomial theorem for integral index:
$(x+y)^n={}^nC_0x^n+{}^nC_1x^{n-1}y+{}^nC_2x^{n-2}y^2+....+{}^nC_rx^{n-r}y^r+....+{}^nC_ny^n,\ n\in N$

1.2) The **general term** of the binomial expansion is $T_{r+1}={}^nC_rx^{n-r}y^r$.

2) Standard Binomial expansion:

$(1+x)^n=1+nx+{}^nC_2x^2+....+{}^nC_rx^r+....+x^n=C_0+C_1x+C_2x^2+....+C_rx^r+......+C_nx^n$

3) In the expansion of $(1+x)^n$ (i) the coefficient of x^r is ${}^nC_r$=the coefficient of $(r+1)^{th}$ term
(ii) the coefficient of r^{th} term is ${}^nC_{r-1}$

4) Middle term and greatest coefficient:

(a) If n is even then (i) the middle term of $(1+x)^n$ is $T_{\frac{n}{2}+1}$ (ii)greatest coefficient is ${}^nC_{\frac{n}{2}}$.

(b) If n is odd then (i) two middle terms are $T_{\frac{n+1}{2}}, T_{\frac{n+3}{2}}$

(ii) greatest coefficients are ${}^nC_{\frac{n-1}{2}}, {}^nC_{\frac{n+1}{2}}$

5) Numerically greatest term(s) in the expansion of $(1+\alpha)^n$: Find $\frac{(n+1)|\alpha|}{|\alpha|+1}$

If this value is a mixed fraction with integral part 'r' then the numerically greatest term is T_{r+1}
If the value is an integer, say r then T_r and T_{r+1} are the two numerically greatest terms.

6) Binomial Theorem for Rational Index:

6.1) $(1+X)^{-n}=1-\frac{n}{1!}X+\frac{n(n+1)}{2!}X^2-.........+(-1)^r\frac{n(n+1).....(n+r-1)}{r!}.X^r+......$

6.2) $(1-X)^{-n}=1+\frac{n}{1!}X+\frac{n(n+1)}{2!}X^2+.........+\frac{n(n+1).....(n+r-1)}{r!}.X^r+......$

6.3) $(1-x)^{\frac{-p}{q}}=1+\frac{p}{1}\left(\frac{x}{q}\right)+\frac{p(p+q)}{1.2}\left(\frac{x}{q}\right)^2+.....+\frac{p(p+q).....(p+(r-1)q)}{r!}.\left(\frac{x}{q}\right)^r+......$

6.4) $(1+x)^{\frac{-p}{q}}=1-\frac{p}{1}\left(\frac{x}{q}\right)+\frac{p(p+q)}{1.2}\left(\frac{x}{q}\right)^2-.....+(-1)^r.\frac{p(p+q)....(p+(r-1)q)}{r!}.\left(\frac{x}{q}\right)^r+.......$

7) If x is a real number and |x|<1, then (i) $(1+x)^{-1}=1-x+x^2-x^3+....+(-1)^rx^r+......$

(ii) $(1+x)^{-2}=1-2x+3x^2-4x^3+....+(-1)^r.(r+1)x^r+......$

8) Useful Formulae: (i) ${}^nC_r+{}^nC_{r-1}={}^{(n+1)}C_r$; ${}^nC_r+{}^nC_{r+1}={}^{(n+1)}C_{r+1}$

(ii) $\frac{{}^nC_r}{{}^nC_{r-1}}=\frac{n-r+1}{r}$; $\frac{{}^nC_{r+1}}{{}^nC_r}=\frac{n-r}{r+1}$

(iii) The number of terms in the expansion of $(a+b+c)^n=\frac{(n+1)(n+2)}{2}$