4.THEORY OF EQUATIONS

(1 x 2) + (1 x 7) = 9 Marks

IMP FORMULAS, KEY CONCEPTS

1) A polynomial equation with n roots $\alpha_1,\alpha_2,....\alpha_n$ is $(x-\alpha_1)(x-\alpha_2)....(x-\alpha_n)=0$.

2) If α,β,γ are the roots of $a_0x^3+a_1x^2+a_2x+a_3=0$, then
(i) $S_1=\alpha+\beta+\gamma=-a_1/a_0$ (ii) $S_2=\alpha\beta+\beta\gamma+\gamma\alpha=a_2/a_0$ (iii) $S_3=\alpha\beta\gamma=-a_3/a_0$

3) If $\alpha,\beta,\gamma,\delta$ are the roots of $a_0x^4+a_1x^3+a_2x^2+a_3x+a_4=0$ then
(i) $S_1=\alpha+\beta+\gamma+\delta=-a_1/a_0$ (ii) $S_2=\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta=a_2/a_0$
(iii) $S_3=\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta=-a_3/a_0$ (iv) $S_4=\alpha\beta\gamma\delta=a_4/a_0$

4.1) $\Sigma\alpha^2=\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)$

4.2) $\Sigma\alpha^2\beta^2=\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)$

4.3) $\Sigma\alpha^2\beta+\Sigma\alpha\beta^2=(\alpha\beta+\beta\gamma+\gamma\alpha)(\alpha+\beta+\gamma)-3\alpha\beta\gamma$

4.4) $\Sigma\alpha^3=\alpha^3+\beta^3+\gamma^3=(\alpha+\beta+\gamma)(\alpha^2+\beta^2+\gamma^2-\alpha\beta-\beta\gamma-\gamma\alpha)+3\alpha\beta\gamma$

4.5) $\Sigma\alpha^3\beta^3=(\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2)(\alpha\beta+\beta\gamma+\gamma\alpha)-\alpha\beta\gamma(\Sigma\alpha^2\beta+\Sigma\alpha\beta^2)$

5.1) The roots of a cubic equation in A.P. can be taken as a–d, a, a+d

5.2) The roots of a cubic equation in G.P. can be taken as a/r, a, ar

6.1) The equation whose roots are those of the equation f(x)=0 with signs changed is f(–x)=0

6.2) The equation whose roots are $k(\neq 0)$ times the roots of f(x)=0 is f(x/k)=0

6.3) The equation whose roots are reciprocals of the roots of f(x)=0 is f(1/x)=0

6.4) The equation whose roots are exceed by h than those of f(x)=0 is f(x–h)=0

6.5) The equation whose roots are diminished by h than those of f(x)=0 is f(x+h)=0

6.6) The equation whose roots are the squares of the roots of f(x)=0 is $f(\sqrt{x})=0$ when this is expressed as a polynomial equation (i.e., radical signs should be removed)

7) **Removal of terms:** If $f(x)=a_0x^n+a_1x^{n-1}+....+a_n=0$ then

(i) to remove the second term in f(x)=0, diminish the roots of f(x)=0 by $h=-\frac{a_1}{na_0}$

(ii) to remove the 3rd term, diminish the roots of f(x)=0 by h such that $a_0\frac{n(n-1)}{2}h^2+a_1(n-1)h+a_2=0$

8.1) A polynomial equation $f(x)=a_0x^n+a_1x^{n-1}+....+a_n=0$ is said to a **reciprocal equation** of **Class-I** if $\mathbf{a_k=a_{n-k}}$ ($\forall$ k=0,1,2,....,n) and **Class-II** if $\mathbf{a_k=-a_{n-k}}$ ($\forall$ k=0,1,2,....,n)

8.2) A reciprocal equation of class-I with even degree is called a **S**tandard **R**eciprocal **E**quation

8.3) **Procedure of solving a S.R.E:** To solve a S.R.E of order 2m, divide the equation by x^m and put $x+\frac{1}{x}=y$ and proceed accordingly.

8.4) For a R.E of class-I of odd degree "–1" is a root; For a R.E of class-II of odd degree "1" is a root; For a R.E of class-II of even degree "1, –1" are two roots