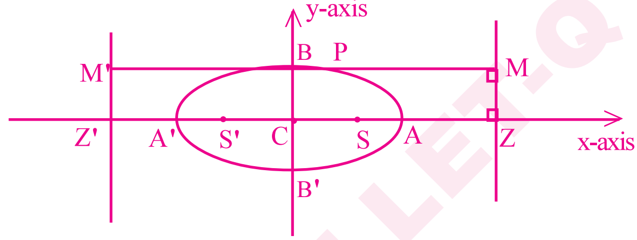
4. ELLIPSE

2 x 4 = 8 Marks

IMP FORMULAS, KEY CONCEPTS

Def: The locus of a point in a plane, which moves such that its distance from a fixed point (focus) bears a constant ratio e, 0<e<1, to its distance from a fixed line (directrix) is called an ellipse.

I. The following terminology holds true w.r.to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a>b:



1) The **eccentricity** of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $e=\frac{\sqrt{a^2-b^2}}{a}$ $\therefore b^2=a^2(1-e^2)$. Also $a^2e^2=a^2-b^2$

2) The **foci** of the ellipse are **S=(ae, 0), S'(−ae,0)** i.e., $(\pm\sqrt{a^2-b^2},0)$

3) The **feet of the directrices** are $Z=\left(\frac{a}{e},0\right)$, $Z'=\left(-\frac{a}{e},0\right)$ & the **equation** of the directrices is $x=\pm\frac{a}{e}$

4.1) **AA'** is called the **major axis**, **BB'** is called the **minor axis** of the ellipse

4.2) The **length** of the **major axis** is **2a** and the **equation** of the **major axis** is **y=0**

The **length** of **minor axis** is **2b** and the **equation** of the **minor axis** is **x=0**

5.1) **A(a,0), A'(-a,0)** are called the **vertices of the ellipse.**

B(0,b), B'(0, -b) are called the **vertices on the minor axis.**

5.2) The **equation** of the **tangents** at the **vertices** is **x=±a**

6.1) The **equation** of the **latus recta** is **x=±ae**

6.2) The **ends** of the **latusrecta** are $\left(ae,\pm\frac{b^2}{a}\right)$ and $\left(-ae,\pm\frac{b^2}{a}\right)$

6.3) The **length** of the **latus rectum** is $\frac{2b^2}{a}$

7) The **focal distance** of the point $P(x_1,y_1)$ on the ellipse w.r.t the focus **S** is $SP=a-ex_1$ and the focal distance of the point **P** on the ellipse w.r.t the focus **S'** is $S'P=a+ex_1$

8) The equation of the **auxiliary circle** of the ellipse is $x^2+y^2=a^2$

9) The equation of the **director circle** of the ellipse is $x^2+y^2=a^2+b^2$

II. The following results hold true w.r.to the horizontal ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$:

1) **Notation:** $S=\frac{x^2}{a^2}+\frac{y^2}{b^2}-1$; $S_1=\frac{x_1x}{a^2}+\frac{y_1y}{b^2}-1$; $S_{11}=\frac{x_1^2}{a^2}+\frac{y_1^2}{b^2}-1$; $S_{12}=\frac{x_1x_2}{a^2}+\frac{y_1y_2}{b^2}-1$

2) **Relative positions** of a point $P(x_1,y_1)$ and the ellipse S=0

(i) The point $P(x_1,y_1)$ lies on the ellipse $S=0 \Leftrightarrow S_{11}=0$

(ii) The point $P(x_1,y_1)$ lies inside the ellipse $S=0 \Leftrightarrow S_{11}<0$

(iii) The point $P(x_1,y_1)$ lies outside the ellipse $S=0 \Leftrightarrow S_{11}>0$

3) The equation of the chord joining the points $A(x_1,y_1)$ & $B(x_2,y_2)$ on the ellipse S=0 is $S_1+S_2=S_{12}$

4) The equation of the tangent at $P(x_1,y_1)$ on the ellipse S=0 is $S_1=0$

5) The equation of the normal at $P(x_1,y_1)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{a^2x}{x_1}-\frac{b^2y}{y_1}=a^2-b^2$

6) The condition for the line y=mx+c to be a tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $c^2=a^2m^2+b^2$.

7) The equation of the tangent to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ having slope m is $y=mx\pm\sqrt{a^2m^2+b^2}$

8) Two tangents can be drawn from an external point $P(x_1,y_1)$ to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ &

if m_1,m_2 are the slopes of the two tangents then $m_1+m_2=\frac{2x_1y_1}{x_1^2-a^2}$, $m_1m_2=\frac{y_1^2-b^2}{x_1^2-a^2}$

9) The combined equation of the pair of tangents drawn from an external point $P(x_1,y_1)$ to the ellipse S=0 is $S_1^2=S_{11}S$.

10) The equation of the chord of contact of $P(x_1,y_1)$ w.r.to the ellipse S=0 is $S_1=0$

11) The equation of the chord of the ellipse S=0 having $P(x_1,y_1)$ as its midpoint is $S_1=S_{11}$.

III. Parametric treatment:

12) The parametric point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $P(a\cos\theta, b\sin\theta)$ and is simply denoted by θ.

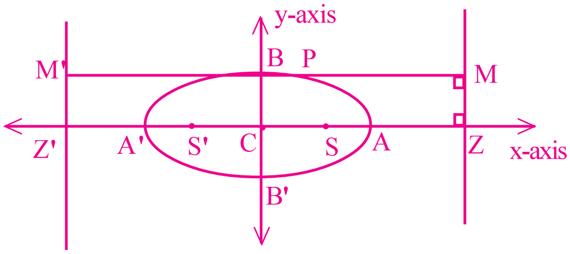
13) The equation of the tangent at $P(\theta)$ on the ellipse S=0 is $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$

14) The equation of the normal at $P(\theta)$ on the ellipse S=0 is $\frac{ax}{\cos\theta}-\frac{by}{\sin\theta}=a^2-b^2$

IV) ELLIPSE IN TWO STANDARD FORMS

HORIZONTAL ELLIPSE

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or $b^2x^2+a^2y^2=a^2b^2$, $a>b$



1) Eccentricity $e=\frac{\sqrt{a^2-b^2}}{a}$

$b^2=a^2(1-e^2)$ or $a^2e^2=a^2-b^2$.

2) Centre C=(0,0)

3) Foci S=(ae, 0) $\Rightarrow S(\sqrt{a^2-b^2},0)$

S'=(–ae,0) $\Rightarrow S'(-\sqrt{a^2-b^2},0)$

4) Vertices, A=(a,0) and A'=(–a,0)

5) Ends of L.R= $\left(\pm ae,\pm\frac{b^2}{a}\right)$

6) Feet of the directrices = (±a/e, 0)

7) Equation of the directrices is x=±a/e (or)

$(\sqrt{a^2-b^2})x=\pm a^2$

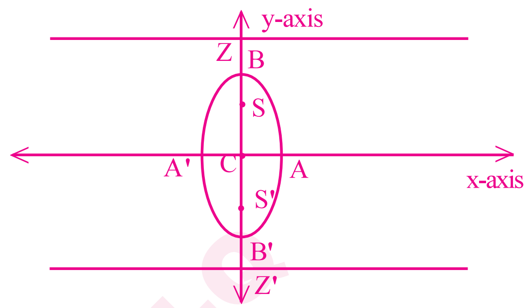
8) Equation of the L.R is x=±ae

9) Length of major axis is 2a and its equation is y=0
Length of minor axis is 2b and its equation is x=0

10) Equation of the tangents at the vertices x=±a

VERTICAL ELLIPSE

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ or $b^2x^2+a^2y^2=a^2b^2$, $a<b$



1) Eccentricity $e=\frac{\sqrt{b^2-a^2}}{b}$

$a^2=b^2(1-e^2)$ or $b^2e^2=b^2-a^2$.

2) Centre C=(0,0)

3) Foci S=(0, be) $\Rightarrow S(0,\sqrt{b^2-a^2})$

S'=(0,–be) $\Rightarrow S'(0,-\sqrt{b^2-a^2})$

4) Vertices, B=(0,b) and B'=(0,–b)

5) Ends of L.R= $\left(\pm\frac{a^2}{b},\pm be\right)$

6) Feet of the directrices = (0, ±b/e)

7) Equation of the directrices is y=±b/e (or)

$(\sqrt{b^2-a^2})y=\pm b^2$

8) Equation of the L.R is y=±be

9) Length of major axis is 2b and its equation is x=0
Length of minor axis is 2a and its equation is y=0

10) Equation of the tangents at the vertices y=±b