3.QUADRATIC EXPRESSIONS

(1 x 2) + (1 x 4) = 6 Marks

IMP FORMULAS, KEY CONCEPTS

1) The **roots** of $ax^2+bx+c=0$ are given by $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

2.1) The **sum** of the roots of $ax^2+bx+c=0$ is given by $\alpha+\beta = \dfrac{-b}{a} = \dfrac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$

2.2) The **product** of the roots of $ax^2+bx+c=0$ is given by $\alpha\beta = \dfrac{c}{a} = \dfrac{\text{constant term}}{\text{coefficient of } x^2}$

3) The quadratic equation with roots α,β is $(x-\alpha)(x-\beta)=0$ i.e., $x^2-(\alpha+\beta)x+\alpha\beta=0$

4) The **discriminant** of $ax^2+bx+c=0$ is $\boldsymbol{\Delta=b^2-4ac}$

5) The **roots** of $ax^2+bx+c=0$ are **equal** $\Leftrightarrow$ $\boldsymbol{\Delta=b^2-4ac=0}$

6.1) If $a>0$ then ax^2+bx+c has the **minimum value** $\boldsymbol{\dfrac{4ac-b^2}{4a}}$ **at** $\boldsymbol{x = -\dfrac{b}{2a}}$

6.2) If $a<0$ then ax^2+bx+c has the **maximum value** $\boldsymbol{\dfrac{4ac-b^2}{4a}}$ **at** $\boldsymbol{x = -\dfrac{b}{2a}}$

7) Tracing the changes in the signs of the quadratic expression ax^2+bx+c:

Sign of Δ	Sign of the expression	Values of x
$\Delta=b^2-4ac<0$	a and ax^2+bx+c have the same sign	$\forall\ x\in R$
$\Delta=b^2-4ac=0$	a and ax^2+bx+c have the same sign	$\forall\ x\in R-\{-b/2a\}$
$\Delta=b^2-4ac>0$ α,β are roots of $ax^2+bx+c=0$, $\alpha<\beta$	a and ax^2+bx+c have the same sign	$x<\alpha$ or $x>\beta$
	a and ax^2+bx+c have opposite signs	$\alpha<x<\beta$

8) **Quadratic inequalities:**

For $a<b$, $(x-a)(x-b)<0 \Leftrightarrow x\in(a,b)$, $(x-a)(x-b)\le 0 \Leftrightarrow x\in[a,b]$

$(x-a)(x-b)>0 \Leftrightarrow x\in(-\infty,a)\cup(b,\infty)$, $(x-a)(x-b)\ge 0 \Leftrightarrow x\in(-\infty, a]\cup[b, -\infty)$

9) **Useful algebraic formulae:**

$a^2+b^2=(a+b)^2-2ab$; $a^3+b^3=(a+b)^3-3ab(a+b)=(a+b)((a+b)^2-3ab)$