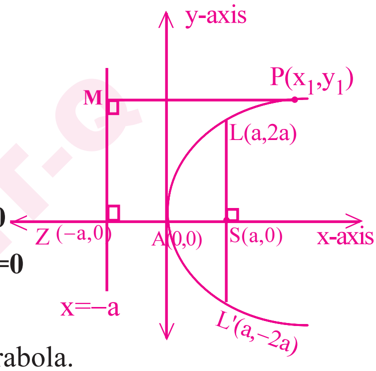
3. PARABOLA

(1 x 2) + (1 x 7) = 9 Marks

IMP FORMULAS, KEY CONCEPTS

1) Parabola: The locus of a point in a plane, which moves such that its distance from a fixed point is equal to its distance from a fixed line, is called a parabola. **AP 23**

The following terminology holds true w.r.to the parabola $y^2=4ax$ in the standard form:



1) The **focus** of the parabola $y^2=4ax$ is **S(a,0)**

2.1) The equation of the **directrix** of the parabola is **x= –a**

2.2) The **foot of the directrix** is **Z(–a,0)**

3) The **vertex** of the parabola is **A(0,0)**

4) The **axis** of the parabola is the **x-axis** and its equation is **y=0**

5) The **tangent** at the vertex is the **y-axis** and its equation is **x=0**

6.1) The line segment passing through the focus S(a,0) and perpendicular to the axis is called the **latusrectum** of the parabola.

6.2) The **equation of the latusrectum** is **x=a**

6.3) The **ends of the latusrectum** of the parabola are **L=(a,2a) and L' =(a,–2a)**

6.4) The **length of the latusrectum LL'** of the parabola $y^2=4ax$ is **4a**

6.5) **LS=*l*** is known as a **semi latusrectum** of the parabola and its length is **2a**

7.1) Any chord passing through the focus S(a,0) is known as the **focal chord** of the parabola

7.2) The **focal distance** of the point $P(x_1,y_1)$ on the parabola $y^2=4ax$ is $\mathbf{SP=x_1+a}$

8. Notation: $S=y^2-4ax$; $S_1=y_1y-2a(x_1+x)$; $S_{11}=y_1{}^2-4ax_1$; $S_{12}=y_1y_2-2a(x_1+x_2)$

9) **Relative positions** of a point and parabola: The point $P(x_1,y_1)$ w.r.to the parabola S=0 lies
(i) on the parabola $\Leftrightarrow S_{11}=0$; (ii) inside the parabola $\Leftrightarrow S_{11}<0$
(iii) outside the parabola S=0 $\Leftrightarrow S_{11}>0$

10) The parametric equations of the parabola $y^2=4ax$ with parameter t are $x=at^2$, $y=2at$
The point $(at^2, 2at)$ is simply denoted by the parameter t

11.1) The equation of the tangent drawn at a point $P(x_1,y_1)$ on the parabola S=0 is $S_1=0$

11.2) The tangential condition for the line y=mx+c and the parabola $y^2=4ax$ is $c=\frac{a}{m}$

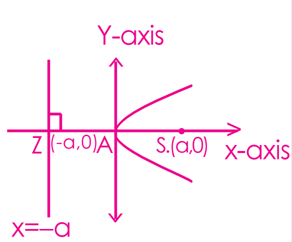
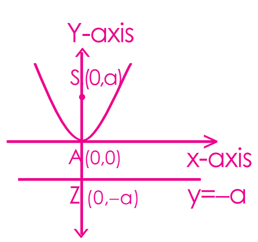
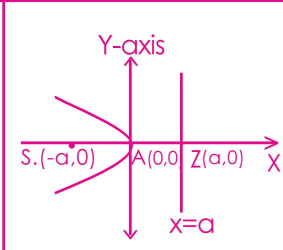
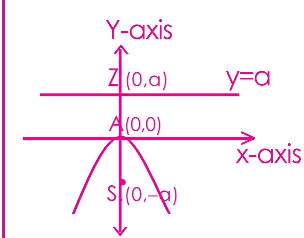
11.3) The equation of the tangent to the parabola $y^2=4ax$ having slope m is $y=mx+\frac{a}{m}$

11.4) The equation of the tangent to the parabola $y^2=4ax$ at the point 't' is $yt=x+at^2$.

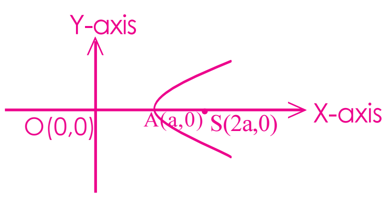
12.1) The equation of the normal at $P(x_1,y_1)$ on the parabola $y^2=4ax$ is $y-y_1=-\frac{y_1}{2a}(x-x_1)$

12.2) The equation of the normal to the parabola $y^2=4ax$ at the point t is $y+xt=2at+at^3$

2) PARABOLAS IN FOUR STANDARD FORMS

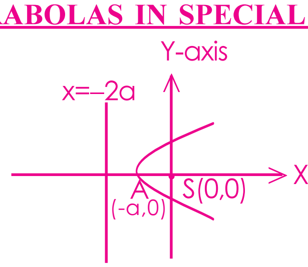
$y^2=4ax$ (Horizontal right side)	$x^2=4ay$ (Vertical upward)	$y^2=-4ax$ (Horizontal left side)	$x^2=-4ay$ (Vertical downward)
			
1) Focus S=(a,0)	1) Focus S=(0,a)	1) Focus S=(–a,0)	1) Focus S=(0,–a)
2) Vertex A=(0,0)	2) Vertex A=(0,0)	2) Vertex A=(0,0)	2) Vertex A=(0,0)
3) Ends of latusrectum L(a,2a), L'(a,–2a)	3) Ends of latusrectum L(2a,a), L'(–2a,a)	3) Ends of latusrectum L(-a,-2a), L'(–a,2a)	3) Ends of latusrectum L(–2a,–a), L'(2a,–a)
4) Foot of the directrix Z=(–a,0)	4) Foot of the directrix Z=(0,–a)	4) Foot of the directrix Z=(a,0)	4) Foot of the directrix Z=(0,a)
5) Equation of the directrix is x= –a	5) Equation of the directrix is y= –a	5) Equation of the directrix is x=a	5) Equation of the directrix is y=a
6) Equation of the latusrectum is x=a	6) Equation of the latusrectum is y=a	6) Equation of the latusrectum is x= –a	6) Equation of the latusrectum is y= –a
7) Equation of the axis is y=0	7) Equation of the axis is x=0	7) Equation of the axis is y=0	7) Equation of the axis is x=0
8) Equation of the tangent at the vertex is x=0	8) Equation of the tangent at the vertex is y=0	8) Equation of the tangent at the vertex is x=0	8) Equation of the tangent at the vertex is y=0

3) PARABOLAS IN SPECIAL CASES



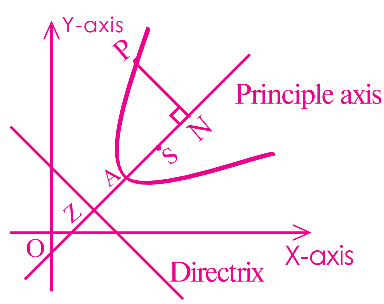
$y^2=4a(x-a)$

Parabola with
Y-axis as directrix



$y^2=4a(x+a)$

Parabola with
focus at the origin



Oblique Parabola:
$PN^2=4AS.AN$