2. SYSTEM OF CIRCLES

(1 x 2) + (1 x 4) = 6 Marks

IMP FORMULAS, KEY CONCEPTS

1.1) If θ is the angle between two intersecting circles of radii r_1, r_2 and d is the distance between the centres then $\cos\theta = \frac{d^2 - r_1^2 - r_2^2}{2r_1r_2}$

1.2 Two circles with radii r_1,r_2 intersect orthogonally if and only if $r_1^2+r_2^2=d^2$.

2.1) If θ is the angle between the two intersecting circles $S=x^2+y^2+2gx+2fy+c=0$ and $S'=x^2+y^2+2g'x+2f'y+c'=0$ then $\cos\theta = \frac{(c+c')-(2gg'+2ff')}{2\sqrt{(g^2+f^2-c)(g'^2+f'^2-c')}}$

2.2) The orthogonal condition for the circles $S=x^2+y^2+2gx+2fy+c=0$, $S'=x^2+y^2+2g'x+2f'y+c'=0$ is $2gg'+2ff'=c+c'$ (or) $2(gg'+ff')=(c+c')$.

3.1) The radical axis of two circles is defined to be the locus of a point which moves so that its powers with respect to the two circles are equal.

3.2) The equation of the radical axis of the circles $S=0$ and $S'=0$ is $S-S'=0$.

4) The radical axis of two circles is
(i) the 'common chord' when the two circles intersect at two distinct points.
(ii) the 'common tangent' at the point of contact when the two circles touch each other.

5) The radical axis of any two circles is perpendicular to the line joining their centres.

6) The point of concurrence of the radical axis of each pair of the three circles whose centres are not collinear is called the radical centre.

7) Let $S'=0$, $S''=0$, $S'''=0$ be three circles whose centres are non-collinear and no two circles of these are intersecting then the circle having
(i) radical centre of these circles as the centre of the circle.
(ii) length of tangent from the radical centre to any one of these three circles as radius cuts the given three circles orthogonally.

8) The equation of any circle passing through the points of intersection of the circle $S=0$ and the line $L=0$ is given by $S+\lambda L=0$, λ is a parameter.