2. De Moivre's Theorem

(1 x 2) + (1 x 7) = 9 Marks

IMP FORMULAS, KEY CONCEPTS

1) Demoivre's theorem:

(i) $(\cos\theta+i\sin\theta)^n = \cos n\theta+i\sin n\theta,\ n\in Z$ (or) $(\text{cis}\theta)^n=\text{cis}(n\theta)$

(ii) $(\cos\theta+i\sin\theta)^{-n} = \cos n\theta-i\sin n\theta,\ n\in Z$ (or) $(\text{cis}\theta)^{-n}=\text{cis}(-n\theta)$

(iii) $(\cos\theta+i\sin\theta)^{p/q} = \cos\frac{p}{q}\theta+i\sin\frac{p}{q}\theta,\ \frac{p}{q}\in R,\ q\neq 0$ (or) $(\text{cis}\theta)^{p/q}=\text{cis}\frac{p}{q}\theta$

2.1) If $x=\cos\theta+i\sin\theta$ then $\frac{1}{x}=\cos\theta-i\sin\theta$ hence (i) $x+\frac{1}{x}=2\cos\theta$ (ii) $x-\frac{1}{x}=2i\sin\theta$

2.2) If $x=\cos\theta+i\sin\theta$ then (i) $x^n+\frac{1}{x^n}=2\cos n\theta$ (ii) $x^n-\frac{1}{x^n}=2i\sin n\theta$

3.1) The roots of $x^3=1$ are called cube roots of unity, which are $1;\ \omega=\frac{-1+i\sqrt{3}}{2};\ \omega^2=\frac{-1-i\sqrt{3}}{2}$

3.2) $1+\omega+\omega^2=0$, hence we have; $1+\omega=-\omega^2,\ 1+\omega^2=-\omega,\ \omega+\omega^2=-1$

3.3) $\omega^3=1,\ \omega^4=(\omega^3)\omega=\omega,\ \omega^5=(\omega^3)\omega^2=\omega^2,\ \omega^6=(\omega^3)^2=1.....$

4.1) The roots of $x^4=1$ are called fourth roots of unity, which are $1, -1, i, -i$

4.2) $1=\text{cis}0=\cos 0+i\sin 0$; $-1=\text{cis}\pi=\cos\pi+i\sin\pi$;

$$i=\text{cis}\frac{\pi}{2}=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2};\quad -i=\text{cis}\left(-\frac{\pi}{2}\right)=\cos\frac{\pi}{2}-i\sin\frac{\pi}{2}$$

5.1) The n^{th} roots of unity are $\text{cis}\frac{2k\pi}{n},\ k=0,1,2,3,...(n-1)$

5.2) The n^{th} roots of $z=r\text{cis}\theta$ are $r^{1/n}\text{cis}\left(\frac{2k\pi+\theta}{n}\right),\ k=0,1,2,....(n-1)$