1. COMPLEX NUMBERS

(2 x 2) + (1 x 4) = 8 Marks

IMP FORMULAS, KEY CONCEPTS

1) A complex number is expressed in the form z=a+ib (or) (a, b) where a, b $\in$ R, $i=\sqrt{-1}$

Here, a is called the real part of z and b is called the imaginary part of z.

2) A complex number is said to be (i) purely real if its imaginary part is zero.

(ii) purely imaginary if its real part is zero.

3) Equality of complex numbers: a+ib=c+id $\Leftrightarrow$ a = c and b = d

4) If z_1=a+ib, z_2 = c+id then (i) z_1+z_2 = (a+c) + i(b+d)

(ii) $z_1.z_2$ = (a+ib) (c+id) = (ac–bd) + i(bc+ad); (a+ib)(c–id)=(ac+bd)+i(bc–ad)

(iii) $\frac{z_1}{z_2}=\frac{a+ib}{c+id}=\frac{(a+ib)(c-id)}{(c+id)(c-id)}=\left(\frac{ac+bd}{c^2+d^2}\right)+i\left(\frac{bc-ad}{c^2+d^2}\right)$

Hints: (i) $\sqrt{-a}=i\sqrt{a}, a\in R^+$ (ii) $(a+ib)^2=(a^2-b^2)+i2ab$

(iii) $(a+ib)(a-ib)=a^2+b^2$ (iv) $\frac{1+i}{1-i}=i$ (v) $\frac{1-i}{1+i}=\frac{1}{i}=-i$

5) If z=a+ib then its (i) conjugate is $\bar{z}$ =a–ib (ii) additive inverse is –z = –a–ib

(iii) Multiplicative inverse is $z^{-1}=\frac{1}{a+ib}=\frac{a-ib}{a^2+b^2}$

(iv) Modulus is $|z|=\sqrt{a^2+b^2}$ (v) $\text{Arg } z=\text{Tan}^{-1}\left(\frac{b}{a}\right)$

6.1) The square root of a+bi when b>0 is $\sqrt{a+ib}=\pm\left(\sqrt{\frac{r+a}{2}}+i\sqrt{\frac{r-a}{2}}\right)$, where $r=\sqrt{a^2+b^2}$

6.2) The square root of a–bi when b>0 is $\sqrt{a-ib}=\pm\left(\sqrt{\frac{r+a}{2}}-i\sqrt{\frac{r-a}{2}}\right)$, where $r=\sqrt{a^2+b^2}$

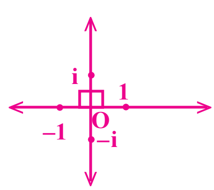
7) Mod amp.form (or) polar form:

If z = x+iy then z=r(cosθ+isinθ) is called Mod-amplitude form, where

(i) Modulus=|z|=$r=\sqrt{x^2+y^2}$

(ii) Amplitude= $\theta=\text{Tan}^{-1}\left(\frac{y}{x}\right)\in(-\pi,\pi]$

Arg (1) = 0;
Arg (i)=π/2;
Arg (–1) = π;
Arg (–i) = –π/2



8.1) If Arg z=θ then Arg $\bar{z}=-\theta$

8.2) If Arg $z_1=\theta_1$ and Arg $z_2=\theta_2$ then (i) Arg $(z_1.z_2)=\theta_1+\theta_2$ (ii) $\text{Arg}(z_1/z_2)=\theta_1-\theta_2$

9) cosθ+isinθ is simply written as cisθ; (i) $\text{cis}\theta_1.\text{cis}\theta_2=\text{cis}(\theta_1+\theta_2)$ (ii) $\frac{\text{cis}\theta_1}{\text{cis}\theta_2}=\text{cis}(\theta_1-\theta_2)$