1. CIRCLES

(2 x 2) + (1 x 4) + (2 x 7) = 22 Marks

IMP FORMULAS, KEY CONCEPTS

1.1) The equation of the circle in the **standard form** is $x^2+y^2=r^2$

The **parametric point** is θ $(r\cos\theta, r\sin\theta)$

1.2) The equation of the circle in the **centre form** (centre (a,b), radius r) is $(x-a)^2+(y-b)^2=r^2$
The **parametric equations** are $x=a+r\cos\theta,\ y=b+r\sin\theta$.

1.3) The equation of the circle in the **general form** is $x^2+y^2+2gx+2fy+c=0$.

Centre $C=(-g,-f)$; **radius** $r=\sqrt{g^2+f^2-c}$

The **Parametric equations** are $x=-g+r\cos\theta,\ y=-f+r\sin\theta$.

2) The equation of the circle with $(x_1,y_1), (x_2,y_2)$ as **ends of a diameter** is
$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$ (or) $x^2+y^2-(x_1+x_2)x-(y_1+y_2)y+x_1x_2+y_1y_2=0$

3) **Notation:** $S\equiv x^2+y^2+2gx+2fy+c$;

$S_{11}\equiv x_1^2+y_1^2+2gx_1+2fy_1+c$

$S_{12}\equiv x_1x_2+y_1y_2+g(x_1+x_2)+f(y_1+y_2)+c$

$S_1\equiv x_1x+y_1y+g(x_1+x)+f(y_1+y)+c$ (or) $S_1\equiv(x_1+g)x+(y_1+f)y+x_1g+y_1f+c$

4) A point $P(x_1,y_1)$ and a **circle S=0** are given then the **point P lies**

(i) **on the circle** if $S_{11}=0$ (ii) **inside** the circle if $S_{11}<0$ (iii) **out side** the circle if $S_{11}>0$

5) The **Power** of the point $P(x_1,y_1)$, w.r.to the circle S=0 is S_{11}

6.1) The equation of the **chord of contact of (x_1,y_1) w.r.to S=0 is $S_1=0$**

6.2) The equation of the chord with (x_1,y_1) as the **mid-point** to the circle S=0 is $S_1=S_{11}$

6.3) If **p** is the **perpendicular distance** from the centre of circle of **radius r**, to a chord then the **length of the chord** is $2\sqrt{r^2-p^2}$

7.1) The **length** of the **tangent** from an external point $P(x_1,y_1)$ to S=0 is $\sqrt{S_{11}}$

7.2) The **equation of tangent at (x_1,y_1)** on the circle S =0 is $S_1=0$

7.3) The **equation of tangent at (x_1,y_1)** on the circle $x^2+y^2=r^2$ **is** $x_1x+y_1y-r^2=0$

7.4) The **equation of the tangent** at $(r\cos\theta, r\sin\theta)$ on $x^2+y^2=r^2$ is $x\cos\theta+y\sin\theta=r$

7.5) The equation of the tangent **with slope m** to the circle $x^2+y^2=r^2$ is $y=mx\pm r\sqrt{1+m^2}$

7.6) The equation of the tangent **with slope m** to the circle $x^2+y^2+2gx+2fy+c=0$ is

$y+f=m(x+g)\pm r\sqrt{1+m^2}$

7.7) The **equation of the tangent** at θ on **the cirlce S=0 is** $(x+g)\cos\theta+(y+f)\sin\theta=r$

7.8) The **tangential condition** for a circle and a line is **p=r**, where p is the perpendicular distance from the centre of the circle to the given line and r is the radius of the circle.

7.9) The **tangential condition** for the line
(i) $\mathbf{y=mx+c}$ and the circle $\mathbf{x^2+y^2=r^2}$ is $\mathbf{c^2=r^2(1+m^2)}$
(ii) $\mathbf{lx+my+n=0}$ and the circle $\mathbf{x^2+y^2=r^2}$ is $\mathbf{n^2=r^2(l^2+m^2)}$

7.10) The condition for the cirlce $x^2+y^2+2gx+2fy+c=0$ to touch
(i) the X-axis is $c=g^2$ (ii) the Y-axis $c=f^2$ (iii)both the axes is $c=g^2=f^2$.

7.11) If θ is the **angle** between pair of tangents from $P(x_1,y_1)$ to $S=0$ then $\tan\frac{\theta}{2}=\frac{r}{\sqrt{S_{11}}}$

7.12) The equation of **pair of tangents** drawn from (x_1,y_1) to $S=0$ is $\mathbf{S_1^2=S_{11}S}$

8.1) The equation of the **normal** at $P(x_1,y_1)$ on $S=0$ is $(x-x_1)(y_1+f)-(y-y_1)(x_1+g)=0$

8.2) The **normal** at $P(x_1,y_1)$ on $S=0$ always passes through the centre of the circle.

9.1) The equation of the **polar** of $P(x_1,y_1)$ w.r.to $S=0$ is $S_1=0$

9.2) The **pole** of the **line** $\mathbf{lx+my+n=0}$ w.r.to the circle $\mathbf{x^2+y^2=r^2}$ is $\left(\frac{-lr^2}{n},\frac{-mr^2}{n}\right)$

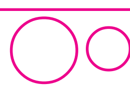
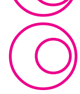
9.3) The **pole** of the **line** $\mathbf{lx+my+n=0}$ w.r.to $\mathbf{S\equiv x^2+y^2+2gx+2fy+c=0}$ is $\left(-g+\frac{lr^2}{N},-f+\frac{mr^2}{N}\right)$
Where, r = radius of the circle, $N=lg+mf-n$

10.1) The points (x_1,y_1), (x_2,y_2) are **conjugate** w.r.to $S=0$ if $S_{12}=0$

10.2) The condition for the lines $\mathbf{l_1x+m_1y+n_1=0}$ and $\mathbf{l_2x+m_2y+n_2=0}$ to be **conjugate** w.r.t the circle $\mathbf{x^2+y^2=r^2}$ is $\mathbf{n_1n_2=r^2(l_1l_2+m_1m_2)}$

11) Two points P,Q are said to be **inverse points** w.r.to S=0 if $CP.CQ=r^2$;C,P,Q are collinear.

12) **Relative positions of two circles:**
1) Determine C_1,C_2 and r_1,r_2
2) Determine C_1C_2, r_1+r_2, r_1-r_2
3) If $C_1C_2>r_1+r_2$ then one circle lies completely outside the other
4) If $C_1C_2=r_1+r_2$ then the two circles touch externally
5) If $C_1C_2<r_1+r_2$ then compare C_1C_2 also with r_1-r_2 and according to the following order determine the relative positions of the circles.
5.1) For $C_1C_2<r_1+r_2$ and if $C_1C_2>|r_1-r_2|$ then the two circles intersect
5.2) For $C_1C_2<r_1+r_2$ and if $C_1C_2=|r_1-r_2|$ then the two circles touch internally
5.3) For $C_1C_2<r_1+r_2$ and if $C_1C_2<r_1-r_2$ i.e, for $C_1C_2<|r_1-r_2|<|r_1+r_2|$ then one circle lies completely inside the other circle.

S.No.	Condition	Relative positions	No.of tangents	centres of similitude
1.	$C_1C_2>r_1+r_2$	one circle lies outside the other 	4 (2 transverse and 2 direct)	both I and E exist
2.	$C_1C_2=r_1+r_2$	two circles touch externally 	3 (1 transverse and 2 direct)	both I and E exist
3.	$\lvert r_1-r_2\rvert<C_1C_2<r_1+r_2$	two circles intersect 	2 (direct)	E exist
4.	$C_1C_2=\lvert r_1-r_2\rvert$	two circles touch internally 	1 (direct)	E exist
5.	$C_1C_2<\lvert r_1-r_2\rvert$	one circle lies inside the other 	0	