SOLVED PAPER-3

MATHS- IB

Time: 3 Hours **Max. Marks : 75**

SECTION -A

I. Answer ALL the following Very Short Answer Questions: 10 × 2 = 20

1. Find the ratio in which the straight line 2x+3y–20=0 divides the join of the points (2,3) and (2,10)
2. Find the length of the perpendicular from the point (–2,–3) to the straight line 5x–2y+4=0
3. Show that the points (1,2,3), (7,0,1), (–2,3,4) are collinear.
4. Find the equation of the plane through the point (α,β,γ) and parallel to the plane ax+by+cz=0
5. Find $\underset{x\to 1}{Lt}\ \dfrac{\sin(x-1)}{x^2-1}$
6. Evaluate $\underset{x\to 0}{Lt}\ \dfrac{e^{\sin x}-1}{x}$
7. If $f(x)=2x^2+3x-5$, then prove that $f'(0)+3f'(-1)=0$.
8. If $x = e^{\sinh y}$ then find $\dfrac{dy}{dx}$
9. Find the length of subtangent at a point on the curve $y = b\sin\left(\dfrac{x}{a}\right)$
10. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 6$ in the interval [–3, 8]

SECTION-B

II. Answer any FIVE of the following Short Answer Questions: 5 × 4 = 20

11. Find the equation of locus of a point , which forms a triangle of area 2 with the points A(1,1,), B(–2,3)
12. When the origin is shifted to the point (–1, 2), the transformed equation of a curve is $x^2 + 2y^2 + 16 = 0$. Find the original equation of the curve.
13. Find the equation of the straight line parallel to the line 3x+4y=7 and passing through the point of intersection of the lines x–2y–3=0, x+3y–6=0.
14. Is f defined by $f(x)=\begin{cases}\dfrac{\sin 2x}{x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$ continuous at 0?
15. If $y=\dfrac{x\sin^{-1}x}{\sqrt{1-x^2}}$ then find $\dfrac{dy}{dx}$
16. The radius of an air bubble is increasing at the rate of 1/2 cm/sec. At what rate is the volume of the bubble increasing when the radius is 1cm?
17. Find the equation of tangent and normal to the curve $y=2e^{\frac{-x}{3}}$ at the point where the curve meets the Y-axis

SECTION-C

III. Answer any FIVE of the following Long Answer Questions: 5 × 7 = 35

18. Find the orthocenter of the triangle whose sides are given by x+y+10=0, x–y–2=0 and 2x+y–7=0
19. If (α,β) is the centroid of the triangle formed by the lines $ax^2+2hxy+by^2=0$, $lx+my=1$ then $\dfrac{\alpha}{bl-hm}=\dfrac{\beta}{am-hl}=\dfrac{2}{3(bl^2-2hlm+am^2)}$
20. Find the condition for the lines joining the origin to the points of intersection of the circle $x^2+y^2=a^2$ and the line $lx+my=1$ to coincide.
21. A(1,8,4), B(0,–11,4), C(2,–3,1) and 3 points and D is the foot of the perpendicular from A to BC. Find the coordinates of D.
22. Find the derivative of $\sin^{-1}\left(\dfrac{b+a\sin x}{a+b\cos x}\right)$ w.r.to x
23. Show that the square of the length of subtangent at any point on the curve $by^2=(x+a)^3$, b≠0 varies with the length of the subnormal at the point.
24. Find the maximum area of the rectangle that can be formed with fixed perimeter 20.

SOLUTIONS

SECTION-A

1. Find the ratio in which the straight line 2x+3y–20=0 divides the join of the points (2,3), (2,10)

Sol: The given points are $(x_1,y_1)=(2,3)$ and $(x_2,y_2)=(2,10)$; Let $L=2x+3y-20$

Now, $L_{11}=2(2)+3(3)-20=14-20=-7$ and $L_{22}=2(2)+3(10)-20=34-20=14$

$\therefore$ the required ratio is $-L_{11} : L_{22}=-(-7) : 14 = 7 : 14 = 1:2$

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2. Find the length of the perpendicular from the point (–2,–3) to the straight line 5x–2y+4=0

Sol: Perpendicular distance from (x_1,y_1) to ax+by+c=0 is $\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$\therefore$ Perpendicular distance from (–2,–3) to 5x–2y+4=0 is

$$=\frac{|5(-2)-2(-3)+4|}{\sqrt{5^2+2^2}}=\frac{|-10+6+4|}{\sqrt{29}}=\frac{0}{\sqrt{29}}=0$$

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3. Show that the points (1,2,3), (7,0,1), (–2,3,4) are collinear.

Sol: Let A=(1,2,3), B=(7,0,1), C=(–2,3,4)

$$AB=\sqrt{(1-7)^2+(2-0)^2+(3-1)^2}=\sqrt{36+4+4}=\sqrt{44}=2\sqrt{11}$$

$$BC=\sqrt{(7+2)^2+(0-3)^2+(1-4)^2}=\sqrt{81+9+9}=\sqrt{99}=3\sqrt{11}$$

$$CA=\sqrt{(-2-1)^2+(3-2)^2+(4-3)^2}=\sqrt{9+1+1}=\sqrt{11}$$

Now, $AB+CA=2\sqrt{11}+\sqrt{11}=3\sqrt{11}=BC$ $\therefore$ A,B,C are collinear

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4. Find the equation of the plane through the point (α,β,γ) and parallel to the plane ax+by+cz=0

Sol: The equation of any plane parallel to ax+by+cz=0 is ax+by+cz+k=0 ......(1)

If (1) passes through (α,β,γ) then $a\alpha+b\beta+c\gamma+k=0 \Rightarrow k=-a\alpha-b\beta-c\gamma$

$\therefore$ (1) $\Rightarrow ax+by+cz-a\alpha-b\beta-c\gamma=0 \Rightarrow a(x-\alpha)+b(y-\beta)+c(z-\gamma)=0$

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5. Find $\underset{x\to1}{Lt}\dfrac{\sin(x-1)}{x^2-1}$

Sol: $$\underset{x\to1}{Lt}\frac{\sin(x-1)}{x^2-1}=\underset{(x-1)\to0}{Lt}\frac{\sin(x-1)}{(x-1)(x+1)}=\underset{(x-1)\to0}{Lt}\frac{\sin(x-1)}{x-1}.\underset{x\to1}{Lt}\frac{1}{x+1}=1.\frac{1}{1+1}=\frac{1}{2}$$

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6. Find $\underset{x\to\infty}{Lt}\dfrac{3^x-1}{4^x-1}$

Sol: $$\underset{x\to\infty}{Lt}\frac{3^x-1}{4^x-1}=\underset{x\to\infty}{Lt}\frac{3^x\left(1-\frac{1}{3^x}\right)}{4^x\left(1-\frac{1}{4^x}\right)}=\underset{x\to\infty}{Lt}\left(\frac{3}{4}\right)^x.\underset{x\to\infty}{Lt}\frac{1-\frac{1}{3^x}}{1-\frac{1}{4^x}}=0.\frac{1-0}{1-0}=0.1=0\ \left(\because\frac{3}{4}<1\right)$$

7. If $f(x)=2x^2+3x-5$, then prove that $f'(0)+3f'(-1)=0$.

Sol: Given $f(x)=2x^2+3x-5 \Rightarrow f'(x) = 4x+3$

Then $f'(0) = 0+3=3$ and $f'(-1) = -4+3=-1$

$\therefore f'(0)+3f'(-1) =3+3(-1)=3-3=0$

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8. If $x=e^{\sinh y}$ then find $\dfrac{dy}{dx}$

Sol: Given that $x=e^{\sinh y}$; differentiating w.r.to y

$$\frac{dx}{dy} = e^{\sinh y}\frac{d}{dy}\sinh y = e^{\sinh y}.\cosh y = x\cosh y \Rightarrow \frac{dy}{dx} = \frac{1}{x\cosh y}$$

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9. Find the length of subtangent at a point on the curve $y = b\sin\left(\dfrac{x}{a}\right)$.

Sol: Let P(x,y) be any point on the curve $y = b\sin\left(\dfrac{x}{a}\right)$ ....(1)

On differentiating (1) w.r.to x, we get $\dfrac{dy}{dx} = b\left(\cos\dfrac{x}{a}\right)\dfrac{1}{a}$

$$\therefore m = \left(\frac{dy}{dx}\right)_P = \frac{b}{a}\cos\left(\frac{x}{a}\right)$$

$$\text{Length of sub-tangent} = \left|\frac{y_1}{m}\right| = \left|\frac{y}{\frac{b}{a}\cos\frac{x}{a}}\right| = \left|\frac{b\sin\left(\frac{x}{a}\right)}{\frac{b}{a}\cos\left(\frac{x}{a}\right)}\right| = \left|\frac{b\sin\left(\frac{x}{a}\right)}{\frac{b}{a}\cos\left(\frac{x}{a}\right)}\right| = \left|a\tan\frac{x}{a}\right|$$

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10. Verify Rolle's theorem for the function $f(x) = x^2 - 5x + 6$ in the interval $[-3,8]$

Sol : Being a Polynomial function, the given function f(x) is continuous and differentiable on R

$\therefore f(x) = x^2-5x+6$ is (i) continuous on $[-3,8]$ and (ii) differentiable in $(-3,8)$

Now, $f(-3) =x^2-5x+6= (-3)^2-5(-3)+6 = 9+15+6 =30$

$f(8) = (8)^2-5(8)+6 = 64-40+6 =24+6 =30$

$\therefore f(-3) = f(8)$

Hence, f(x) satisfies all the 3 conditions of Rolle's theorem.

$\therefore$ By Rolle's theorem, there exists $c\in(-3,8)$ such that $f'(c)=0$

Now, $f(x) = x^2-5x+6 \Rightarrow f'(x) = 2x-5$

Hence, $f'(c)=0 \Rightarrow 2c=5 \Rightarrow c=5/2=2.5$

Here, $c=2.5\in(-3,8)$

Thus, Rolle's theorem is verified.

SECTION-B

11. Find the equation of locus of a point, which forms a triangle of area 2 with the points A(1,1,), B(–2,3)

Sol: Given that A=(1,1), B=(–2,3) and P(x.,y) be a point on the locus.
From the given condition, area of ΔPAB=2

$$\Rightarrow \frac{1}{2}\begin{vmatrix} x-1 & x+2 \\ y-1 & y-3 \end{vmatrix} = 2 \Rightarrow \left|(x-1)(y-3)-(y-1)(x+2)\right| = 2(2)$$

$$\Rightarrow |xy-3x-y+3-(xy+2y-x-2)| = 4 \Rightarrow |xy-3x-y+3-xy-2y+x+2|$$

$$\Rightarrow |-2x-3y+5| = 4 \Rightarrow |2x+3y-5| = 4 \Rightarrow 2x+3y-5 = \pm 4$$

$$\Rightarrow 2x+3y-5=4 \text{ (or) } 2x+3y-5=-4$$

$$\Rightarrow 2x+3y-9=0 \text{ (or) } 2x+3y-1=0$$

$$\Rightarrow (2x+3y-9)(2x+3y-1)=0$$

∴ The equation of locus of P(x,y) is (2x+3y–9)(2x+3y–1)=0

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12. When the origin is shifted to the point (–1,2), the transformed equation of a curve is $x^2+2y^2+16=0$.Find the original equation of the curve.

Sol: From the transformed(new) equation we have to find original(old) equation.
Let the new coordinates of any point (x,y) be denoted by (X,Y).
Then the given equation be written as $X^2+2Y^2+16=0$
Here, the new origin (h,k) = (–1,2)
∴ X=x–h=x+1 and Y=y–k=y–2
∴ the original equation of $X^2+2Y^2+16=0$ is $(x+1)^2+2(y-2)^2+16=0$
$\Rightarrow x^2+2x+1+2(y^2-4y+4)+16=0 \Rightarrow x^2+2x+1+2y^2-8y+8+16=0 \Rightarrow x^2+2y^2+2x-8y+25=0$

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13. Find the equation of the straight line parallel to the line 3x+4y=7 and passing through the point of intersection of the lines x–2y–3=0, x+3y–6=0.

Sol: First we find the Point of intersection of x–2y–3=0, x+3y–6=0.

$$\frac{x}{(-2)(-6)-3(-3)} = \frac{y}{(-3)(1)-1(-6)} = \frac{1}{1(3)-1(-2)}$$

$$\Rightarrow \frac{x}{12+9} = \frac{y}{-3+6} = \frac{1}{3+2} \Rightarrow \frac{x}{21} = \frac{y}{3} = \frac{1}{5} \Rightarrow x = \frac{21}{5}, y = \frac{3}{5}$$

∴ Point of Intersection= $\left(\frac{21}{5}, \frac{3}{5}\right)$

Now, the equation of any line parallel to 3x+4y=7 is of the form 3x+4y=k.

If this line passes through $\left(\frac{21}{5}, \frac{3}{5}\right)$ then $3\left(\frac{21}{5}\right)+4\left(\frac{3}{5}\right)=k$

$$\Rightarrow \frac{63}{5}+\frac{12}{5} = k \Rightarrow \frac{75}{5} = k \Rightarrow k = 15$$

∴ The equation of the required line is 3x+4y=15 ⇒3x+4y–15 =0

14. Is f defined by $f(x)=\begin{cases}\frac{\sin 2x}{x} & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$ continuous at 0?

Sol: Given that f(0)=1

Now $\underset{x\to 0}{Lt} f(x)=\underset{x\to 0}{Lt}\frac{\sin 2x}{x}=2 \therefore \underset{x\to 0}{Lt} f(x)\neq f(0)$ $\therefore$ f is not continuous at x=0

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15. If $y=\frac{x\sin^{-1}x}{\sqrt{1-x^2}}$ then find $\frac{dy}{dx}$

Sol: $\frac{dy}{dx}=\frac{d}{dx}\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right)=\frac{\sqrt{1-x^2}\frac{d}{dx}x\sin^{-1}x-(x\sin^{-1}x)\frac{d}{dx}\sqrt{1-x^2}}{(\sqrt{1-x^2})^2}$

$$=\frac{\sqrt{1-x^2}\left(x.\frac{1}{\sqrt{1-x^2}}+\sin^{-1}x(1)\right)-(x\sin^{-1}x)\frac{1}{2\sqrt{1-x^2}}(-2x)}{1-x^2}$$

$$=\frac{\left(\sqrt{1-x^2}\right)\left(\frac{x+\sin^{-1}x.\sqrt{1-x^2}}{\sqrt{1-x^2}}\right)+\frac{x^2\sin^{-1}x}{\sqrt{1-x^2}}}{1-x^2}$$

$$=\frac{x\sqrt{1-x^2}+\sin^{-1}x(1-x^2)+x^2\sin^{-1}x}{(\sqrt{1-x^2})(1-x^2)}=\frac{x\sqrt{1-x^2}+\sin^{-1}x}{(1-x^2)^{3/2}}$$

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16. The radius of an air bubble is increasing at the rate of 1/2 cm/sec. At what rate is the volume of the bubble increasing when the radius is 1cm?

Ans: Let r be the radius of the air bubble and V be its volume

Given that $\frac{dr}{dt}=\frac{1}{2}cm/sec$, we have to find $\frac{dV}{dt}$ at r=1

Volume of the spherical air bubble $V=\frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt}=\frac{4}{\not 3}\pi \not 3 r^2\frac{dr}{dt}=4\pi(1)^2\frac{1}{2}=2\pi$ cm^3/sec

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17. Find the equation of tangent and normal to the curve $y=2.e^{\frac{-x}{3}}$ at the point where the curve meets the Y-axis

Sol: Given curve is $y=2.e^{\frac{-x}{3}}$ ....(1)

Equation of Y-axis is x=0...........(2)

Solving (1) and (2), we get $y=2e^0=2$

The point of intersection is P(0,2)

Differentiating $y=2.e^{\frac{-x}{3}}$ both sides w.r.to x, we get $\frac{dy}{dx}=2\left(-e^{\frac{-x}{3}}.\frac{1}{3}\right)=-\frac{2}{3}e^{\frac{-x}{3}}$

$$\Rightarrow m = \left(\frac{dy}{dx}\right)_{P(0,2)} = -\frac{2}{3}e^0 \Rightarrow m = -\frac{2}{3}$$

Equation of the tangent to the curve at P(0,2) is $y-y_1=m(x-x_1)$

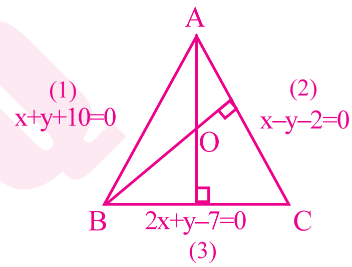
$$\Rightarrow y-2 = \frac{-2}{3}(x-0) \Rightarrow 3y-6=-2x \Rightarrow 2x+3y-6=0$$

Equation of normal to the curve at P(0,2) is

$$y-y_1 = -\frac{1}{m}(x-x_1) \Rightarrow y-2 = \frac{3}{2}(x-0) \Rightarrow 2y-4=3x \Rightarrow 3x-2y+4=0$$

SECTION-C

18. Find the orthocenter of the triangle whose sides are given by x+y+10=0, x–y–2=0, 2x+y–7=0



Sol: Let x+y+10=0....(1), x–y–2=0 ....(2) 2x+y–7=0 .....(3) represent the sides of ΔABC.

Let O be the orthocentre of ΔABC

First we find the equation of the altitude through A

Solving (1) and (2), we get A; x+y+10=0
x–y–2=0

$$\Rightarrow \frac{x}{-2+10} = \frac{y}{10+2} = \frac{1}{-1-1} \Rightarrow \frac{x}{8} = \frac{y}{12} = \frac{-1}{2}$$

$\Rightarrow x=-4, y=-6 \quad \therefore A=(-4,-6)$

The slope of the opposite side BC, 2x+y–7=0 is –2

⇒ the slope of its perpendicular is 1/2

The equation of the altitude passing through A(–4,–6) and with slope 1/2 is

y+6=(1/2)(x+4) ⇒ 2y+12= x+4=0 ⇒ x–2y–8=0 .... (4)

Now, we find the equation of the altitude through B

Solving (1) & (3) we get B; x+y+10=0
2x+y–7=0

$$\Rightarrow \frac{x}{-7-10} = \frac{y}{20+7} = \frac{1}{1-2} \Rightarrow \frac{x}{-17} = \frac{y}{27} = -1$$

$\Rightarrow x=17, y=-27 \quad \therefore B=(17,-27)$

The slope of the opposite side AC, x–y–2=0 is 1

⇒ the slope of its perpendicular is –1

The equation of the altitude through B(17,–27) and with slope –1 is y+27=–1(x–17)

⇒ x+y+10=0 .....(5)

Solving (4) & (5) we get orthocentre O;

(4)–(5) ⇒ –3y –18 =0 ⇒3y = –18⇒ y = –6

From (4), x =2y+8= 2(–6)+8 = –12+8 = –4.

∴ Orthocentre = (–4, –6)

19. If (α,β) is the centroid of the triangle formed by the lines $ax^2+2hxy+by^2=0$, $lx+my=1$ then $\dfrac{\alpha}{bl-hm}=\dfrac{\beta}{am-hl}=\dfrac{2}{3(bl^2-2hlm+am^2)}$

Proof: Let the 2 lines of $ax^2+2hxy+by^2=0$ be $y=m_1x$, $y=m_2x$.

Then we know that $m_1+m_2=-\dfrac{2h}{b}$, $m_1m_2=\dfrac{a}{b}$

Now, solving $lx+my-1=0$, $m_1x-y+0=0$, we get A

$$\Rightarrow \frac{x}{m(0)-(-1)(-1)}=\frac{y}{(-1)(m_1)-l(0)}=\frac{1}{l(-1)-m(m_1)} \Rightarrow \frac{x}{-1}=\frac{y}{-m_1}=\frac{1}{-l-mm_1}$$

$$\Rightarrow A(x,y)=A\left(\frac{1}{l+mm_1},\frac{m_1}{l+mm_1}\right); B=\left(\frac{1}{l+mm_2},\frac{m_2}{l+mm_2}\right)$$

Similarly, solving $lx+my-1=0$, $m_2x-y=0$, we get

Also, the point of intersection of $ax^2+2hxy+by^2=0$ is O(0,0)

and it was given that the centroid of ΔOAB is (α,β)

$$\Rightarrow (\alpha,\beta)=\left(\frac{0+\dfrac{1}{l+mm_1}+\dfrac{1}{l+mm_2}}{3},\frac{0+\dfrac{m_1}{l+mm_1}+\dfrac{m_2}{l+mm_2}}{3}\right)$$

$$\Rightarrow (3\alpha,3\beta)=\left(\frac{1}{l+mm_1}+\frac{1}{l+mm_2},\frac{m_1}{l+mm_1}+\frac{m_2}{l+mm_2}\right)$$

$$=\left(\frac{l+mm_2+l+mm_1}{(l+mm_1)(l+mm_2)},\frac{m_1(l+mm_2)+m_2(l+mm_1)}{(l+mm_1)(l+mm_2)}\right)$$

$$=\left(\frac{2l+m(m_1+m_2)}{l^2+lm(m_1+m_2)+m^2m_1m_2},\frac{l(m_1+m_2)+m_1m_2(m+m)}{l^2+lm(m_1+m_2)+m^2m_1m_2}\right)$$

$$=\left(\frac{2l+m\left(-\dfrac{2h}{b}\right)}{l^2+lm\left(-\dfrac{2h}{b}\right)+m^2\left(\dfrac{a}{b}\right)},\frac{l\left(-\dfrac{2h}{b}\right)+\dfrac{a}{b}(2m)}{l^2-\left(\dfrac{2h}{b}\right)lm+m^2\left(\dfrac{a}{b}\right)}\right)=\left(\frac{\dfrac{2lb-2hm}{b}}{\dfrac{bl^2-2hlm+am^2}{b}},\frac{\dfrac{-2hl+2am}{b}}{\dfrac{bl^2-2hlm+am^2}{b}}\right)$$

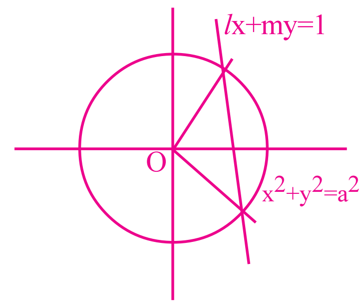
$$\Rightarrow (3\alpha,3\beta)=\left(\frac{2(lb-hm)}{bl^2-2hlm+am^2},\frac{2(am-hl)}{bl^2-2hlm+am^2}\right)$$

$$\Rightarrow 3\alpha=\frac{2(lb-hm)}{bl^2-2hlm+am^2} \Rightarrow \frac{\alpha}{lb-nm}=\frac{2}{3(bl-2hlm+am^2)}$$

and $3\beta=\dfrac{2(am-hl)}{bl^2-2hlm+am^2} \Rightarrow \dfrac{\beta}{am-hl}=\dfrac{2}{3(bl^2-2hlm+am^2)}$

$$\Rightarrow \frac{\alpha}{lb-nm}=\frac{\beta}{am-hl}=\frac{2}{3(bl^2-2hlm+am^2)}$$

20. Find the condition for the lines joining the origin to the points of intersection of the circle $x^2+y^2=a^2$ and the line $lx+my=1$ to coincide.



Sol: The equation of the given chord is $lx+my=1$ ...(1)

Now, we homogenise the circle $x^2+y^2=a^2$ using (1)

$\Rightarrow x^2+y^2=a^2(1^2) \Rightarrow x^2+y^2=a^2(lx+my)^2$

$\Rightarrow x^2+y^2-a^2(l^2x^2+2lmxy+m^2y^2)=0$

$\Rightarrow x^2(1-a^2l^2)-2a^2lmxy+y^2(1-a^2m^2)=0$

If above pair of lines are coincident then $h^2=ab$

$\Rightarrow (a^2lm)^2=(1-a^2l^2)(1-a^2m^2)=1-a^2m^2-a^2l^2+a^4l^2m^2$

$\Rightarrow a^2l^2+a^2m^2=1$

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21. A(1,8,4), B(0,–11,4), C(2,–3,1) and 3 points and D is the foot of the perpendicular from A to BC. Find the coordinates of D.

Sol: Given that A(1,8,4), B(0,–11,4), C(2,–3,1)

Let D divides $\overline{BC}$ in the ratio $k:1 \Rightarrow D=\left(\frac{2k}{k+1},\frac{-3k-1}{k+1},\frac{k+4}{k+1}\right)$

$\because$ A=(1,8,4) we have Dr's of $AD=\left(\frac{2k}{k+1}-1,\frac{-3k-11}{k+1}-8,\frac{k+4}{k+1}-4\right)$

$=\left(\frac{2k-k-1}{k+1},\frac{-3k-11-8k-8}{k+1},\frac{k+4-4k-4}{k+1}\right)=\left(\frac{k-1}{k+1},\frac{-11k-19}{k+1},\frac{-3k}{k+1}\right)$ .....(1)

Also, B=(0,–11,4), C=(2,–3,1) $\Rightarrow$ Dr's of BC=(0–2, –11+3, 4–1) = (–2. –8, 3) ....(2)

But $\overline{AD}\perp\overline{BC}\Rightarrow a_1a_2+b_1b_2+c_1c_2=0$

$\Rightarrow\left(\frac{k-1}{k+1}\right)(-2)-\left(\frac{11k+19}{k+1}\right)(-8)-\left(\frac{3k}{k+1}\right)(3)=0 \Rightarrow (k-1)(-2)+(11k+19)8-(3k)(3)=0$

$\Rightarrow -2k+2+88k+152-9k=0 \Rightarrow 77k+154=0 \Rightarrow 77k=-154 \Rightarrow k=-2$

$\therefore D=\left(\frac{2k}{k+1},\frac{-3k+1}{k+1},\frac{k+4}{k+1}\right)=\left(\frac{2(-2)}{-2+1},\frac{-3(-2)-11}{-2+1},\frac{-2+4}{-2+1}\right)=\left(\frac{-4}{-1},\frac{-5}{-1},\frac{2}{-1}\right)=(4,5,-2)$

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22. Find the derivative of $\mathrm{Sin}^{-1}\left(\frac{b+a\sin x}{a+b\cos x}\right)$ w.r.to x

Sol: $\frac{d}{dx}\mathrm{Sin}^{-1}\left(\frac{b+a\sin x}{a+b\sin x}\right)=\frac{1}{\sqrt{1-\left(\frac{b+a\sin x}{a+b\sin x}\right)^2}}\frac{d}{dx}\left(\frac{b+a\sin x}{a+b\sin x}\right)$

$$=\frac{1.(a+b\sin x)}{\sqrt{(a+b\sin x)^2-(b+a\sin)^2}}\left(\frac{(a+b\sin x)\frac{d}{dx}(b+a\sin x)-(b+a\sin x)\frac{d}{dx}(a+b\sin x)}{(a+b\sin x)^2}\right)$$

$$=\frac{1}{\sqrt{(a^2+b^2\sin^2+2ab\sin x)-(b^2+a^2\sin^2x+2ab\sin x)}}\left(\frac{(a+b\sin x)(a\cos x)-(b+a\sin x)(b\cos x)}{a+b\sin x}\right)$$

$$= \frac{a^2\cos x + ab\sin x\cos x - b^2\cos x - ab\sin x\cos x}{\sqrt{(a^2-b^2) - \sin^2 x(a^2-b^2)}.(a+b\sin x)} = \frac{(a^2-b^2)\cos x}{\sqrt{(a^2-b^2)(1-\sin^2 x)}.(a+b\sin x)}$$

$$= \frac{(a^2-b^2)\cos x}{\sqrt{(a^2-b^2).\cos^2 x}.(a+b\sin x)} = \frac{(a^2-b^2)\cos x}{\sqrt{a^2-b^2}\cos x.(a+b\sin x)} = \frac{\sqrt{a^2-b^2}}{a+b\sin x}$$

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23. Show that the square of the length of subtangent at any point on the curve $by^2=(x+a)^3$, $b\neq 0$ varies with the length of the subnormal at the point.

Sol: The given equation is $by^2=(x+a)^3$; differntiating w.r.to x

$$2by\frac{dy}{dx} = 3(x+a)^2 \Rightarrow \frac{dy}{dx} = \frac{3(x+a)^2}{2by}$$

$\therefore$ Length of the subnormal at any point P(x,y) is $PN = \left|y\frac{dy}{dx}\right| = \left|\frac{3(x+a)^2}{2by}y\right| = \frac{3(x+a)^2}{2b}$ ....(1)

Length of the subtangent is $PT = \left|\frac{y}{dy/dx}\right| = \left|y\frac{2by}{3(x+a)^2}\right| = \frac{2}{3}\frac{by^2}{(x+a)^2} = \frac{2}{3}\frac{(x+a)^3}{(x+a)^2} = \frac{2}{3}(x+a)$ ...(2)

From (1) &(2); $\frac{PT^2}{PN} = \frac{4}{9}(x+a)^2.\frac{2b}{3(x+a)^2} = \frac{8b}{27} = \text{a constant}$

$\therefore PT^2 \propto PN$

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24. Find the maximum area of the rectangle that can be formed with fixed preimeter 20.

Sol: Let x and y denote the length and the breadth of a rectangle respectively.

Given that the perimeter of the rectangle is 20.

$\Rightarrow 2(x+y)=20 \Rightarrow x+y=10$ ....(1)

Let A denote the area of rectangle. Then $A=xy$ ....(2) which is to be minimized.

From (1) , $y=10-x$ ....(3)

From (3) and (2), we have $A=x(10-x) \Rightarrow A=10x-x^2$ .....(4)

Differentiating (4) w.r.to x we get $\frac{dA}{dx} = 10-2x$ ....(5)

The stationary point is a root of $10-2x=0 \Rightarrow x=5$

$\therefore$ x=5 is the stationary point.

Differentiating (5) w.r.to x, we get $\frac{d^2A}{dx^2} = -2$ which is negative.

Therefore by second derivative test, the area A is maximized at x=5.

Here y=10–5=5 and the maximum area is A=5(5) = 25 sq.units