SOLVED PAPER-3

Time: 3 Hours **MATHS-1A** Max. Marks : 75

SECTION-A

I. Answer ALL the following Very Short Answer Questions: 10 × 2 = 20

1. Find the inverse of the real function of $f(x) = ax + b, a \neq 0 \ \ a, b \in R$
2. Find the domain of the real function $f(x)=\dfrac{1}{\sqrt{1-x^2}}$
3. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}3 & 8\\ 7 & 2\end{bmatrix}$ and $2X + A = B$ then find X.
4. Find the cofactors of 2 and –5 in the matrix $\begin{bmatrix}-1 & 0 & 5\\ 1 & 2 & -2\\ -4 & -5 & 3\end{bmatrix}$
5. Show that the points $A(2\bar{i}-\bar{j}+\bar{k}), B(\bar{i}-3\bar{j}-5\bar{k}), C(3\bar{i}-4\bar{j}-4\bar{k})$ are the vertices of a right angled triangle.
6. Find the angle between the vectors $\bar{i}+2\bar{j}+3\bar{k}$ and $3\bar{i}-\bar{j}+2\bar{k}$
7. Let $\bar{a}=\bar{i}+\bar{j}+\bar{k}$,and $\bar{b}=2\bar{i}+3\bar{j}+\bar{k}$ find projection vector of $\bar{b}$ on $\bar{a}$ and its magnitude.
8. Eliminate 'θ' from $x = a\cos^3\theta, y = b\sin^3\theta$.
9. Find a cosine function whose period is 7.

10. Prove that $\cosh^2 x - \sinh^2 x = 1$

SECTION-B

II. Answer any FIVE of the following Short Answer Questions: 5 × 4 = 20

11. Show that $\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}=(a-b)(b-c)(c-a)$
12. Find λ in order that the four points A(3, 2, 1), B(4, λ, 5), C(4, 2, –2) and D(6, 5, –1) be coplanar.
13. If $[\bar{b}\bar{c}\bar{d}]+[\bar{c}\bar{a}\bar{d}]+[\bar{a}\bar{b}\bar{d}]=[\bar{a}\bar{b}\bar{c}]$, then show that the points with p.v's $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ are coplanar.
14. Show that $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\dfrac{1+\sin\theta}{\cos\theta}$
15. Solve the equation $2\cos^2\theta + 11\sin\theta = 7$ and write general solution.
16. Find the value of $\tan\left(\operatorname{Sin}^{-1}\dfrac{3}{5}+\operatorname{Cos}^{-1}\dfrac{5}{\sqrt{34}}\right)$

17. If $C = 60^\circ$, then show that $\dfrac{a}{b+c}+\dfrac{b}{c+a}=1$

SECTION-C

III. Answer any FIVE of the following Long Answer Questions: 5 × 7 = 35

18. If $f : A\to B$ is a function and I_A, I_B are identity functions on A,B respectively then prove that $foI_A = f = I_B of$
19. By Mathematical Induction, show that $49^n + 16n - 1$ is divisible by 64 for all positive Integer n.
20. If A is a non-singular matrix then prove that $A^{-1}=\dfrac{1}{\det A}(\text{Adj } A)$
21. Solve the system of equations by Matrix inverse method, 2x–y+3z=8,–x+2y+z=4,3x+y–4z=0
22. For any four vectors $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$, prove that

    (i) $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=[\bar{a}\bar{c}\bar{d}]\bar{b}-[\bar{b}\bar{c}\bar{d}]\bar{a}$ and (ii) $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=[\bar{a}\bar{b}\bar{d}]\bar{c}-[\bar{a}\bar{b}\bar{c}]\bar{d}$
23. If A, B, C are angles in a triangle, then prove that $\cos A+\cos B-\cos C=-1+4\cos\dfrac{A}{2}.\cos\dfrac{B}{2}.\sin\dfrac{C}{2}$
24. If $a=(b-c)\sec\theta$, prove that $\tan\theta=\dfrac{2\sqrt{bc}}{b-c}\sin\dfrac{A}{2}$

SOLUTIONS

SECTION-A

1. Find the inverse of the real function of $f(x) = ax + b,\ a \neq 0\ \ a,b \in R$

Sol: Let $f(x) = y = ax + b \Rightarrow ax = y - b$

$$\Rightarrow x = \frac{y-b}{a} \Rightarrow f^{-1}(y) = \frac{y-b}{a}$$

$$\therefore f^{-1}(x) = \frac{x-b}{a} \qquad \text{Since } a \neq 0$$

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2. Find the domain of the real function $f(x)=\frac{1}{\sqrt{1-x^2}}$

Sol: Given f(x) is defined when $1 - x^2 > 0$

$\Rightarrow -(x^2 - 1) > 0 \Rightarrow x^2 - 1 < 0 \Rightarrow (x+1)(x-1) < 0 \Rightarrow x \in (-1,1)$ $\therefore$ Domain is $(-1, 1)$

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3. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix}$ and 2X+A=B then find X.

Sol: Given that $2X + A = B \Rightarrow 2X = B - A$

$$= \begin{bmatrix} 3 & 8 \\ 7 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3-1 & 8-2 \\ 7-3 & 2-4 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix}$$

$$\therefore X = \frac{1}{2}\begin{bmatrix} 2 & 6 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$$

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4. Find the cofactors of 2 and –5 in the matrix $\begin{bmatrix} -1 & 0 & 5 \\ 1 & 2 & -2 \\ -4 & -5 & 3 \end{bmatrix}$

Sol: Cofactor of 2 is $(-1)^{2+2}\begin{vmatrix} -1 & 5 \\ -4 & 3 \end{vmatrix} = \begin{vmatrix} -1 & 5 \\ -4 & 3 \end{vmatrix} = -3 + 20 = 17$

Cofactor of -5 is $(-1)^{3+2}\begin{vmatrix} -1 & 5 \\ 1 & -2 \end{vmatrix} = -\begin{vmatrix} -1 & 5 \\ 1 & -2 \end{vmatrix} = -(2-5) = 3$

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5. Show that the points $A(2\bar{i} - \bar{j} + \bar{k}), B(\bar{i} - 3\bar{j} - 5\bar{k}), C(3\bar{i} - 4\bar{j} - 4\bar{k})$ are the vertices of a right angled triangle.

Sol: $\overline{AB} = \overline{OB} - \overline{OA} = (1-2)\bar{i} + (-3+1)\bar{j} + (-5-1)\bar{k} = -\bar{i} - 2\bar{j} - 6\bar{k} \Rightarrow |\overline{AB}| = \sqrt{1+4+36} = \sqrt{41}$

$\overline{BC} = \overline{OC} - \overline{OB} = (3-1)\bar{i} + (-4+3)\bar{j} + (-4+5)\bar{k} = 2\bar{i} - \bar{j} + \bar{k} \Rightarrow |\overline{BC}| = \sqrt{4+1+1} = \sqrt{6}$

$\overline{CA} = \overline{OA} - \overline{OC} = (2-3)\bar{i} + (-1+4)\bar{j} + (1+4)\bar{k} = -\bar{i} + 3\bar{j} + 5\bar{k} = \sqrt{1+9+25} = \sqrt{35}$

Here, $|\overline{AB}|^2 = 41; |\overline{BC}|^2 + |\overline{CA}|^2 = 6 + 35 = 41 \Rightarrow |\overline{AB}|^2 = |\overline{BC}|^2 + |\overline{CA}|^2$

$\therefore$ A, B, C Form a Right angled triangle.

6. Find the angle between the vectors $\bar{i}+2\bar{j}+3\bar{k}$ and $3\bar{i}-\bar{j}+2\bar{k}$

Sol: We take $\bar{a}=\bar{i}+2\bar{j}+3\bar{k}$, $\bar{b}=3\bar{i}-\bar{j}+2\bar{k}$

$$\therefore \cos\theta=\frac{\bar{a}.\bar{b}}{|\bar{a}||\bar{b}|}=\frac{(\bar{i}+2\bar{j}+3\bar{k}).(3\bar{i}-\bar{j}+2\bar{k})}{\sqrt{1^2+2^2+3^2}.\sqrt{3^2+(-1)^2+2^2}}=\frac{1(3)+2(-1)+3(2)}{\sqrt{14}\sqrt{14}}$$

$$=\frac{3-2+6}{14}=\frac{7}{14}=\frac{1}{2}=\cos 60^{\circ}$$

$\therefore$ Angle $\theta=60^{\circ}$

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7. Let $\bar{a}=\bar{i}+\bar{j}+\bar{k}$,and $\bar{b}=2\bar{i}+3\bar{j}+\bar{k}$ find projection vector of $\bar{b}$ on $\bar{a}$ and its magnitude.

Sol: Let $\bar{a}=\bar{i}+\bar{j}+\bar{k}$, $\bar{b}=2\bar{i}+3\bar{j}+\bar{k} \Rightarrow \bar{a}.\bar{b}=1(2)+1(3)+1(1)=2+3+1=6$

Also, $|\bar{a}|=\sqrt{1^2+1^2+1^2}=\sqrt{3}$

$\therefore$ projection vector of $\bar{b}$ on $\bar{a}$ is $\left(\frac{\bar{b}.\bar{a}}{|\bar{a}|^2}\right)\bar{a}=\left(\frac{6}{3}\right)(\bar{i}+\bar{j}+\bar{k})=2(\bar{i}+\bar{j}+\bar{k})$

Also, Magnitude of the projection vector $=\frac{|\bar{b}.\bar{a}|}{|\bar{a}|}=\frac{6}{\sqrt{3}}=\frac{2\times 3}{\sqrt{3}}=2\sqrt{3}$

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8. Eliminate 'θ' from $x=a\cos^3\theta$, $y=b\sin^3\theta$.

Sol: Given that $x=a\cos^3\theta$, $y=b\sin^3\theta$.

$$\Rightarrow \cos^3\theta=\frac{x}{a},\sin^3\theta=\frac{y}{b}\Rightarrow \cos\theta=\left(\frac{x}{a}\right)^{1/3},\sin\theta=\left(\frac{y}{b}\right)^{1/3}$$

$$\text{Now, } \cos^2\theta+\sin^2\theta=1\Rightarrow\left(\left(\frac{x}{a}\right)^{1/3}\right)^2+\left(\left(\frac{y}{b}\right)^{1/3}\right)^2=1\Rightarrow\left(\frac{x}{a}\right)^{2/3}+\left(\frac{y}{b}\right)^{2/3}=1$$

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9. Find a cosine function whose period is 7.

Sol: We take coskx as the required cosine function

$\therefore$ Period $\frac{2\pi}{k}=7 \Rightarrow k=\frac{2\pi}{7}$ $\left(\because \text{Period of } \cos kx=\frac{2\pi}{k}\right)$

$\therefore \cos\left(\frac{2\pi}{7}\right)x$ is the required cosine function

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10. Prove that $\cosh^2x-\sinh^2x=1$

Sol: L.H.S= $\cosh^2x-\sinh^2x=\left(\frac{e^x+e^{-x}}{2}\right)^2-\left(\frac{e^x-e^{-x}}{2}\right)^2$ $\left[\because \cosh x=\left(\frac{e^x+e^{-x}}{2}\right); \sinh x=\left(\frac{e^x-e^{-x}}{2}\right)\right]$

$$=\frac{\left[(e^x+e^{-x})^2-(e^x-e^{-x})^2\right]}{4}=\frac{(4.e^x.e^{-x})}{4} \quad [\because (a+b)^2-(a-b)^2=4ab]$$

$$=\frac{4}{4}=1=\text{R.H.S}$$

SECTION-B

11. Show that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Sol: $L.H.S = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix} (\because R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1)$

$= 1.\begin{vmatrix} b-a & b^2-a^2 \\ c-a & c^2-a^2 \end{vmatrix} = \begin{vmatrix} b-a & (b-a)(b+a) \\ c-a & (c-a)(c+a) \end{vmatrix} = (b-a)(c-a)\begin{vmatrix} 1 & b+a \\ 1 & c+a \end{vmatrix}$

$= (b-a)(c-a)((c+a)-(b+a)) = (a-b)(b-c)(c-a) = R.H.S$

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12. Find λ in order that the four points A(3,2,1), B(4,λ,5), C(4,2,–2) and D(6,5,–1) be coplanar.

Sol: If O is the origin of reference then the position vectors of the given points are

$\overline{OA} = 3\overline{i} + 2\overline{j} + \overline{k}$, $\overline{OB} = 4\overline{i} + \lambda\overline{j} + 5\overline{k}$, $\overline{OC} = 4\overline{i} + 2\overline{j} - 2\overline{k}$, $\overline{OD} = 6\overline{i} + 5\overline{j} - \overline{k}$

$\therefore \overline{AB} = \overline{OB} - \overline{OA} = (4\overline{i} + \lambda\overline{j} + 5\overline{k}) - (3\overline{i} + 2\overline{j} + \overline{k}) = \overline{i} + (\lambda - 2)\overline{j} + 4\overline{k}$

$\overline{AC} = \overline{OC} - \overline{OA} = (4\overline{i} + 2\overline{j} - 2\overline{k}) - (3\overline{i} + 2\overline{j} + \overline{k}) = \overline{i} - 3\overline{k}$

$\overline{AD} = \overline{OD} - \overline{OA} = (6\overline{i} + 5\overline{j} - \overline{k}) - (3\overline{i} + 2\overline{j} + \overline{k}) = 3\overline{i} + 3\overline{j} - 2\overline{k}$

A, B, C, D are coplanar $\Leftrightarrow [\overline{AB}\ \ \overline{AC}\ \ \overline{AD}] = 0$

$\Rightarrow \begin{vmatrix} 1 & \lambda-2 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0 \Rightarrow 1(0+9) - (\lambda-2)(-2+9) + 4(3-0) = 0 \Rightarrow 9 - 7\lambda + 14 + 12 = 0 \Rightarrow 7\lambda = 35 \Rightarrow \lambda = 5$

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13. If $[\overline{b}\overline{c}\overline{d}] + [\overline{c}\overline{a}\overline{d}] + [\overline{a}\overline{b}\overline{d}] = [\overline{a}\overline{b}\overline{c}]$, then show that the points with position vectors $\overline{a}, \overline{b}, \overline{c}, \overline{d}$ are coplanar.

Sol: Let $A(\overline{a}), B(\overline{b}), C(\overline{c}), D(\overline{d})$ represent the P.V's of the given points w.r.to O

$A(\overline{a}), B(\overline{b}), C(\overline{c}), D(\overline{d})$ are coplanar $\Rightarrow \overline{AB}, \overline{AC}, \overline{AD}$ are coplanar

$\Leftrightarrow [\overline{AB}, \overline{AC}, \overline{AD}] = 0 \Leftrightarrow [\overline{OB} - \overline{OA}\ \ \overline{OC} - \overline{OA}\ \ \overline{OD} - \overline{OA}] = 0$

$\Leftrightarrow [\overline{b} - \overline{a}, \overline{c} - \overline{a}, \overline{d} - \overline{a}] = 0 \Leftrightarrow [\overline{b} - \overline{a}].[\overline{c} - \overline{a}] \times [\overline{d} - \overline{a}] = 0\ \ [\because [\overline{a}\overline{b}\overline{c}] = \overline{a}.(\overline{b} \times \overline{c})]$

$\Leftrightarrow [\overline{b} - \overline{a}].[\overline{c} \times [\overline{d} - \overline{a}] - \overline{a} \times [\overline{d} - \overline{a}] = 0 \Leftrightarrow [\overline{b} - \overline{a}].[\overline{c} \times \overline{d} - \overline{c} \times \overline{a} - \overline{a} \times \overline{d} + \overline{a} \times \overline{a}] = 0$

$\Leftrightarrow [\overline{b} - \overline{a}].[\overline{c} \times \overline{d} - \overline{c} \times \overline{a} - \overline{a} \times \overline{d} + \overline{0}] = 0\ \ [\because \overline{a} \times \overline{a} = \overline{0}]$

$$\Leftrightarrow \bar{b}.\bar{c}\times\bar{d}-\bar{b}.[\bar{c}\times\bar{a}]-\bar{b}.[\bar{a}\times\bar{d}]-\bar{a}.[\bar{c}\times\bar{d}]+\bar{a}[\bar{c}\times\bar{a}]+\bar{a}.[\bar{a}\times\bar{d}]=0$$

$$\Leftrightarrow [\bar{b}\ \bar{c}\ \bar{d}]-[\bar{b}\ \bar{c}\ \bar{a}]-[\bar{b}\ \bar{c}\ \bar{d}]-[\bar{a}\ \bar{c}\ \bar{d}]+[\bar{a}\ \bar{c}\ \bar{a}]+[\bar{a}\ \bar{a}\ \bar{d}]=0,\ [[\bar{a}\ \bar{c}\ \bar{a}]=0,[\bar{a}\ \bar{a}\ \bar{d}]=0]$$

$$\Leftrightarrow [\bar{b}\ \bar{c}\ \bar{d}]-[\bar{a}\ \bar{b}\ \bar{c}]+[\bar{a}\ \bar{b}\ \bar{d}]+[\bar{c}\ \bar{a}\ \bar{d}]=0$$

$$[\because [\bar{b}\ \bar{c}\ \bar{a}]=[\bar{a}\ \bar{b}\ \bar{c}],[\bar{b}\ \bar{a}\ \bar{d}]-[\bar{a}\ \bar{b}\ \bar{d}],\ [\bar{a}\ \bar{c}\ \bar{d}]=-[\bar{c}\ \bar{a}\ \bar{d}]]$$

$$\Leftrightarrow [\bar{b}\ \bar{c}\ \bar{d}]+[\bar{c}\ \bar{a}\ \bar{d}]+[\bar{a}\ \bar{b}\ \bar{d}]=[\bar{a}\ \bar{b}\ \bar{c}]$$ Hence the given result is proved.

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14. Show that $\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\dfrac{1+\sin\theta}{\cos\theta}$

Sol: L.H.S $=\dfrac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\dfrac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}\ (\because 1=\sec^2\theta-\cos^2\theta)$

$$=\frac{(\tan\theta+\sec\theta)-(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)(1-(\sec\theta-\tan\theta))}{\tan\theta-\sec\theta+1}=\frac{(\tan\theta+\sec\theta)\,\cancel{(1-\sec\theta+\tan\theta)}}{\cancel{1-\sec\theta+\tan\theta}}=\tan\theta+\sec\theta$$

$$=\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}=\frac{\sin\theta+1}{\cos\theta}=\text{R.H.S}$$

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15. Solve the equation $2\cos^2\theta + 11\sin\theta = 7$ and write general solution.

Sol: Given equation $2\cos^2\theta+11\sin\theta=7$

$$\Rightarrow 2(1-\sin^2\theta)+11\sin\theta=7 \Rightarrow 2-2\sin^2\theta+11\sin\theta=7$$

$$\Rightarrow 2\sin^2\theta-11\sin\theta+5=0 \Rightarrow 2\sin^2\theta-10\sin\theta-\sin\theta+5=0$$

$$\Rightarrow 2\sin\theta(\sin\theta-5)-(\sin\theta-5)=0 \Rightarrow (2\sin\theta-1)(\sin\theta-5)=0$$

$$\Rightarrow 2\sin\theta-1=0,\ \sin\theta-5=0 \Rightarrow 2\sin\theta=1,\ \sin\theta=5\ \text{(no solution)}$$

$$\Rightarrow \sin\theta=\frac{1}{2}=\sin\frac{\pi}{6},\ \text{here P.V is}\ \alpha=\frac{\pi}{6}$$

$\therefore$ General solution is given by $\theta=n\pi+(-1)^n\alpha,\ n\in Z \Rightarrow \theta=n\pi+(-1)^n\dfrac{\pi}{6},\ n\in Z$

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16. Find the value of $\tan\left(\text{Sin}^{-1}\dfrac{3}{5}+\text{Cos}^{-1}\dfrac{5}{\sqrt{34}}\right)$

Sol: Let $\text{Sin}^{-1}\left(\dfrac{3}{5}\right)=\alpha \Rightarrow \sin\alpha=\dfrac{3}{5}\Rightarrow \tan\alpha=\dfrac{3}{4}$; $\text{Cos}^{-1}\dfrac{5}{\sqrt{34}}=\beta\Rightarrow \cos\beta=\dfrac{5}{\sqrt{34}}\Rightarrow \tan\beta=\dfrac{3}{5}$

Now, $\tan\left(\text{Sin}^{-1}\dfrac{3}{5}+\text{Cos}^{-1}\dfrac{5}{\sqrt{34}}\right)=\tan(\alpha+\beta)=\dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\dfrac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}.\frac{3}{5}}=\dfrac{15+12}{20-9}=\dfrac{27}{11}$

17. If C=60°, then show that $\frac{a}{b+c}+\frac{b}{c+a}=1$

Sol: Given that C=60°, then from the cosine rule we know that $c^2=a^2+b^2-2ab\cos C$

$\Rightarrow c^2=a^2+b^2-2ab\cos 60^\circ \Rightarrow c^2=a^2+b^2-2ab.\frac{1}{2} \Rightarrow c^2=a^2+b^2-ab$ ....(1)

Now L.H.S $=\frac{a}{b+c}+\frac{b}{c+a}=\frac{a(c+a)+b(b+c)}{(b+c)(c+a)}=\frac{ac+a^2+b^2+bc}{bc+ba+c^2+ca}$

$=\frac{ac+a^2+b^2+bc}{bc+ba+(a^2+b^2-ab)+ca}=\frac{ac+a^2+b^2+bc}{bc+a^2+b^2+ca}=1=\text{R.H.S}$

SECTION-C

18. If f:A→B is a function and I_A, I_B are identity functions on A,B respectively then prove that $foI_A=f=I_Bof$

Sol: (i) To prove that $foI_A=f$

Part–1: Given f:A→B is a function.

We know I_A:A→A

∴ foI_A:A→B

So, foI_A and f, both have same domain A

Part–2: For a∈A, $(foI_A)(a)=f[I_A(a)]$

$=f(a)[\because I_A(a)=a$ for all a∈A]

Hence, we proved that $foI_A=f$

(ii) To prove that $I_Bof=f$

Part–1: Given f:A→B is a function.

We know I_B:B→B

∴ I_Bof:A→B

So, I_Bof and f, both have same domain A

Part–2: For a∈A, $(I_Bof)(a)=I_B[f(a)]$

$=f(a)$ [∵ $I_B(b)=b$ for all b∈B]

Hence, we proved that $I_Bof=f$

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19. By Mathematical Induction, show that $49^n+16n-1$ is divisible by 64 for all positive Integer n.

Sol: Given S(n) : $49^n+16n-1=64q$, q∈Z

Step 1: L.H.S of S(1)=$49^{(1)}$+16 (1)–1 = 49+16 –1 = 64 = 64(1)

So, S(1) is true

Step 2: Assume that S(k) is true for k∈N

$S(k): 49^k + 16k - 1 = 64q$ ....(1)

Step 3: We show that S(k+1) is true

Writing $(k+1)^{th}$ term from (1), we get

L.H.S $= 49^{k+1} + 16(k+1) - 1 = 49^k.49 + 16k + 16 - 1$

$= (64q - 16k + 1).49 + 16k + 15$ (since from (1))

$= 64q.49 - 16k.49 + 1.49 + 16k + 15 = 64q.49 - 16k.(49-1) + (49+15)$

$= 64q.49 - 16k.(48) + 64 \quad = 64q.49 - 16k.(4.12) + 64$

$= 64q.49 - 64k(12) + 64 = 64(49q - 12k + 1) = 64$(an integer)

So, S(k+1) is true whenever S(k) is true

Hence, by P.M.I the given statement is true, for all $n \in N$

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20. If A is a non-singular matrix then prove that $A^{-1} = \frac{1}{\det A}(\text{Adj } A)$

Sol: We take $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

We take cofactors of a_1, b_1, c_1, ...as A_1, B_1, C_1 ......

$$\therefore \text{Adj } A = \begin{bmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{bmatrix}^T \Rightarrow \text{Adj}A = \begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix}$$

$$A.(\text{Adj}A) = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}\begin{bmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{bmatrix}$$

$$= \begin{bmatrix} a_1A_1 + b_1B_1 + c_1C_1 & a_1A_2 + b_1B_2 + c_1C_2 & a_1A_3 + b_1B_3 + c_1C_3 \\ a_2A_1 + b_2B_1 + c_2C_1 & a_2A_2 + b_2B_2 + c_2C_2 & a_2A_3 + b_2B_3 + c_2C_3 \\ a_3A_1 + b_3B_1 + c_3C_1 & a_3A_2 + b_3B_2 + c_3C_2 & a_3A_3 + b_3B_3 + c_3C_3 \end{bmatrix}$$

$$= \begin{bmatrix} \det A & 0 & 0 \\ 0 & \det A & 0 \\ 0 & 0 & \det A \end{bmatrix} \text{(From properties of determinants)}$$

$$= (\det A)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (\det A).I$$

$\therefore A(\text{Adj } A) = (\det A)\, I$; Similarly, we can prove that $(\text{Adj } A)A = (\det A)\, I$

$\therefore A\left(\frac{\text{Adj } A}{\det A}\right) = I \quad (\because \det A \neq 0, \text{as } A \text{ is non singular})$

$\therefore A^{-1} = \frac{\text{Adj } A}{\det A} \quad [\because AB = I \Rightarrow A^{-1} = B]$

21. Solve the system of equations by Matrix inverse method, 2x–y+3z=8, –x+2y+z=4, 3x+y–4z=0

Sol: Given equations in the matrix equation form: AX = D, where

$$A=\begin{bmatrix}2 & -1 & 3\\ -1 & 2 & 1\\ 3 & 1 & -4\end{bmatrix}, D=\begin{bmatrix}8\\4\\0\end{bmatrix}, X=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

In the Matrix Inverstion Method, the solution is $X=A^{-1}D$

First we find A^{-1}

$$|A|=\begin{vmatrix}2 & -1 & 3\\ -1 & 2 & 1\\ 3 & 1 & -4\end{vmatrix}=2(-8-1)+1(4-3)+3(-1-6)=2(-9)+1(1)+3(-7)=-18+1-21=-38$$

$$\text{Adj}A=\begin{bmatrix}+\begin{vmatrix}2 & 1\\ 1 & -4\end{vmatrix} & -\begin{vmatrix}-1 & 1\\ 3 & -4\end{vmatrix} & +\begin{vmatrix}-1 & 2\\ 3 & 1\end{vmatrix}\\ -\begin{vmatrix}-1 & 3\\ 1 & -4\end{vmatrix} & +\begin{vmatrix}2 & 3\\ 3 & -4\end{vmatrix} & -\begin{vmatrix}2 & -1\\ 3 & 1\end{vmatrix}\\ +\begin{vmatrix}-1 & 3\\ 2 & 1\end{vmatrix} & -\begin{vmatrix}2 & 3\\ -1 & 1\end{vmatrix} & +\begin{vmatrix}2 & -1\\ -1 & 2\end{vmatrix}\end{bmatrix}^T=\begin{bmatrix}(-8-1) & -(4-3) & (-1-6)\\ -(4-3) & (-8-9) & -(2+3)\\ (-1-6) & -(2+3) & (4-1)\end{bmatrix}^T=\begin{bmatrix}-9 & -1 & -7\\ -1 & -17 & -5\\ -7 & -5 & 3\end{bmatrix}^T$$

$$\Rightarrow \text{Adj }A=\begin{bmatrix}-9 & -1 & -7\\ -1 & -17 & -5\\ -7 & -5 & 3\end{bmatrix}$$

$$\text{Now, } A^{-1}=\frac{1}{|A|}(\text{adj}A)=\frac{1}{-38}\begin{bmatrix}-9 & -1 & -7\\ -1 & -17 & -5\\ -7 & -5 & 3\end{bmatrix}$$

$$\therefore X=A^{-1}.D=\frac{-1}{38}\begin{bmatrix}-9 & -1 & -7\\ -1 & -17 & -5\\ -7 & -5 & 3\end{bmatrix}\begin{bmatrix}8\\4\\0\end{bmatrix}=\frac{-1}{38}\begin{bmatrix}-72-4-0\\ -8-68-0\\ -56-20-0\end{bmatrix}=\frac{-1}{38}\begin{bmatrix}-76\\-76\\-76\end{bmatrix}=\begin{bmatrix}2\\2\\2\end{bmatrix}$$

$$\therefore X=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2\\2\\2\end{bmatrix}$$

Hence the solution is x=2, y=2, z=2

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22. For any four vectors $\bar{a},\bar{b},\bar{c}$ and $\bar{d}$, prove that

(i) $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=[\bar{a}\bar{c}\bar{d}]\bar{b}-[\bar{b}\bar{c}\bar{d}]\bar{a}$ and (ii) $(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=[\bar{a}\bar{b}\bar{d}]\bar{c}-[\bar{a}\bar{b}\bar{c}]\bar{d}$

Sol: Take $\bar{c}\times\bar{b}=\bar{p}$

$$(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=(\bar{a}\times\bar{b})\times\bar{p}=(\bar{a}.\bar{p})\bar{b}-(\bar{b}.\bar{p})\bar{a}=(\bar{a}.(\bar{c}\times\bar{d}))\bar{b}-(\bar{b}.(\bar{c}\times\bar{d}))\bar{a}$$

$$=\left[\bar{a}\ \bar{c}\ \bar{d}\right]\bar{b}-\left[\bar{b}\ \bar{c}\ \bar{d}\right]\bar{a}$$

Take $\bar{a}\times\bar{b}=\bar{q}$

$$(\bar{a}\times\bar{b})\times(\bar{c}\times\bar{d})=\bar{q}\times(\bar{c}\times\bar{d})=(\bar{q}.\bar{d})\bar{c}-(\bar{q}.\bar{c})\bar{d}=((\bar{a}\times\bar{b}).\bar{d}.)\bar{c}-(\bar{a}\times\bar{b}).\bar{c}.)\bar{d}$$

$$=\left[\bar{a}\ \bar{b}\ \bar{d}\right]\bar{c}-\left[\bar{a}\bar{b}\ \bar{c}\right]\bar{d}$$

23. If A, B, C are angles in a triangle, then P.T $\cos A + \cos B - \cos C = -1 + 4\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2}$

Sol: Given A,B,C are angles of a triangle, then A+B+C=180º $\Rightarrow \frac{A+B+C}{2} = 90^{o}$

L.H.S $= (\cos A + \cos B) - \cos C = 2\cos\frac{A+B}{2}.\cos\frac{A-B}{2} - \cos C$

$= 2\cos\left(90^{o} - \frac{C}{2}\right).\cos\frac{A-B}{2} - \cos C$

$= 2\sin\frac{C}{2}.\cos\frac{A-B}{2} - \left(1 - 2\sin^2\frac{C}{2}\right)$ $\left[\because \cos\theta = 1 - 2\sin^2\frac{\theta}{2}\right]$

$= -1 + 2\sin\frac{C}{2}.\cos\frac{A-B}{2} + 2\sin^2\frac{C}{2} = -1 + 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} + \sin\frac{C}{2}\right)$

$= -1 + 2\sin\frac{C}{2}\left[\cos\frac{A-B}{2} + \sin\left(90^{o} - \frac{A+B}{2}\right)\right]$

$= -1 + 2\sin\frac{C}{2}\left(\cos\frac{A-B}{2} + \cos\frac{A+B}{2}\right)$ $[\because \sin(90^{o} - \theta) = \cos\theta]$

$= -1 + 2\sin\frac{C}{2}\left(2\cos\frac{A}{2}.\cos\frac{B}{2}\right)$ $[\because \cos(A+B) + \cos(A-B) = 2\cos A\cos B]$

$= -1 + 4\cos\frac{A}{2}.\cos\frac{B}{2}.\sin\frac{C}{2} = \text{R.H.S}$

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24. If a=(b–c)secθ, prove that $\tan\theta = \frac{2\sqrt{bc}}{b-c}\sin\frac{A}{2}$

Sol: Given a = (b–c)secθ, then $\sec\theta = \frac{a}{b-c} \Rightarrow \sec^2\theta = \frac{a^2}{(b-c)^2}$

$\therefore \tan^2\theta = \sec^2\theta - 1$ $[\because \sec^2\theta - \tan^2\theta = 1]$

$= \frac{a^2}{(b-c)^2} - 1 = \frac{a^2 - (b-c)^2}{(b-c)^2}$

$= \frac{a^2 - (b^2 + c^2 - 2bc)}{(b-c)^2} = \frac{a^2 - b^2 - c^2 + 2bc}{(b-c)^2}$

$= \frac{2bc - (b^2 + c^2 - a^2)}{(b-c)^2} = \frac{2bc - (2bc\cos A)}{(b-c)^2}$ $\left(\because \frac{b^2 + c^2 - a^2}{2bc} = \cos A\right)$

$= \frac{2bc(1 - \cos A)}{(b-c)^2} = \frac{2bc.\left(2\sin^2\frac{A}{2}\right)}{(b-c)^2} = \frac{4bc.\sin^2\frac{A}{2}}{(b-c)^2}$

$\therefore \tan\theta = \frac{2\sqrt{bc}}{b-c}\sin\left(\frac{A}{2}\right)$